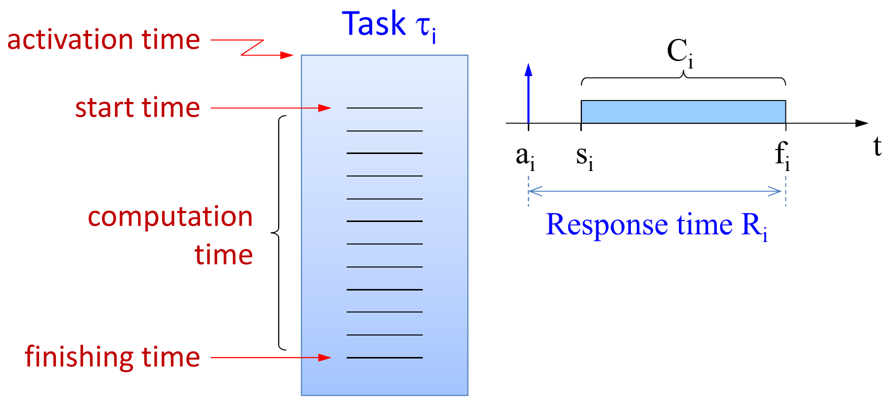
A brief summary of main results for Uniprocessor Real-Time Systems

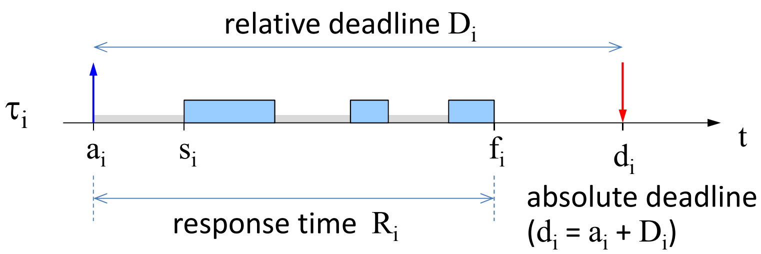
Task model

- Sequence of instructions that in the absence of other activities is continuously executed by the processor until completion.



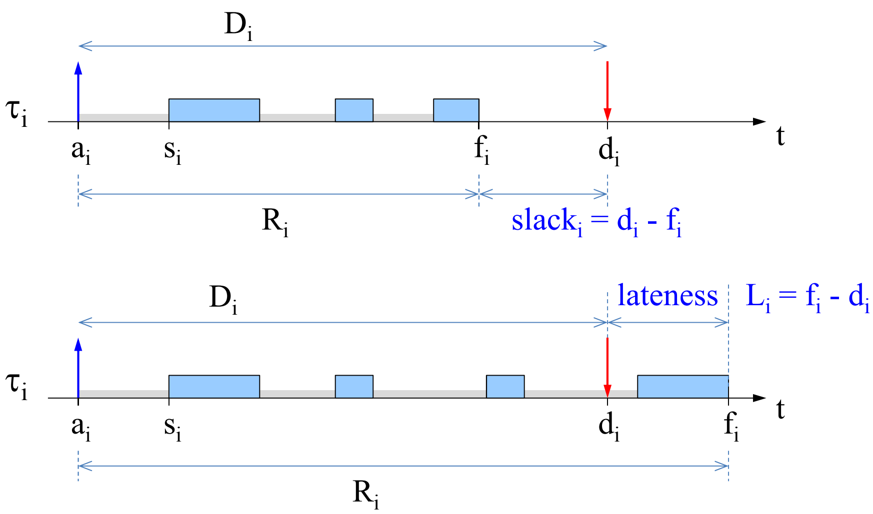
Real-Time Task

- It is a task characterized by a timing constraint on its response time, called deadline:



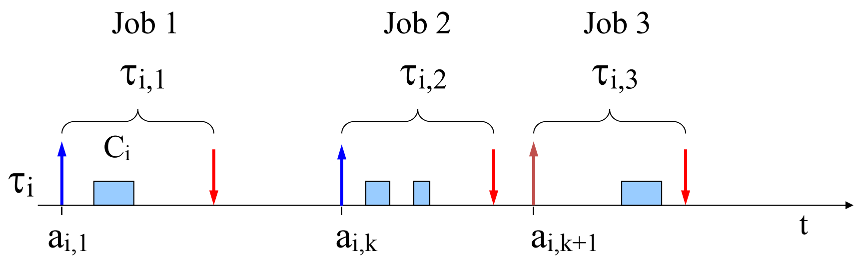
A real-time task τ_i is said to be feasible if it completes within its absolute deadline, that is, if $f_i \le d_i$, o equivalently, if $R_i \le D_i$

Slack and Lateness



Tasks and jobs

A task running several times on different input data generates a sequence of instances (jobs):



Activation modes

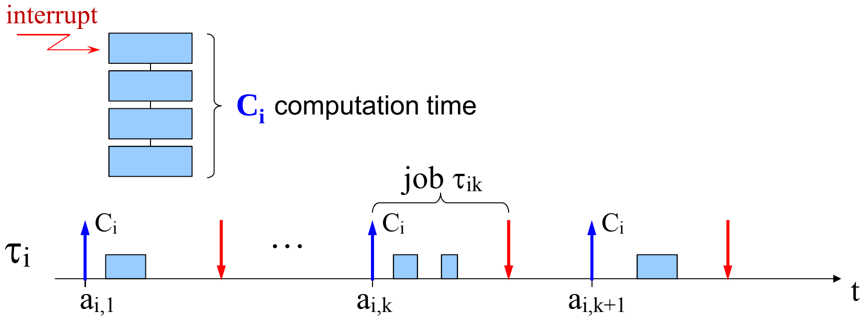
- **Time driven**
  The task is automatically activated by the operating system at predefined time instants.

- **Event driven**
  The task is activated at an event arrival or by explicitly invocating a system call.

Types of tasks

- **Aperiodic**
  Activated by events. Task activation times are unknown and unbounded.
- **Sporadic**
  Activated by events. Task activation times are unknown and bounded: consecutive activations are separated by a minimum interarrival time.
- **Periodic**
  Activated by a timer. Task activation times are known and bounded: Consecutive jobs are separated by a constant interval (period).

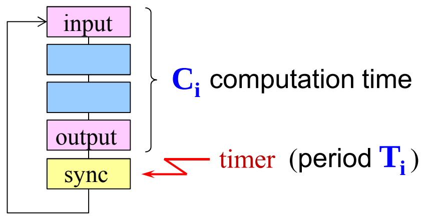
Aperiodic/Sporadic task



- **Aperiodic:** (C_i, D_i) $\quad a_{i,k+1} > a_{i,k}$
- **Sporadic:** (C_i, D_i, T_i) $\quad a_{i,k+1} \geq a_{i,k} + T_i$

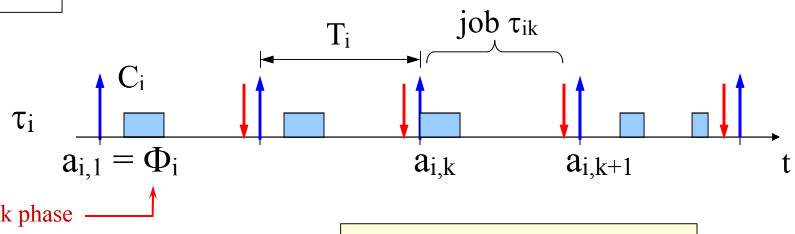
minimum interarrival time

Periodic task



$$U_i = \frac{C_i}{T_i}$$

utilization factor



$\tau_i\,(\Phi_i\,, C_i, T_i, D_i\,)$

$$a_{i,k} = \Phi_i + (k-1)\,T_i$$
$$d_{i,k} = a_{i,k} + D_i$$

Assumptions

- **Implicit deadlines**

$$\forall i \quad D_i = T_i$$

- **Constrained deadlines**

$$\forall i \quad D_i \leq T_i$$

- **Arbitrary deadlines**

Deadlines can be less than, greater than, or equal to periods

Analysis under fixed priority

Let $\Gamma = \{\tau_1, \ldots, \tau_n\}$ be a set of n periodic tasks.

Implicit deadlines

Utilization-based test [Liu & Layland, 1973]	$\sum_{i=1}^{n} U_i \leq n(2^{1/n} - 1)$	only sufficient
Hyperbolic Bound [Bini - Buttazzo[2], 2001]	$\prod_{i=1}^{n} (U_i + 1) \leq 2$	only sufficient

Analysis under fixed priority

Constrained deadlines

Response Time Analysis [Audsley et al., 1993]	$\forall i \quad R_i \leq D_i$	necessary and sufficient

Iterative solution:

$$\begin{cases} R_i^{(0)} = \sum_{k=1}^{i} C_k \\ R_i^{(s)} = C_i + \sum_{k=1}^{i-1} \left\lceil \frac{R_i^{(s-1)}}{T_k} \right\rceil C_k \end{cases}$$

iterate while

$R_i^{(s)} > R_i^{(s-1)}$

Analysis under EDF

Constrained deadlines

Processor Demand
[Baruah et al., 1990]

$$\forall t \in \mathcal{D} \quad dbf(t) \leq t$$

necessary and sufficient

- $dbf(t)$ is called **demand bound function** and denotes the computation time of tasks with deadlines $\leq t$

$$dbf(t) = \sum_{i=1}^{n} \left\lfloor \frac{t + T_i - D_i}{T_i} \right\rfloor C_k$$

- $\mathcal{D}$ is the set of points where the test has to be performed

$$\mathcal{D} = \{d_k \mid d_k \leq L_b\} \qquad L_b = max\{D_{max},\ min(\mathrm{H}, \mathrm{L}^*)\}$$

$$H = lcm(T_1, \ldots, T_n) \qquad L^* = \frac{\sum_{i=1}^{n}(T_i - D_i)U_i}{1-U}$$

EDF example

task	C_i	T_i	D_i	T_i - D_i	U_i
τ_1	1	4	2	2	1/4
τ_2	3	6	5	1	1/2
τ_3	2	14	9	5	1/7

$$U = \frac{1}{4} + \frac{1}{2} + \frac{1}{7} = \frac{25}{28}$$

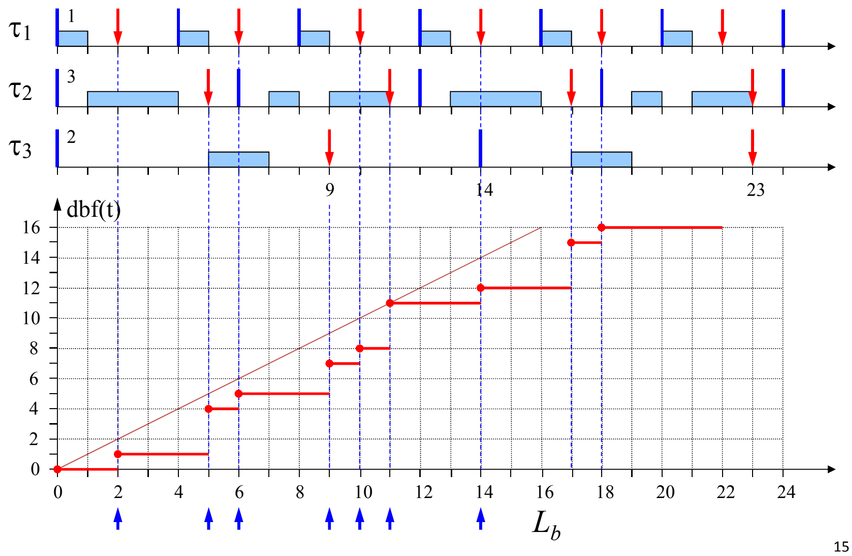
$$H = 4 \cdot 3 \cdot 7 = 84$$

$$L^* = \frac{\sum_{i=1}^{n}(T_i - D_i)U_i}{1-U} = \frac{\frac{2}{4} + \frac{1}{2} + \frac{5}{7}}{\frac{3}{28}} = \frac{12}{7} \cdot \frac{28}{3} = 16$$

$$D_{max} = 9 \qquad L_b = max\,(9,\ min(84,16)) = 16$$

$$\mathcal{D} = \{d_k \mid d_k \leq L_b\} = \{2, 5, 6, 9, 10, 11, 14\}$$

EDF example



Workload Analysis under FP

Arbitrary deadlines

Workload Analysis
[Lehoczky et al., 1989]

$$\forall i \quad \exists t \in A_i \quad W_i(t) \leq t$$

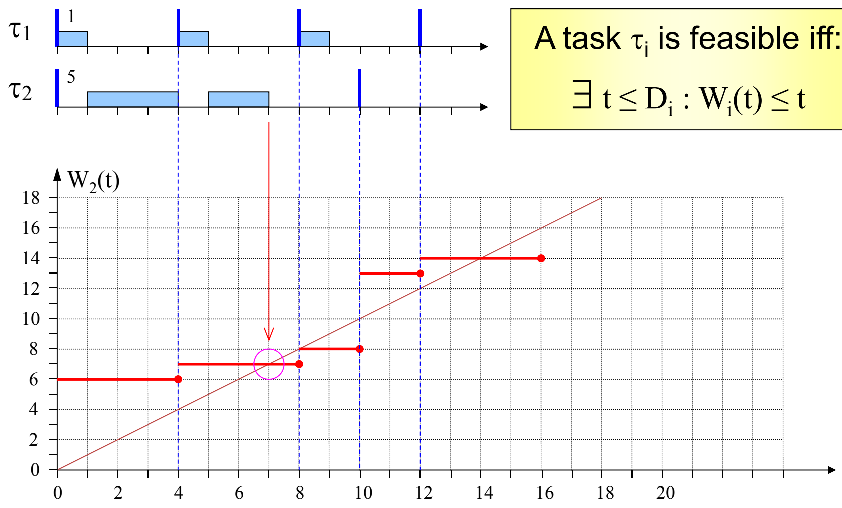
necessary and sufficient

- $W_i(t)$ is called **workload** in $(0, t]$ at priority level P_i and denotes the computation time requested in $(0, t]$ by tasks with priority higher than or equal to P_i

$$W_i(t) = C_i + \sum_{k=1}^{i-1} \left\lceil \frac{t}{T_k} \right\rceil C_k$$

- A_i is the set of points where the test has to be performed, equal to the activation times ≤ D_i, including D_i

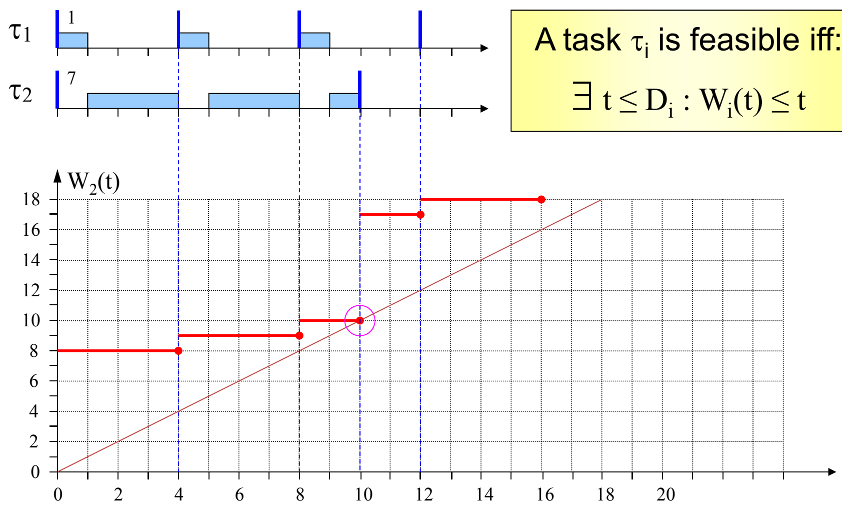
Workload Analysis under FP



A task τ_i is feasible iff:

$\exists\, t \leq D_i : W_i(t) \leq t$

Workload Analysis under FP



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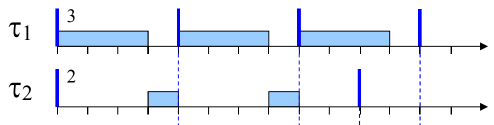
Workload Analysis under FP



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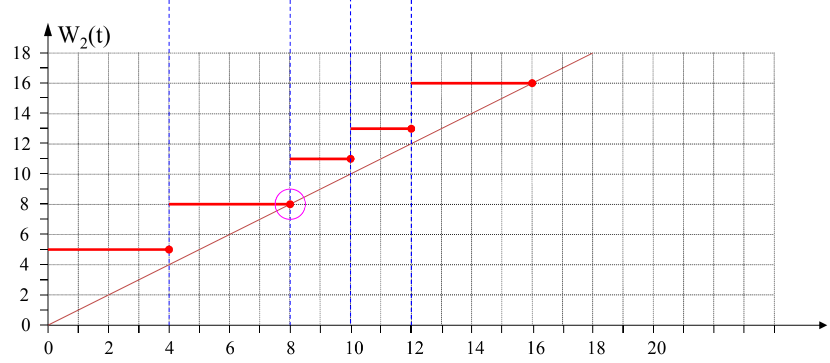
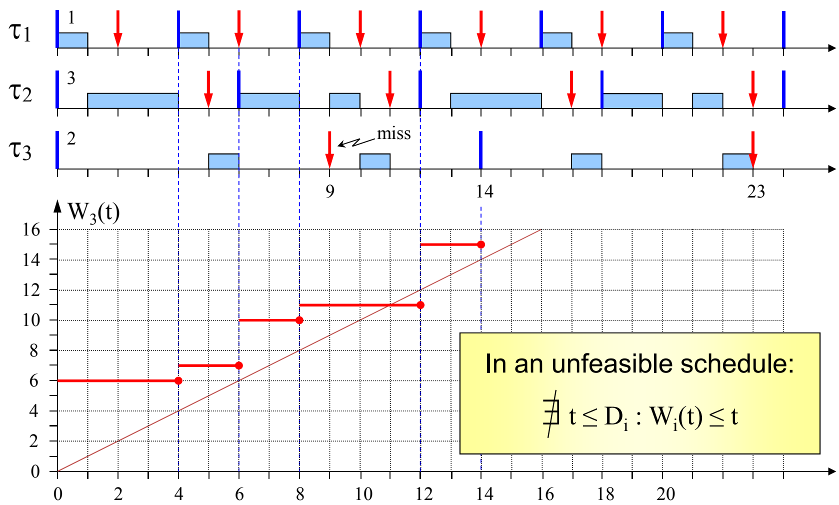


Workload Analysis under FP



In an unfeasible schedule:

$$\nexists\, t \le D_i : W_i(t) \le t$$

Workload Analysis under FP

Theorem [Lehoczky-Sha-Ding, 1989]

A task set is feasible under fixed priorities iff:

$$\forall\, i = 1 \ldots n \quad \exists\, t \le D_i : W_i(t) \le t$$

Problem

How many points need to be tested?

When checking τ_i feasibility, we need to verify $W(t) \le t$ for D_i and for all release times $r_{hk} \le D_i$ of jobs τ_{hk} with priority $P_h \ge P_i$, that is, for all t in A_i:

$$A_i = \left\{ r_{hk} \mid r_{hk} = kT_h,\ h = 1 \ldots i,\ k = 1 \ldots \left\lfloor \frac{T_i}{T_h} \right\rfloor \right\} \cup \{D_i\}$$

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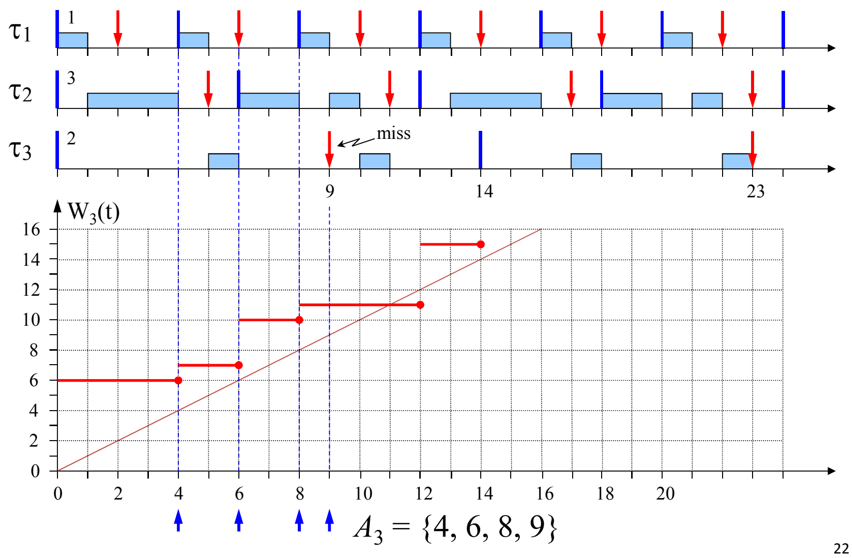
Workload Analysis under FP



$A_3 = \{4, 6, 8, 9\}$

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Workload Analysis under FP

Theorem [Bini-Buttazzo, 2002] $\bigwedge$ AND $\bigvee$ OR

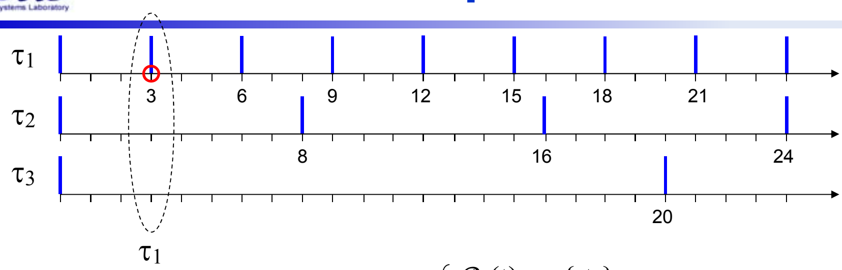
A task set is feasible under fixed priorities iff:

$$\bigwedge_{i=1\ldots n} \bigvee_{t \in \mathcal{P}_{i-1}(D_i)} W_i(t) \le t$$

Where $\mathcal{P}_i(t)$ is defined by:

$$\begin{cases} \mathcal{P}_0(t) = \{ t \} \\ \mathcal{P}_i(t) = \mathcal{P}_{i-1}\left(\left\lfloor \frac{t}{T_i} \right\rfloor T_i\right) \cup \mathcal{P}_{i-1}(t) \end{cases}$$

Example



$$\begin{cases} \mathcal{P}_0(t) = \{ t \} \\ \mathcal{P}_i(t) = \mathcal{P}_{i-1}\left(\left\lfloor \frac{t}{T_i} \right\rfloor T_i\right) \cup \mathcal{P}_{i-1}(t) \end{cases}$$

τ_1: $\mathcal{P}_0(D_1) = \mathcal{P}_0(3) = \{3\}$

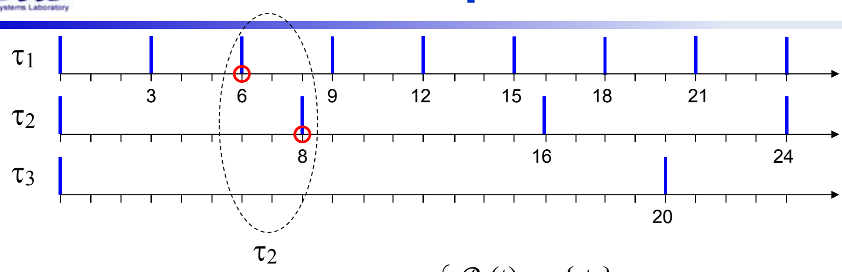
τ_2: $\mathcal{P}_1(D_2) = \mathcal{P}_0(6) \cup \mathcal{P}_0(8) = \{6, 8\}$

τ_3: $\mathcal{P}_2(D_3) = \mathcal{P}_1(16) \cup \mathcal{P}_1(20)$

$= [\mathcal{P}_0(15) \cup \mathcal{P}_0(16)] \cup [\mathcal{P}_0(18) \cup \mathcal{P}_0(20)]$

$= \{15, 16, 18, 20\}$

Example



$$\begin{cases} \mathcal{P}_0(t) = \{ t \} \\ \mathcal{P}_i(t) = \mathcal{P}_{i-1}\left(\left\lfloor \frac{t}{T_i} \right\rfloor T_i \right) \cup \mathcal{P}_{i-1}(t) \end{cases}$$

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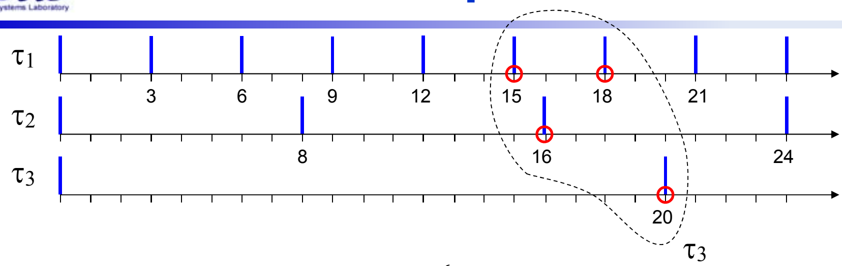
Example



$$\begin{cases} \mathcal{P}_0(t) = \{ t \} \\ \mathcal{P}_i(t) = \mathcal{P}_{i-1}\left(\left\lfloor \frac{t}{T_i} \right\rfloor T_i \right) \cup \mathcal{P}_{i-1}(t) \end{cases}$$

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$= \{15, 16, 18, 20\}$

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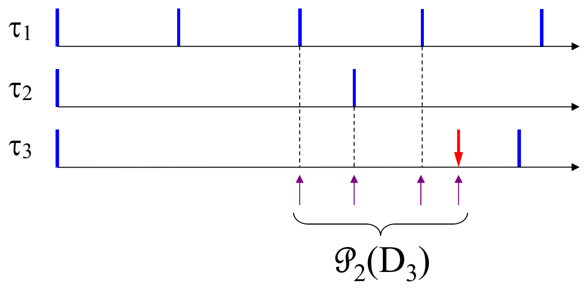
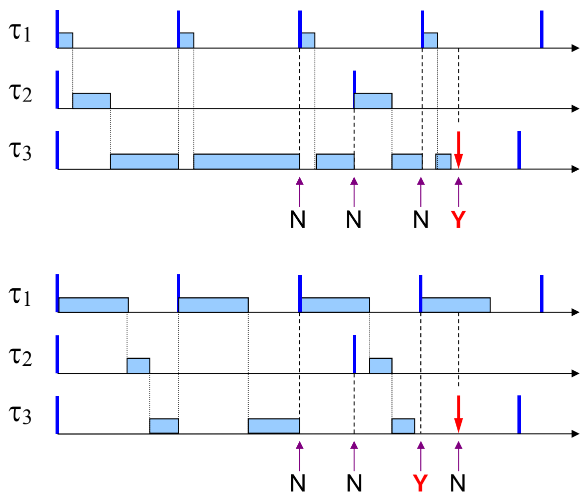
Workload Analysis under FP

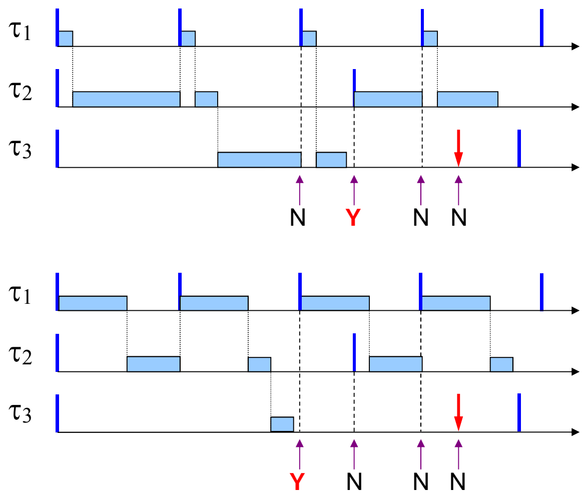


$\mathcal{P}_{i-1}(D_i)$ is the minimum set of points for checking the feasibility of τ_i.

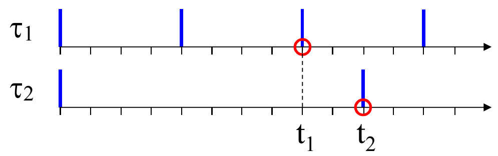
Example



Example



Workload Analysis under FP

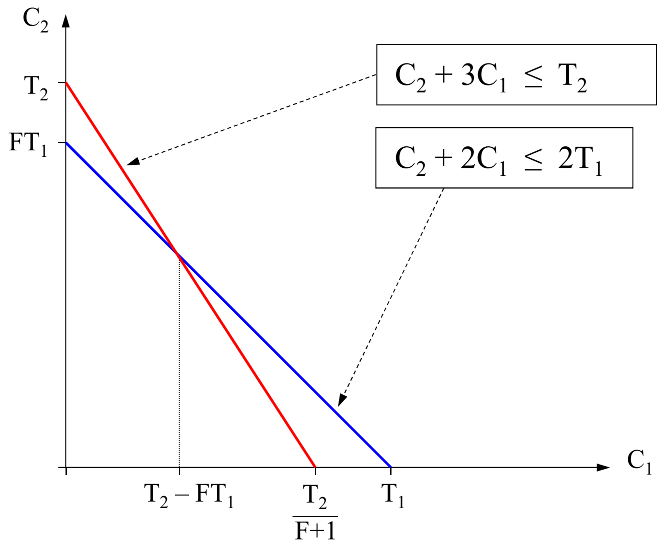


$$\bigwedge_{i=1\ldots n} \bigvee_{t \in \mathcal{P}_{i-1}(D_i)} W_i(t) \le t$$

$$i = 1 \qquad C_1 \le T_1$$

$$i = 2 \qquad \left\| \begin{array}{l} C_2 + \left\lceil \frac{t_1}{T_1} \right\rceil C_1 \le t_1 \\ C_2 + \left\lceil \frac{t_2}{T_1} \right\rceil C_1 \le t_2 \end{array} \right. \quad \Rightarrow \quad \left\| \begin{array}{l} C_2 + 2C_1 \le 2\,T_1 \\ C_2 + 3C_1 \le T_2 \end{array} \right.$$

Workload Analysis under FP



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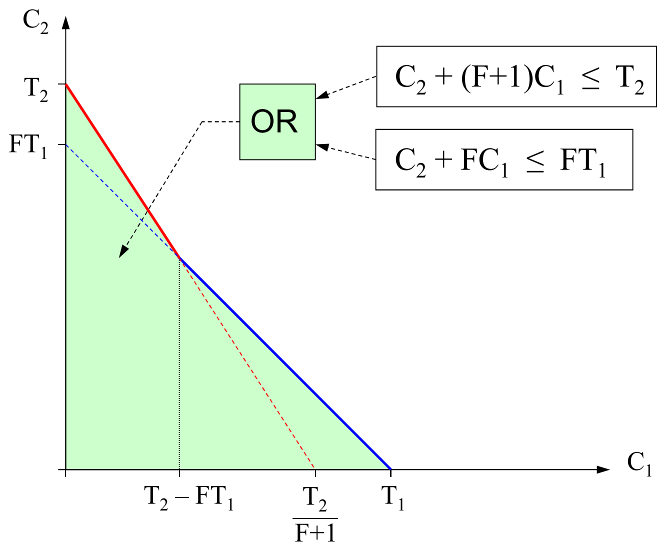
Workload Analysis under FP



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Analysis summary

Under EDF (Processor Demand Criterion):

$$\forall\, t \in \mathcal{D} \quad dbf(t) \leq t$$

$$dbf(t) = \sum_{i=1}^{n} \left\lfloor \frac{t + T_i - D_i}{T_i} \right\rfloor C_k$$

Under Fixed Priorities (Workload Analysis):

$$\forall i = 1,\ldots,n \quad \exists\, t \in A_i \,:\, W_i(t) \leq t$$

$$W_i(t) = C_i + \sum_{k=1}^{i-1} \left\lceil \frac{t}{T_k} \right\rceil C_k$$