Global Scheduling in Multiprocessor Real-Time Systems

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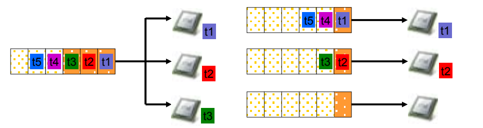
Global vs Partitioned scheduling

- Single shared queue instead of multiple dedicated queues

Global scheduling — Partitioned scheduling



Bin-packing problem + Uniprocessor scheduling problem

NP-hard in the strong sense; various heuristics adopted

Well-known

Pros and cons

- **Global** scheduling
  - ✓ Automatic load balancing
  - ✓ Lower avg. response time
  - ✓ Simpler implementation
  - ✓ *Optimal* schedulers exist
  - ✓ More efficient reclaiming
  - ✗ Migration costs
  - ✗ Inter-core synchronization
  - ✗ Loss of cache affinity
  - ✗ Weak scheduling framework
- **Partitioned** scheduling
  - ✓ Supported by automotive industry (e.g., AUTOSAR)
  - ✓ No migrations
  - ✓ Isolation between cores
  - ✓ Mature scheduling framework
  - ✗ Cannot exploit unused capacity
  - ✗ Rescheduling not convenient
  - ✗ NP-hard allocation

Main (negative) results

- **Weak theoretical framework**
  - Unknown critical instant
  - G-EDF is not optimal
  - Any G-JLFP scheduler is not optimal
  - Optimality only for implicit deadlines
  - Many sufficient tests (most of them incomparable)

Unknown critical instant

- **Critical instant**
  - Job release time such that **response-time is maximized**
- **Uniprocessor**
  - Liu & Layland: synchronous release sequence yields worst-case response-times
    - Synchronous: all tasks release a job at time 0
    - Assuming constrained deadlines and no deadline misses
- **Multiprocessors**
  - **No general critical instant is known!**
  - It is not necessarily the synchronous release sequence…

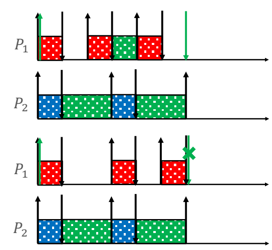
Unknown critical instant

- Synchronous periodic arrival of jobs is not a critical instant for multiprocessors

C_i, D_i, T_i
$\tau_1 = (1, 1, 2)$
$\tau_2 = (1, 1, 3)$
$\tau_3 = (5, 6, 6)$

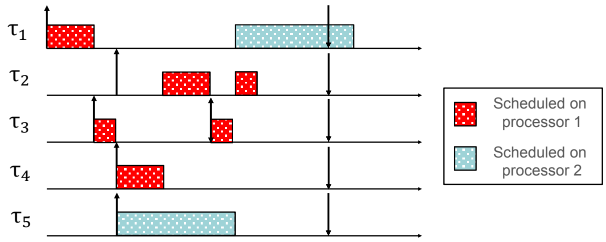
Synchronous periodic situation

The second job of τ_1 is delayed by one unit



We need to find pessimistic situations to derive **sufficient** schedulability tests

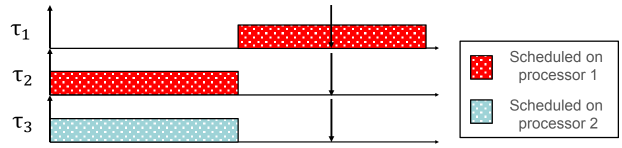
G-EDF is not optimal



- **Uniprocessors**
  - EDF is optimal
- **Multiprocessors**
  - G-EDF is not optimal
  - Key problem: **sequentiality** of tasks
  - Two processors available for τ_1, but it can only use one

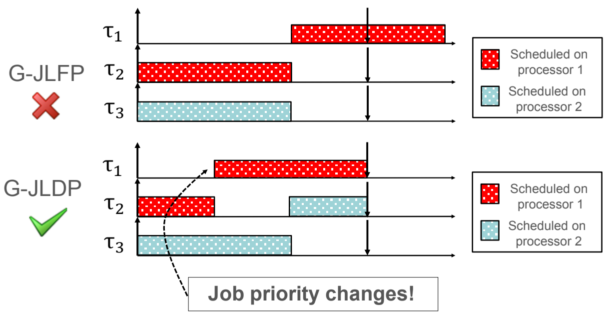
Any G-JLFP scheduler is not optimal

Two processors, three tasks, $T_i = 15,\ C_i = 10$



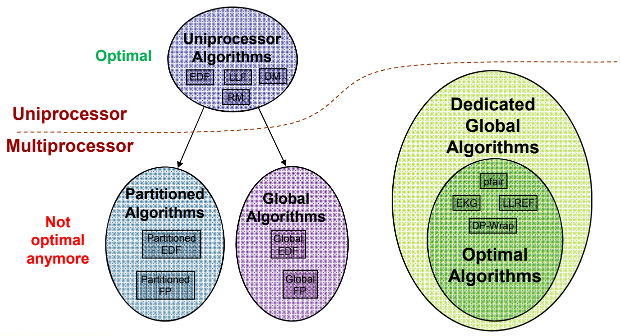
- **Any job-level fixed-priority scheduler is not optimal**
  - Synchronous release time
  - One of the three jobs is scheduled last under any JLFP policy
  - Deadline miss unavoidable!

G-JLDP required for optimality



- **G-JLDP:** *Global Job Level Dynamic Priority*; the priority of each job may change over time

Taxonomy of multiprocessor scheduling algorithms



Proportionate fairness

- P-fair: notion of "fair share of processor"
- If a schedule is P-fair, no **implicit** deadline will be missed → **optimal** algorithm

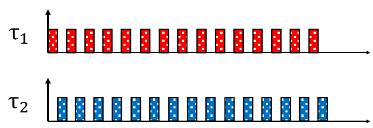
Basic principle:

- Timeline is divided into **equal length slots**
- Task period and execution time are multiples of the slot size
- Each task receives amount of slots **proportional to its task utilization**
  - If a task has utilization $U = \frac{C_i}{T_i}$, then it will have been allocated $U \cdot t$ time slots for execution in the interval $[0, t]$

Proportionate fairness

Example:

- $C_1 = C_2 = 3;\ T_1 = T_2 = 6\ (U_1 = U_2 = \frac{1}{2})$



- Quantum-based: $C_i \in \mathbb{Z}^+, T_i \in \mathbb{Z}^+$; scheduling decisions can only occur at integers
- A task executes during a whole time slot or not execute at all in that time slot

Proportionate fairness

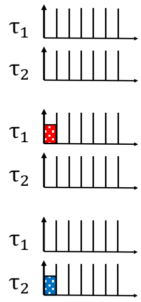
$$lag(\tau_i, t) = t \cdot \left(\frac{C_i}{T_i}\right) - allocated(\tau_i, t)$$

- Error: $lag(\tau_i, t)$
- "Fluid" execution: should have executed in $[0, t)$: $t \cdot \left(\frac{C_i}{T_i}\right)$
- Real execution in $[0, t)$: $allocated(\tau_i, t)$

- **Goal:** find an algorithm that minimizes $\max_t |lag(\tau_i, t)|$
- **Which are the values that $lag(\tau_i)$ can take?**

Proportionate fairness

- Example: $\tau = \{(T_1 = 5,\ C_1 = 2),\ (T_2 = 7,\ C_2 = 4)\}$, 1 processor



No task executes in $[0,1)$
$lag(\tau_1, 1) = 1 \cdot \left(\frac{2}{5}\right) - 0 \neq 0$
$lag(\tau_2, 1) = 1 \cdot \left(\frac{4}{7}\right) - 0 \neq 0$

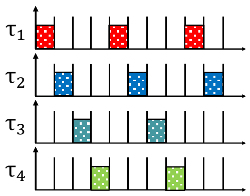
Task τ_1 executes in $[0,1)$
$lag(\tau_1, 1) = 1 \cdot \left(\frac{2}{5}\right) - 1 \neq 0$
$lag(\tau_2, 1) = 1 \cdot \left(\frac{4}{7}\right) - 0 \neq 0$

Task τ_2 executes in $[0,1)$
$lag(\tau_1, 1) = 1 \cdot \left(\frac{2}{5}\right) - 0 \neq 0$
$lag(\tau_2, 1) = 1 \cdot \left(\frac{4}{7}\right) - 1 \neq 0$

$lag(\tau_i, 1) = 0$ is impossible

Proportionate fairness

- Example: $\tau = \{(T_1 = 4,\ C_1 = 1),\ (T_2 = 4,\ C_2 = 1), (T_3 = 4,\ C_3 = 1), (T_2 = 4,\ C_2 = 1)\}$, one processor



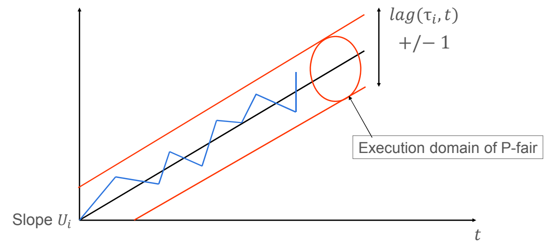
$lag(\tau_1, 1) = 1 \cdot \left(\frac{1}{4}\right) - 1 = -\frac{3}{4}$

$lag(\tau_4, 3) = 3 \cdot \left(\frac{1}{4}\right) - 0 = \frac{3}{4}$

$-1 < lag(\tau_i, t) < 1$ seems to be the worst-case lag

Proportionate fairness

- Definition (P-fair schedule):
  a schedule is P-fair if and only if $\forall\ \tau_i$ and $\forall\ t$: $-1 < lag(\tau_i, t) < 1$



Proportionate fairness

- Theorem
  A P-fair schedule is optimal in the sense of feasibility for a set of periodic tasks with implicit deadlines
- Proof
  A schedule S is P-fair
  $\Rightarrow -1 < lag(\tau_i, t) < 1$
  $\Rightarrow -1 < lag(\tau_i, kT_i) < 1$
  $\Rightarrow -1 < kT_i \frac{C_i}{T_i} - allocated(\tau_i, kT_i) < 1$
  $\Rightarrow -1 < kC_i - allocated(\tau_i, kT_i) < 1$
  $\Rightarrow kC_i - allocated(\tau_i, kT_i) = 0$
  $\Rightarrow kC_i = allocated(\tau_i, kT_i)$
  $\Rightarrow allocated(\tau_i, (k+1)T_i) - allocated(\tau_i, kT_i) = C_i$
  $\Rightarrow \tau_i$ executes C_i time-units during $[kT_i,\ (k+1)T_i]$
  $\Rightarrow \tau_i$ meets every deadline in periodic scheduling

The algorithm PF

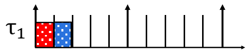
- **How to generate a P-fair schedule?**
  - Execute all *urgent* tasks
    - A task τ_i is urgent at time t if
      $lag(\tau_i, t) > 0$ and $lag(\tau_i, t+1) \geq 0$ if τ_i executes
  - Do not execute *tnegru* tasks
    - A task τ_i is tnegru at time t if
      $lag(\tau_i, t) < 0$ and $lag(\tau_i, t+1) \leq 0$ if τ_i does not execute
  - For the other tasks, execute the task that has the least t such that $lag(\tau_i, t) > 0$

The algorithm PF

- **Results**
  - The algorithm PF assigns priorities to tasks at every time slot → Job-level dynamic priority (JLDP) scheduling policy
  - **Theorem**: the schedule generated by algorithm PF is P-fair
    - Proof: [Baruah et al., '96]

The algorithm PF

- Example: $\tau = \{(T_1 = 5, C_1 = 2),\ (T_2 = 5,\ C_2 = 3)\}$, one processor



At time 0, any of the two tasks may be scheduled

At time 1:

$$lag(\tau_1, 1) = 1 \cdot \left(\frac{2}{5}\right) - 1 = -\frac{3}{5}$$

$$lag(\tau_2, 1) = 1 \cdot \left(\frac{3}{5}\right) - 0 = \frac{3}{5}$$

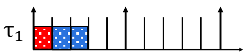
At time 2 if τ_2 executes:

$$lag(\tau_2, 2) = 2 \cdot \left(\frac{3}{5}\right) - 1 = \frac{1}{5}$$

τ_2 is urgent at time 1!!

The algorithm PF

- Example: $\tau = \{(T_1 = 5, C_1 = 2),\ (T_2 = 5,\ C_2 = 3)\}$, one processor



At time 2:

$$lag(\tau_1, 2) = 2 \cdot \left(\frac{2}{5}\right) - 1 = -\frac{1}{5}$$

$$lag(\tau_2, 2) = 2 \cdot \left(\frac{3}{5}\right) - 1 = \frac{1}{5}$$

At time 3 if τ_2 executes:

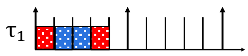
$$lag(\tau_1, 3) = 3 \cdot \left(\frac{2}{5}\right) - 1 = \frac{1}{5}$$

$$lag(\tau_2, 3) = 3 \cdot \left(\frac{3}{5}\right) - 2 = -\frac{1}{5}$$

τ_2 is scheduled since it has the least t such that lag is positive

The algorithm PF

- Example: $\tau = \{(T_1 = 5, C_1 = 2),\ (T_2 = 5,\ C_2 = 3)\}$, one processor



At time 3:

$$lag(\tau_1, 3) = 3 \cdot \left(\frac{2}{5}\right) - 1 = \frac{1}{5}$$

$$lag(\tau_2, 3) = 3 \cdot \left(\frac{3}{5}\right) - 2 = -\frac{1}{5}$$

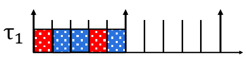
At time 4 if τ_1 executes:

$$lag(\tau_1, 4) = 4 \cdot \left(\frac{2}{5}\right) - 2 = -\frac{2}{5}$$

τ_1 is scheduled since it has the least t such that lag is positive

The algorithm PF

- Example: $\tau = \{(T_1 = 5, C_1 = 2),\ (T_2 = 5,\ C_2 = 3)\}$, one processor



At time 4:

$$lag(\tau_1, 4) = 4 \cdot \left(\frac{2}{5}\right) - 2 = -\frac{2}{5}$$

$$lag(\tau_2, 4) = 4 \cdot \left(\frac{3}{5}\right) - 2 = \frac{2}{5}$$

At time 5 if τ_2 executes:

$$lag(\tau_2, 5) = 5 \cdot \left(\frac{3}{5}\right) - 3 = 0$$

τ_2 is urgent at time 4!!

…and so on…

Proportionate fairness

- Exact test of existence of a P-fair schedule:

$$\sum_{i=1}^{n} U_i \le m$$

- Full processor utilization!

Disadvantages

- High number of preemptions
- High number of migrations
- Optimal only for implicit deadlines

(Other) negative results

- No optimal algorithm is known for constrained or arbitrary deadline systems
- No optimal online algorithm is possible for arbitrary collections of jobs [Leung and Whitehead]
- Even for sporadic task systems, optimality requires **clairvoyance** [Fisher et al., 2009]

⇒ Many **sufficient** schedulability tests exist, according to different metrics of evaluation

- Percentage of schedulable task-sets detected ⟹ **RTA-based** test

RTA-based test

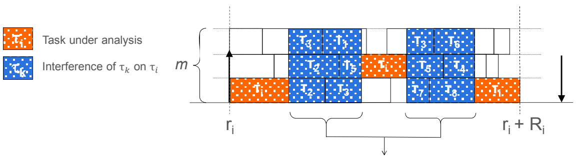
- Response time analysis
  - In a **uniprocessor system**, it provides a **necessary and sufficient** test for fixed-priority preemptive scheduling with constrained deadlines

$$\begin{cases} R_i^{(0)} = \sum_{k=1}^{i} C_k \\ R_i^{(s)} = C_i + \sum_{k=1}^{i-1} \left\lceil \frac{R_i^{(s-1)}}{T_k} \right\rceil C_k \end{cases}$$

Exact interference from higher-priority tasks

  - In a **multiprocessor system**, it provides an **only sufficient** schedulability test
  - How to compute interference from higher-priority tasks?

Introducing the interference

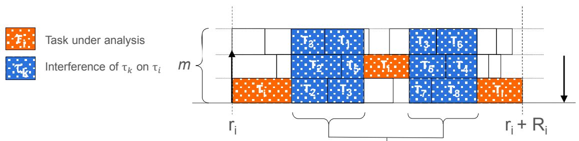


$$I_i = \frac{1}{m} \sum_{k \neq i} I_{i,k}$$

Global FP and Global EDF are **work-conserving** schedulers

Work-conserving scheduler: it never idles a core if there is workload ready to be executed

Introducing the interference



$$I_i = \frac{1}{m} \sum_{k \neq i} I_{i,k}$$

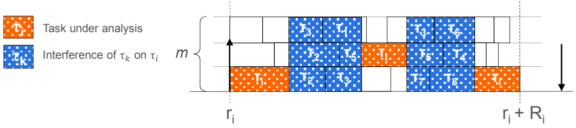
For work-conserving schedulers: a ready job cannot execute **only if all m processors are busy**

$$R_i = C_i + I_i(R_i) = C_i + \frac{1}{m} \sum_{k \neq i} I_i^k(R_i)$$

We can safely assume that the interference is distributed across all *m* processors

Limiting the interference

It is sufficient to consider at most the portion $(D_i - C_i + 1)$ of each term I_i^k in the sum



It can be proved that R_i is given by the fixed point iteration of:

$$R_i = C_i + \frac{1}{m} \sum_{k \neq i} \min(I_i^k(R_i), D_i - C_i + 1)$$

Bounding the interference

- Exactly computing the interference is complex
  - No critical instant scenario
- Pessimistic assumptions:
  1. Bound the interference of a task with the **workload**

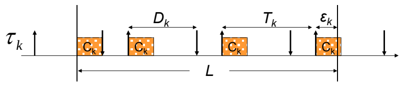
$$I_i^k(R_i) \leq W_k(R_i)$$

  2. Use an **upper-bound** to the workload

Bounding the workload

Consider a pessimistic situation in which:

- The first job executes as close as possible to its deadline
- Successive jobs execute as soon as possible



$$W_k(L) \le w_k(L) = N_k(L) \cdot C_k + \varepsilon_k(L)$$

Where:

$$N_k(L) = \left\lfloor \frac{L + D_k - C_k}{T_k} \right\rfloor$$ Number of jobs excluding the last one

$$\varepsilon_k(L) = \min(C_k, L + D_k - C_k - N_k(L) \cdot T_k)$$ Last job

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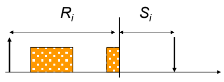
RTA for generic global schedulers

An upper-bound on the worst-case response time of τ_i is given by the fixed point iteration of:

$$R_i \leftarrow C_i + \frac{1}{m} \sum_{k \ne i} \min(w_k(R_i), D_i - C_i + 1)$$



The **slack** of τ_i is at least: $S_i = D_i - R_i$

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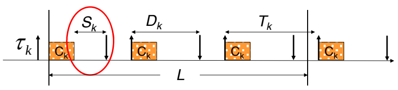
Improvement using slack values

Consider a pessimistic situation in which:

- The first job executes as close as possible to its deadline
- Successive jobs execute as soon as possible



$$W_k(L) \le w_k(L, S_k) = N_k(L, S_k) \cdot C_k + \varepsilon_k(L, S_k)$$

Where:

$$N_k(L, S_k) = \left\lfloor \frac{L + D_k - S_k - C_k}{T_k} \right\rfloor$$ Number of jobs excluding the last one

$$\varepsilon_k(L, S_k) = \min(C_k, L + D_k - C_k - S_k - N_k(L, R_k) \cdot T_k)$$ Last job

33

Improvement using slack values

Consider a pessimistic situation in which:

- The first job executes as close as possible to its deadline
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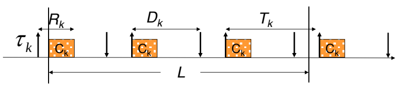

$$W_k(L) \le w_k(L, R_k) = N_k(L, R_k) \cdot C_k + \varepsilon_k(L, R_k)$$

Where:

$$N_k(L, R_k) = \left\lfloor \frac{L + R_k - C_k}{T_k} \right\rfloor$$ Number of jobs excluding the last one

$$\varepsilon_k(L, R_k) = \min(C_k, L + R_k - C_k - N_k(L, R_k) \cdot T_k)$$ Last job

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RTA for generic global schedulers

An upper-bound on the worst-case response time of τ_i is given by the fixed point iteration of:

$$R_i \leftarrow C_i + \frac{1}{m} \sum_{k \ne i} \min(w_k(R_i, R_k), D_i - C_i + 1)$$

If a fixed point $R_i \le D_i$ is reached for every task in the system, the task set is schedulable with **any work-conserving** global scheduler

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Iterative schedulability test

1. All response times R_i initialized to C_i
2. Compute response time bound for tasks $1, \ldots, n$
   - If larger than old value ⟶ update R_i
   - If $R_i > D_i$, mark as temporarily not schedulable
3. If no response time has been updated for tasks $1, \ldots, n$ and all tasks have $R_i \le D_i$ ⟶ return **success**
4. If no response time has been updated for tasks $1, \ldots, n$ and $R_i > D_i$ for some task ⟶ return **fail**
5. Otherwise, return to point 2

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RTA refinement for Fixed Priority

- The interference from lower priority tasks is always null

$$I_i^k(R_i) = 0, \forall k > i$$

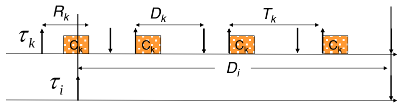
- An upper bound on the worst-case response time of τ_i can be given by the fixed point iteration of

$$R_i \leftarrow C_i + \frac{1}{m} \sum_{k<i} \min(w_k(R_i, R_k), D_i - C_i + 1)$$

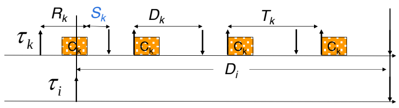
RTA refinement for EDF

- A different bound can be derived analyzing the worst-case workload in a situation in which:
  - The interfering and interfered tasks have a common deadline
  - All jobs execute as late as possible

$$I_i^k(R_i) \leq w_k{}'(D_i, R_k)$$



RTA refinement for EDF



$$w_k'(D_i, R_k) \leq \left\lfloor \frac{D_i}{T_k} \right\rfloor C_k + \min\left(C_k, \left(D_i - S_k - \left\lfloor \frac{D_i}{T_k} \right\rfloor T_k\right)_0\right)$$

- An upper-bound on the worst-case response time of τ_i is given by the fixed point iteration of

$$R_i \leftarrow C_i + \frac{1}{m} \sum_{k \neq i} \min(w_k(R_i, R_k), D_i - C_i + 1, w_k'(D_i, R_k))$$

Complexity

- Pseudo-polynomial complexity
- Fast average behavior
- Lower complexity for Fixed Priority systems
  - Response times are updated in decreasing priority order
- Multiple rounds may be needed in the general case

Thank you!

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