Task Scheduling

Definitions

A schedule σ is said to be **feasible** if it satisfies a set of constraints.

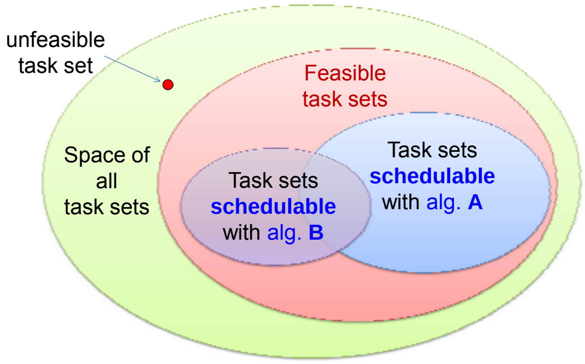
A task set Γ is said to be **feasible**, if there exists an algorithm that generates a feasible schedule for Γ.

A task set Γ is said to be **schedulable** with an algorithm A, if A generates a feasible schedule.

Examples of constraints

- Timing constraints: activation, period, deadline, jitter.
- Precedence: order of execution between tasks.
- Resources: synchronization for mutual exclusion.

Feasibility vs. schedulability



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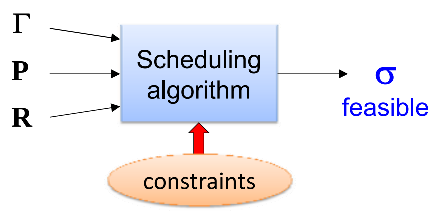
The scheduling problem

Given a set Γ of n tasks, a set **P** of p processors, and a set **R** of r resources, find an assignment of **P** and **R** to Γ that produces a feasible schedule under a set of constraints.



Complexity

- In 1975, Garey and Johnson showed that the general scheduling problem is **NP hard**.

  In practice, it means that the time for finding a feasible schedule grows exponentially with the number of tasks.

Fortunately, polynomial time algorithms can be found under particular conditions.

Why do we care about complexity?

- Let's consider an application with **n = 30** tasks on a processor in which the elementary step takes **1 μs**
- Consider 3 algorithms with the following complexity:

A_1: ***O(n)***	A_2: ***$O(n^8)$***	A_3: ***$O(8^n)$***
⬇	⬇	⬇
30 μs	**182 hours**	**40.000 billion years**

Simplifying assumptions

- Single processor
- Homogeneous task sets
- Fully preemptive tasks
- Simultaneous activations
- No precedence constraints
- No resource constraints

Task set assumptions

We consider algorithms for different types of tasks:

- Single-job tasks (one shot)
  tasks with a single activation (not recurrent)
- Periodic tasks
  recurrent tasks regularly activated by a timer (each task potentially generates infinite jobs)
- Aperiodic/Sporadic tasks
  recurrent tasks irregularly activated by events (each task potentially generates infinite jobs)
- Mixed task sets

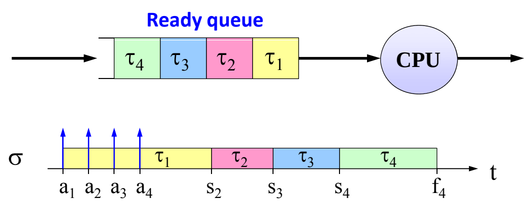
Classical scheduling policies

- First Come First Served
- Shortest Job First
- Priority Scheduling
- Round Robin

Not suited for real-time systems

First Come First Served

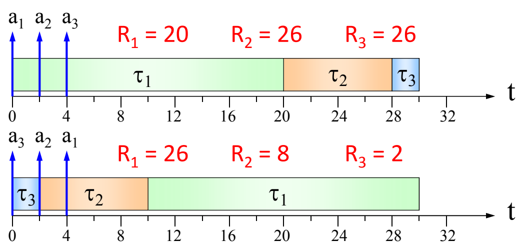
It assigns the CPU to tasks based on their arrival times (intrinsically non preemptive):



First Come First Served

- Very unpredictable

response times strongly depend on task arrivals:



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Shortest Job First (SJF)

It selects the ready task with the shortest computation time.

- Static (C_i is a constant parameter)
- It can be used on line or off-line
- Can be preemptive or non preemptive
- It minimizes the average response time

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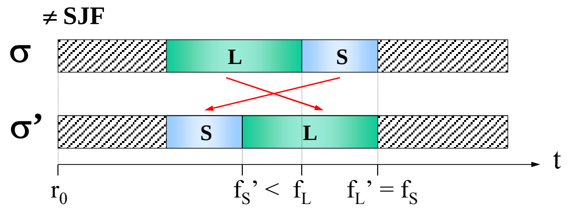
SJF - Optimality



$f_S' + f_L' \le f_S + f_L$

$$\overline{R}(\sigma') = \frac{1}{n}\sum_{i=1}^{n}(f'_i - r_i) \le \frac{1}{n}\sum_{i=1}^{n}(f_i - r_i) = \overline{R}(\sigma)$$

SJF - Optimality

$$\sigma \rightarrow \sigma' \rightarrow \sigma'' \rightarrow \ldots \rightarrow \sigma^*$$

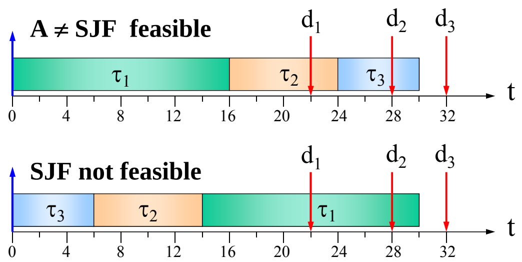
$$\overline{R}(\sigma) \ge \overline{R}(\sigma') \ge \overline{R}(\sigma'') \;\ldots\; \ge \overline{R}(\sigma^*)$$

$\sigma^* = \sigma_{SJF}$

$\overline{R}(\sigma_{SJF})$ is the minimum response time achievable by any algorithm

Is SJF suited for Real-Time?

- It is not optimal in the sense of feasibility



Priority Scheduling

- Each task has a priority P_i, typically $P_i \in [0, 255]$
- The task with the highest priority is selected for execution.
- Tasks with the same priority are served FCFS

NOTE:

$p_i \propto 1/C_i$	$\Rightarrow$	SJF
$p_i \propto 1/a_i$	$\Rightarrow$	FCFS

Priority Scheduling

- **Problem: starvation**

  low priority tasks may experience long delays due to the preemption of high priority tasks.

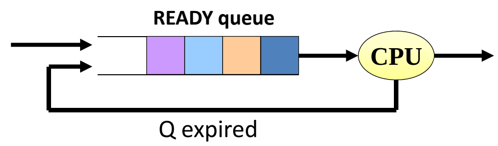
- **A possible solution: aging**

  priority increases with waiting time

Round Robin

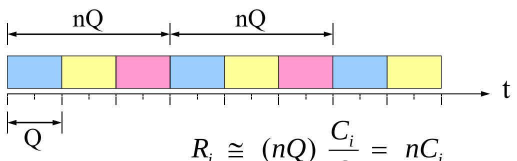
The ready queue is served with FCFS, but ...

- Each task τ_i cannot execute for more than Q time units (Q = time quantum).
- When Q expires, τ_i is put back in the queue.



Round Robin

n = number of task in the system



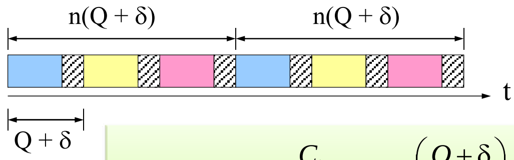
$$R_i \cong (nQ)\,\frac{C_i}{Q} = nC_i$$

Time sharing

Each task runs as it was executing alone on a virtual processor n times slower than the real one.

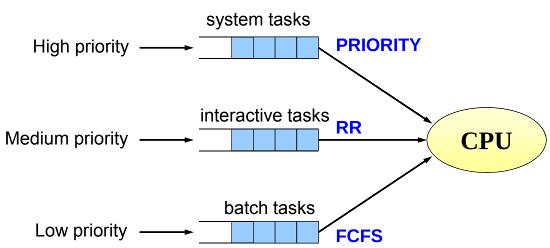
Round Robin

- **if** Q > max(C_i) **then** **RR ≡ FCFS**
- **if** Q ≅ context switch time (δ) **then**

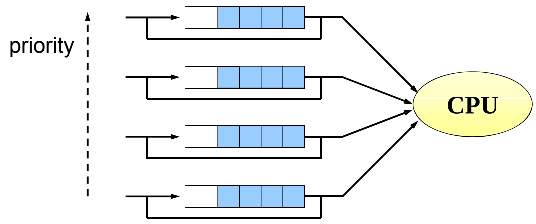


$$R_i \cong n(Q+\delta)\,\frac{C_i}{Q} = nC_i\left(\frac{Q+\delta}{Q}\right)$$

Multi-Level Scheduling



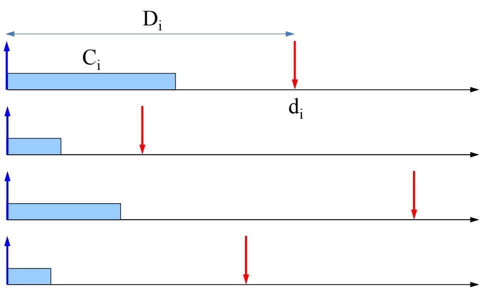
Multi-Level Scheduling



Real-Time Scheduling Algorithms

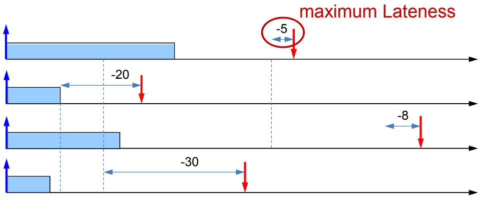
How to schedule RT tasks?

How to schedule RT tasks to maximize feasibility?



Earliest Due Date [Jackson 55]

Given a set of real-time tasks arrived simultaneously, executing them by increasing deadline will minimize the maximum lateness (L_{max}).



NOTE:
- No other scheduler can decrease L_{max}
- Preemption is not required

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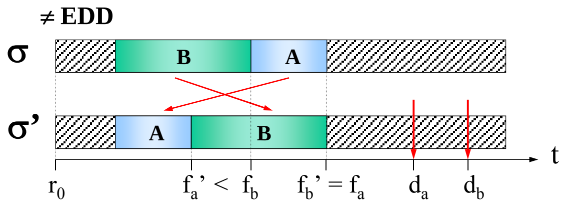
EDD - Optimality



$L_{max} = L_a = f_a - d_a$

$L_a' = f_a' - d_a < f_a - d_a$

$L_b' = f_b' - d_b < f_a - d_a$

$L'_{max} < L_{max}$

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EDD - Optimality

$$\sigma \rightarrow \sigma' \rightarrow \sigma'' \rightarrow \ldots \rightarrow \sigma^*$$

$$L_{max}(\sigma) \geq L_{max}(\sigma') \geq L_{max}(\sigma'') \ldots \geq L_{max}(\sigma^*)$$

$$\sigma^* = \sigma_{EDD}$$

$L_{max}(\sigma_{EDD})$ is the minimum value achievable by any algorithm

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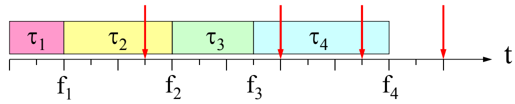
EDD guarantee test (off line)



A task set Γ is feasible iff $\forall i \quad f_i \leq d_i$

$$f_i = \sum_{k=1}^{i} C_k$$

$$\forall i \quad \sum_{k=1}^{i} C_k \leq D_i$$

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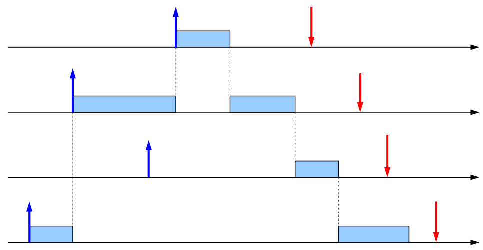
Earliest Deadline First

If tasks arrive dynamically, the maximum lateness can be minimized executing them by increasing absolute deadline, but preemption must be enabled.



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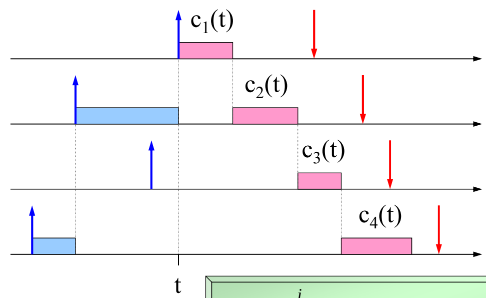
EDF Guarantee test (on line)



$$\forall i \quad \sum_{k=1}^{i} c_k(t) \leq d_i - t$$

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Complexity

EDD

Scheduler (queue ordering): $O(n \log n)$

Feasibility Test (guarantee test): $O(n)$

EDF

Scheduler (insertion in the queue): $O(n)$

Feasibility Test (guarantee single task): $O(n)$

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EDF optimality

⇒ In the sense of feasibility [Dertouzos 1974]

An algorithm A is **optimal** in the sense of feasibility if it generates a feasible schedule, if there exists one.

Demonstration method

It is sufficient to prove that, given an arbitrary feasible schedule, the schedule generated by EDF is also feasible.

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A property of optimal algorithms

If a task set Γ is not schedulable by an optimal algorithm, then Γ cannot be scheduled by any other algorithm.

If an algorithm A minimizes L_{max} then A is also optimal in the sense of feasibility. The opposite is not true.

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Periodic Task Scheduling

Problem formulation

- We consider a computing system that has to execute a set Γ of n periodic real-time tasks:

$$\Gamma = \{ \tau_1, \tau_2, \dots \tau_n \}$$

- Each task τ_i is characterized by:

C_i worst-case computation time

T_i activation period

D_i relative deadline

Φ_i initial arrival time (phase)

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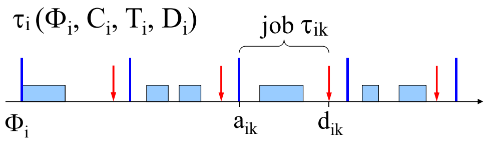
Problem formulation



For each periodic task τ_i we must guarantee that:

- each job τ_{ik} is activated at $a_{ik} = \Phi_i + (k-1)T_i$
- each job τ_{ik} completes within $d_{ik} = a_{ik} + D_i$

There are several wrong ways to achieve this goal.

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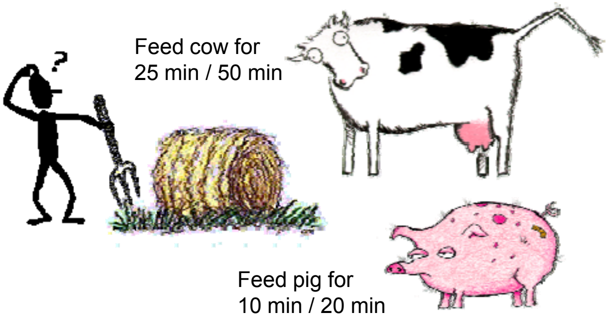
A farm scheduling problem



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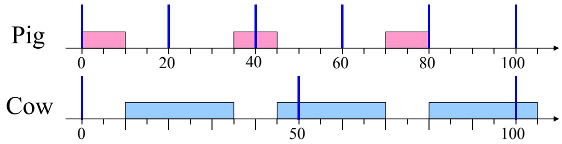
First try

Alternate pig with cow



Evaluation:

Pig gets hungry
Cow gets fat

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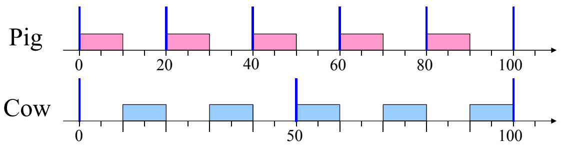
Second try

Feed pig and cow 10 min each



Evaluation:

Pig is OK
Cow is not happy

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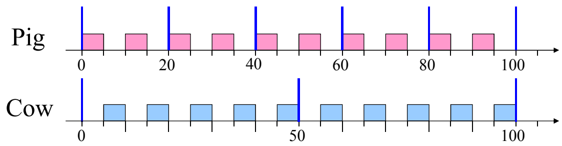
Third try

Feed pig and cow 5 min each



Evaluation:

Pig is OK, Cow is OK
but the farmer is tired

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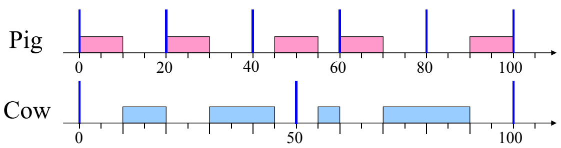
Optimal algorithm

Feed the most starving animal (≡ EDF)



Evaluation: Everybody is happy

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What do we learn?

- Reducing the execution time window, we get closer to a feasible solution.
- The time is split proportionally between the animals.

In the example, each animal required food for 50% of the time, but how can we generalize the solution if the animals require different fraction of time?

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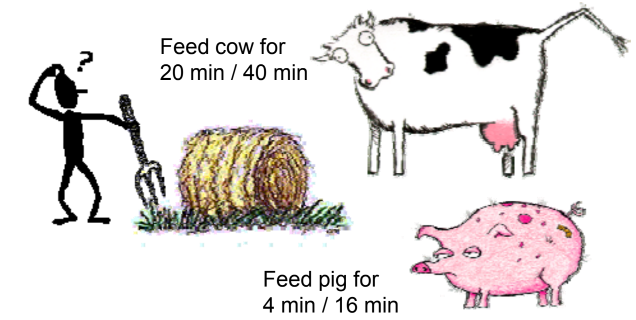
A new scheduling problem

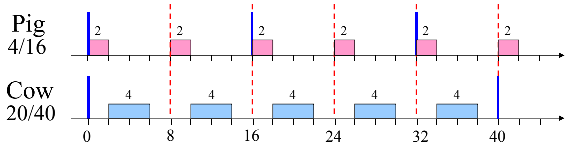


Proportional share algorithm

Basic idea

- Divide the timeline into slots of equal length.
- Within each slot serve each task for a time proportional to its utilization:

Pig utilization factor	=	4/16	= 1/4
Cow utilization factor	=	20/40	= 1/2



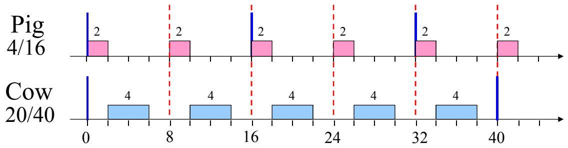
Proportional share algorithm

In general

Let: U_i = required feeding fraction

$\Delta = GCD(T_1, T_2) = 8$

execute each task for $\delta_i = U_i\Delta$ in each slot Δ



NOTE: $U_i\Delta$ ensures C_i in T_i, in fact: $\delta_i(T_i/\Delta) = C_i$

Feasibility test: $\Sigma\delta_i \leq \Delta$ i.e. $\Sigma U_i \leq 1$

Proportional share algorithm

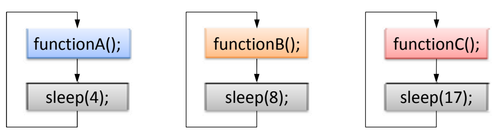
- This method approximates a fluid system, where execution progresses proportionally to U_i
- The major problem is that if periods are not harmonic, $\Delta = GCD(T_1, \ldots, T_n)$ is small and a task is fragmented into many chunks: T_i/Δ of small duration $\delta_i = U_i\Delta$.

too much overhead

Work and Sleep

According to this method, a task executes for C_i units and then suspends for $T_i - C_i$ units:

task	C_i	T_i	Sleep time
A	1	5	4
B	2	10	8
C	3	20	17

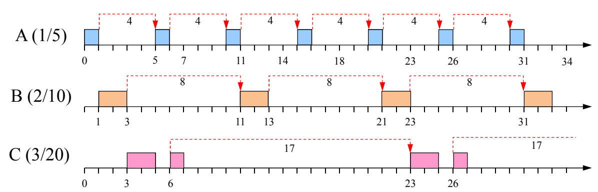


Work and Sleep

Example 1:

task	C_i	T_i	Sleep
A	1	5	4
B	2	10	8
C	3	20	17

It works well for small computation times

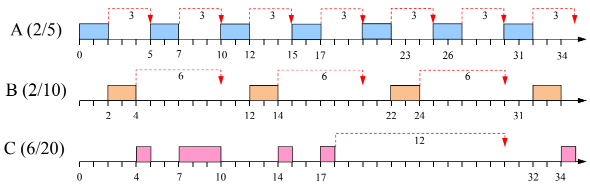


Work and Sleep

Example 2:

task	C_i	T_i	Sleep
A	2	5	3
B	2	8	6
C	6	20	12

Problem
Low priority tasks experience long delays



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Loop Scheduling

It is a simple trick to schedule periodic activities at different rates using a single loop (often used in Arduino):

```
int   count = 0;           // relative time
int   T1 = 20;             // period 1 in ms
int   T2 = 50;             // period 2 in ms
int   T3 = 80;             // period 3 in ms

      while (1) {
            if (count%T1 == 0) function1();
            if (count%T2 == 0) function2();
            if (count%T3 == 0) function3();

            count++;
            if (count == T1*T2*T3) count = 0;

            delay(1);      // wait for 1 ms
      }
```

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Loop Scheduling

Note that the counter must be reset at the least common multiple of the periods, called the hyperperiod (H):

```
      count++;
      if (count == T1*T2*T3) count = 0;
```

Q: How many bits are needed to represent the hyperperiod?

T1 = 10
T2 = 40
T3 = 50
T4 = 100
T5 = 500
T6 = 1000

$H = 10^{12}$

$$\text{bits} = \lceil \log_2 10^{12} \rceil = \lceil 39.86 \rceil = 40$$

It does not fit to a long integer.
We are in trouble!

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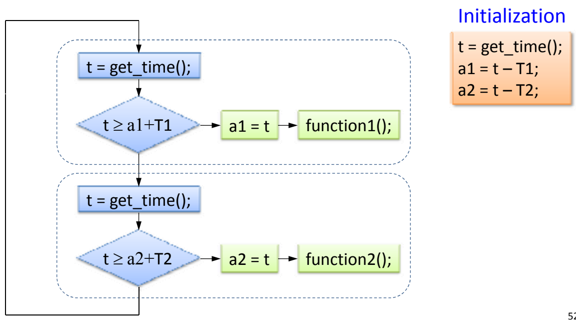
Loop Scheduling

A better way is to rely on a system call that returns the system time:



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Loop Scheduling

Implementation:

```
#define    N     5                  // number of tasks
time       t;                       // current time
time       a[N], T[N];              // act. times, periods

      initialize_periods(T);        // e.g., read from file

      t = get_time();
      for (i=0; i<N; i++) a[i] = t – T[i];

      while (1) {
            for (i=0; i<N; i++) {
                  t = get_time();
                  if (t >= a[i] + T[i]) {
                        a[i] = t;
                        function(i);
                  }
            }
      }
```

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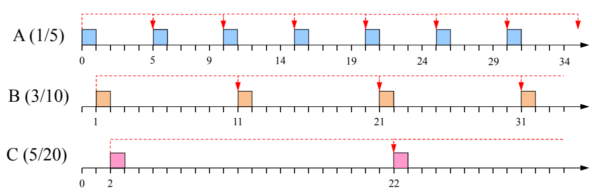
Loop Scheduling

Example 1:

task	C_i	T_i
A	1	5
B	1	10
C	1	20



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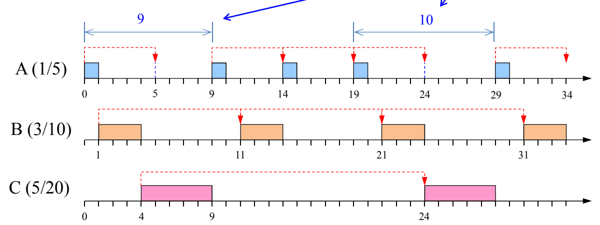
Loop Scheduling

Example 2:

task	C_i	T_i
A	1	5
B	3	10
C	5	20

Problem
Tasks experience delays from the other tasks



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Loop Scheduling

If the scheduler is not the only thread, a sleep must be inserted:

```
#define    N          5           // number of tasks
#define    DELTA      3           // milliseconds
time       t;                     // current time
time       a[N], T[N];            // act. times, periods

      initialize_periods(T);      // e.g., read from file

      t = get_time();
      for (i=0; i<N; i++) a[i] = t - T[i];
      while (1) {
            for (i=0; i<N; i++) {
                  t = get_time();
                  if (t-a[i] >= T[i]) {
                        a[i] = t;
                        function(i);
                  }
            }
            sleep(DELTA);         // suspend for 3 ms
      }
```

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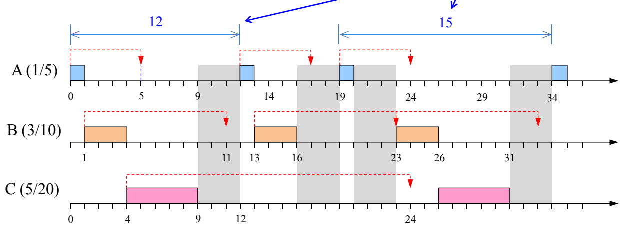
Loop Scheduling

Example 3:

DELTA = 3

task	C_i	T_i
A	1	5
B	3	10
C	5	20

Problem
The experienced delay is given by other tasks + suspension



NOTE: Suspension time can be higher due to other tasks

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Timeline Scheduling

Also known as cyclic scheduling, it has been used for 30 years in military systems, navigation, and monitoring systems.

Examples

- Air traffic control systems
- Space Shuttle
- Boeing 777
- Airbus navigation system

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Timeline Scheduling

Method

- The time axis is divided in intervals of equal length (*time slots*).
- Each task is statically allocated in a slot in order to meet the desired request rate.
- The execution in each slot is activated by a timer.

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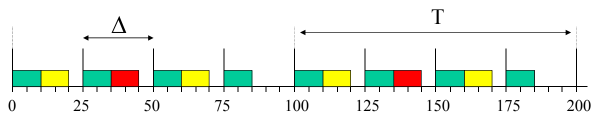
Timeline Scheduling

Example:

task	C_i	T_i
A	10 ms	25 ms
B	10 ms	50 ms
C	10 ms	100 ms

Δ = GCD (minor cycle)

T = lcm (major cycle)



Guarantee: $C_A + C_B \leq \Delta$, $C_A + C_C \leq \Delta$

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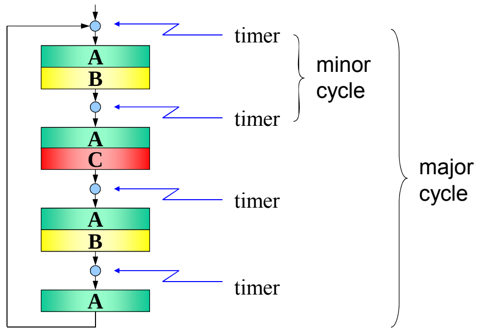
Timeline Scheduling

Implementation:



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Cycling Scheduling

Coding:

```
#define    MINOR      25          // minor cycle = 25 ms

    initialize_timer(MINOR);      // interrupt every 25 ms

    while (1) {
        sync();                   // block until interrupt
        function_A();
        function_B();
        sync();                   // block until interrupt
        function_A();
        function_C();
        sync();                   // block until interrupt
        function_A();
        function_B();
        sync();                   // block until interrupt
        function_A();
    }
```

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Timeline scheduling

Advantages

- Simple implementation (no RTOS is required).
- Low run-time overhead.
- All tasks run with very low jitter.

Disadvantages

- It is not robust during overloads.
- It is difficult to expand the schedule.
- It is not easy to handle aperiodic activities.

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Problems during overloads

What do we do during task overruns?

➢ **Let the task continue**

– we can have a ***domino effect*** on all the other tasks (timeline break)

➢ **Abort the task**

– the system can remain in inconsistent states.

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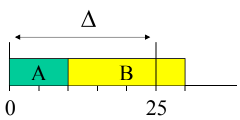
Expandibility

If one or more tasks need to be upgraded, we may have to re-design the whole schedule again.

Example: B is updated so that $C_B = 20$ ms
now $C_A + C_B > \Delta$



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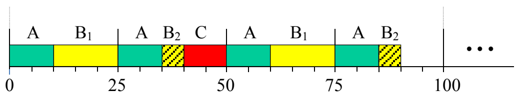
Expandibility

- We have to split task B in two subtasks (B_1, B_2) and re-build the schedule:



Guarantee: $\begin{cases} C_A + C_{B1} \le \Delta \\ C_A + C_{B2} + C_C \le \Delta \end{cases}$

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Expandibility

If the frequency of some task is changed, the impact can be even more significant:

task	T_{old}	T_{new}
A	25 ms	25 ms
B	50 ms	**40 ms**
C	100 ms	100 ms

minor cycle:	$\Delta = 25$	$\Delta = 5$	40 sync. per cycle!
major cycle:	$T = 100$	$T = 200$	

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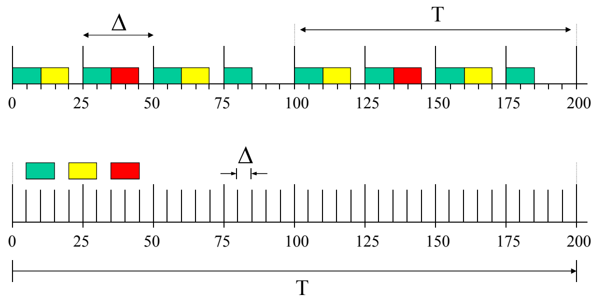
Example



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Priority Scheduling

Priority Scheduling

Method

1. Assign priorities to each task based on its timing constraints.
2. Verify the feasibility of the schedule using analytical techniques.
3. Execute tasks on a priority-based kernel.

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How to assign priorities?

- Typically, task priorities are assigned based on the their relative importance.
- However, different priority assignments can lead to different processor utilization bounds.

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Priority vs. importance

If τ_2 is more important than τ_1 and is assigned higher priority, the schedule may not be feasible:

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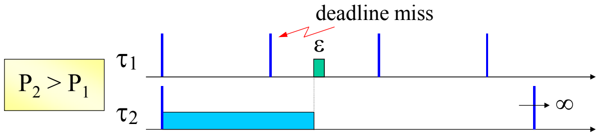
Priority vs. importance

If priority are not properly assigned, the utilization bound can be arbitrarily small:

An application can be unfeasible even when the processor is almost empty!



$$U = \frac{\varepsilon}{T_1} + \frac{C_2}{\infty} \rightarrow 0$$

Optimal priority assignments

- **Rate Monotonic (RM):** (optimal among FP algs for T = D)
  $P_i \propto 1/T_i$ (static)
- **Deadline Monotonic (DM):** (optimal among FP algs for $D \leq T$)
  $P_i \propto 1/D_i$ (static)
- **Earliest Deadline First (EDF):** (optimal among all algs)
  $P_i \propto 1/d_{ik}$ (dynamic)

$$d_{i,k} = r_{i,k} + D_i$$

Rate Monotonic is optimal

RM is **optimal** among all fixed priority algorithms (if $D_i = T_i$):

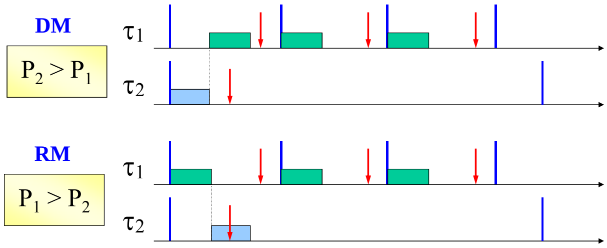
If there exists a fixed priority assignment which leads to a feasible schedule, then the RM schedule is feasible.

⇕

If a task set is not schedulable by RM, then it cannot be scheduled by any fixed priority assignment.

Deadline Monotonic is optimal

If $D_i \leq T_i$ then the **optimal** priority assignment is given by **Deadline Monotonic** (**DM**):



EDF Optimality

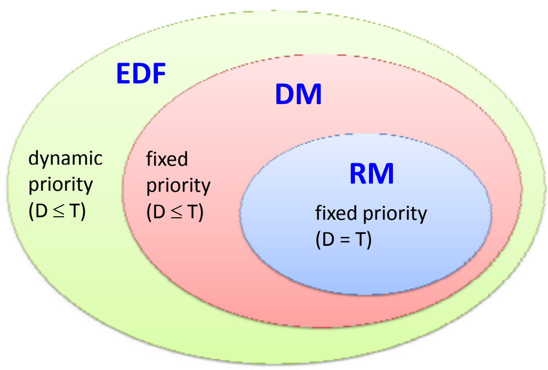
EDF is **optimal** among all algorithms:

If there exists a feasible schedule for a task set, then EDF will generate a feasible schedule.

⇕

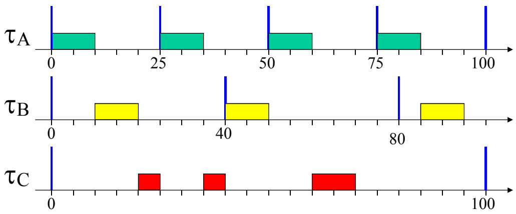
If a task set is not schedulable by EDF, then it cannot be scheduled by any algorithm.

Optimality



Rate Monotonic (RM)

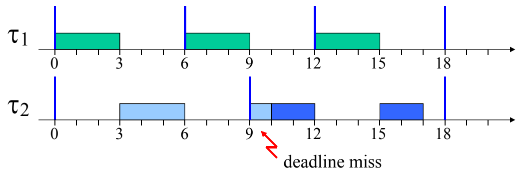
- Each task is assigned a fixed priority proportional to its rate [Liu & Layland '73].



Note that small parameter variations are automatically handled by the scheduler without any intervention.

An unfeasible RM schedule

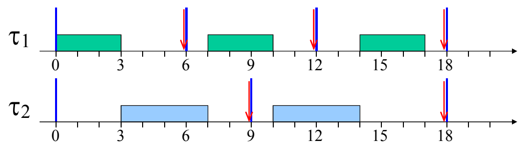
$$U_p = \frac{3}{6} + \frac{4}{9} = 0.944$$



EDF Schedule

$$U_p = \frac{3}{6} + \frac{4}{9} = 0.944$$

$D_i = T_i$



How can we verify feasibility?

- Each task uses the processor for a fraction of time:

$$U_i = \frac{C_i}{T_i}$$

- Hence the total **processor utilization** is:

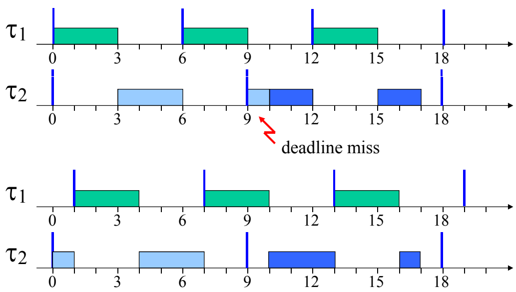
$$U_p = \sum_{i=1}^{n} \frac{C_i}{T_i}$$

- U_p is a measure of the **processor load**.

Identifying the worst case

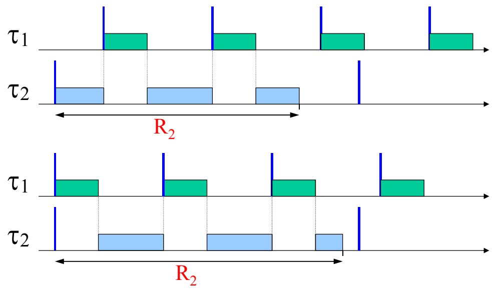
Feasibility may depend on the initial activations (phases):

$$U_p = \frac{3}{6} + \frac{4}{9} = 0.944$$



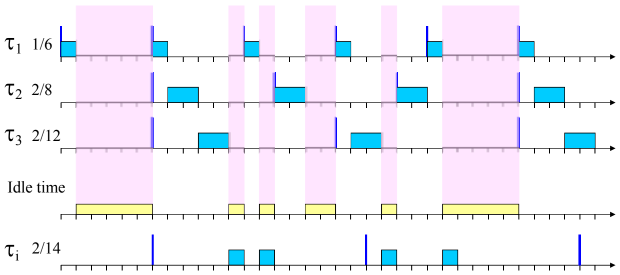
Critical Instant

For any task τ_i, the longest response time occurs when it arrives together with all higher priority tasks.



Critical Instant

For independent preemptive tasks under fixed priorities, the critical instant of τ_i, occurs when it arrives together with all higher priority tasks.



A necessary condition

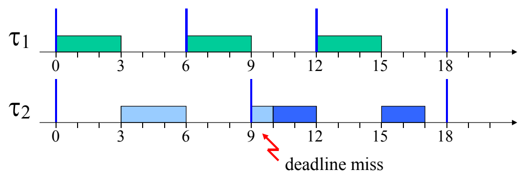
A necessary condition for having a feasible schedule is that $\mathbf{U_p \leq 1}$.

In fact, if $\mathbf{U_p > 1}$ the processor is overloaded hence the task set cannot be schedulable.

However, there are cases in which $\mathbf{U_p \leq 1}$ but the task set is not schedulable by RM.

An unfeasible RM schedule

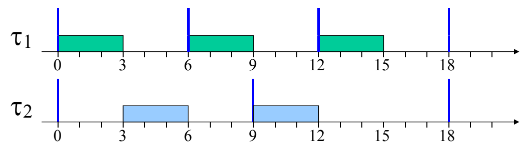
$$U_p = \frac{3}{6} + \frac{4}{9} = 0.944$$



Given this task set (period configuration), what is the higher utilization that guarantees feasibility?

Utilization upper bound

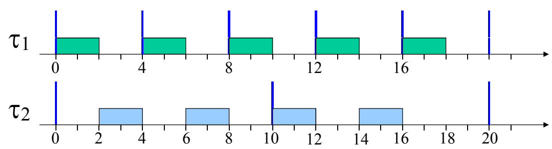
$$U_p = \frac{3}{6} + \frac{3}{9} = 0.833$$



NOTE: If C_1 or C_2 is increased, τ_2 will miss its deadline!

A different upper bound

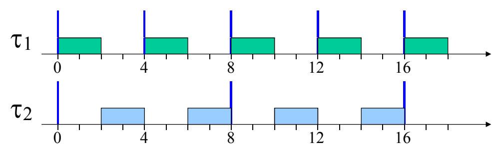
$$U_{ub} = \frac{2}{4} + \frac{4}{10} = 0.9$$



NOTE: The upper bound U_{ub} depends on the specific task set.

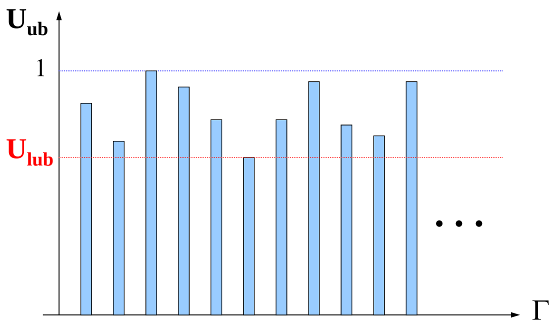
A different upper bound

$$U_p = \frac{2}{4} + \frac{4}{8} = 1$$



NOTE: The upper bound U_{ub} depends on the specific task set.

The least upper bound



A sufficient condition

If $U_p \leq U_{lub}$ the task set is certainly schedulable with the RM algorithm.

NOTE

If $U_{lub} < U_p \leq 1$ we cannot say anything about the feasibility of that task set.

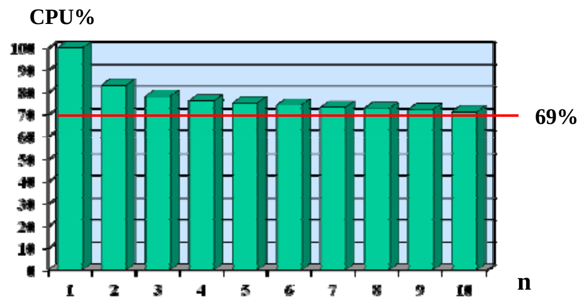
U_{lub} for RM

- In 1973, **Liu and Layland** proved that for a set of n periodic tasks:

$$U_{lub}^{RM} = n\left(2^{1/n} - 1\right)$$

for $n \to \infty$ $U_{lub} \to \ln 2$

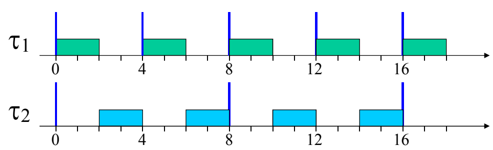
RM Least Upper Bound



A special case

If tasks have harmonic periods U_{lub} = 1.

$$U_p = \frac{2}{4} + \frac{4}{8} = 1$$



RM Guarantee Test

- We compute the processor utilization as:

$$U_p = \sum_{i=1}^{n} \frac{C_i}{T_i}$$

- Guarantee Test (only sufficient):

$$U_p \leq n\left(2^{1/n} - 1\right)$$

Basic Assumptions

A1. C_i is constant for every job of τ_i

A2. T_i is constant for every job of τ_i

A3. For each task, $D_i = T_i$

A4. Tasks are independent:

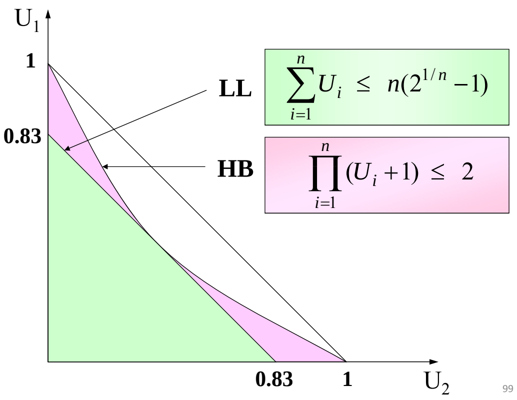
- no precedence relations
- no resource constraints
- no blocking on I/O operations

The Hyperbolic Bound

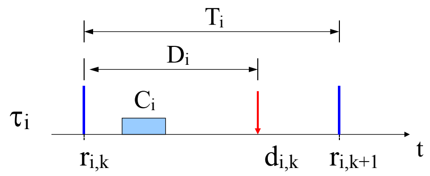
- In 2000, **Bini et al.** proved that a set of *n* periodic tasks is schedulable by RM if:

$$\prod_{i=1}^{n} (U_i + 1) \leq 2$$

HB vs. LL



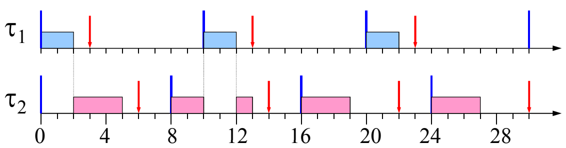
Extension to tasks with D < T



Scheduling algorithms

- Deadline Monotonic: **$p_i \propto 1/D_i$** (static)
- Earliest Deadline First: **$p_i \propto 1/d_i$** (dynamic)

Deadline Monotonic



Problem with the Utilization Bound

$$U_p = \sum_{i=1}^{n} \frac{C_i}{D_i} = \frac{2}{3} + \frac{3}{6} = 1.16 > 1$$

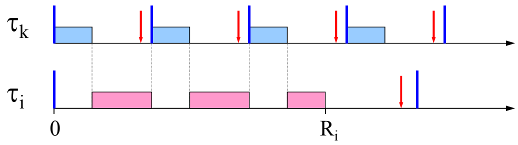
but the task set is schedulable.

Response Time Analysis

[Audsley '90]

- For each task τ_i compute the interference due to higher priority tasks: $I_i = \sum_{D_k < D_i} C_k$
- compute its response time as $R_i = C_i + I_i$
- verify that $R_i \leq D_i$

Computing Interference



Interference of τ_k on τ_i in the interval $[0, R_i]$: $I_{ik} = \left\lceil \frac{R_i}{T_k} \right\rceil C_k$

Interference on τ_i by high-priority tasks: $I_i = \sum_{k=1}^{i-1} \left\lceil \frac{R_i}{T_k} \right\rceil C_k$

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Computing Response Times

$$R_i = C_i + \sum_{k=1}^{i-1} \left\lceil \frac{R_i}{T_k} \right\rceil C_k$$

Iterative solution:

$$\begin{cases} R_i^0 = C_i \\ R_i^s = C_i + \sum_{k=1}^{i-1} \left\lceil \frac{R_i^{(s-1)}}{T_k} \right\rceil C_k \end{cases}$$

iterate while $R_i^s > R_i^{(s-1)}$

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Dynamic Priority Scheduling

Earliest Deadline First (EDF)

- Each job receives an absolute deadline:

$$d_{i,k} = r_{i,k} + D_i$$

- At any time, the processor is assigned to the job with the earliest absolute deadline.
- Under EDF, any task set can utilize the processor up to 100%.

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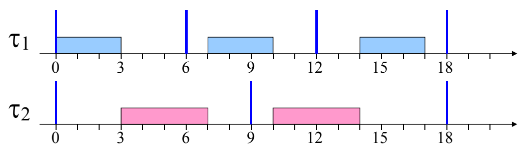
EDF Example

$$U_p = \frac{3}{6} + \frac{4}{9} = 0.944$$

$D_i = T_i$



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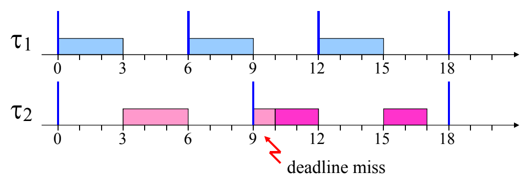
Unfeasible under RM

$$U_p = \frac{3}{6} + \frac{4}{9} = 0.944$$



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EDF Optimality

EDF is **optimal** among all algorithms:

If there exists a feasible schedule for a task set Γ, then EDF will generate a feasible schedule.

⇕

If Γ is not schedulable by EDF, then it cannot be scheduled by any algorithm.

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EDF schedulability

- In 1973, **Liu and Layland** proved that for a set of n periodic tasks:

$$U_{lub}^{EDF} = 1$$

- This means that a task set Γ is schedulable by EDF **if and only if**

$$U_p \leq 1$$

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EDF with D ≤ T

Schedulability Analysis

Processor Demand Criterion [Baruah '90]

In any interval of length L, the computational demand $g(0,L)$ of the task set must be no greater than the available time in that interval.

$$\forall L > 0, \quad g(0,L) \leq L$$

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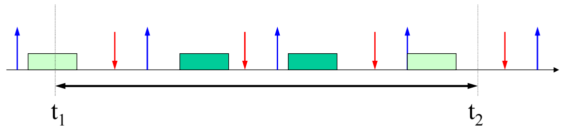
Processor Demand



The demand in [t_1, t_2] is the computation time of those tasks started at or after t_1 with deadline less than or equal to t_2:

$$g(t_1,t_2) = \sum_{r_i \geq t_1}^{d_i \leq t_2} C_i$$

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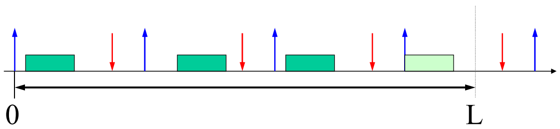
Demand of a periodic task



$$g_i(0,L) = \left\lfloor \frac{L - D_i + T_i}{T_i} \right\rfloor C_i$$

$$g(0,L) = \sum_{i=1}^{n} \left\lfloor \frac{L - D_i + T_i}{T_i} \right\rfloor C_i$$

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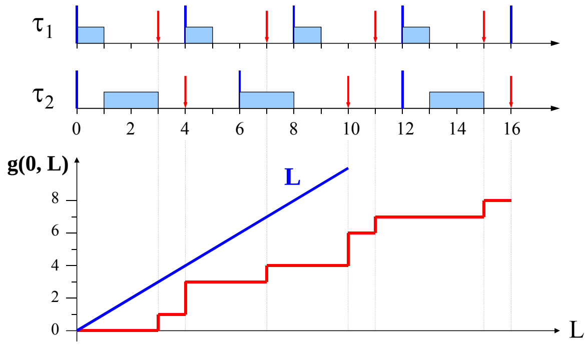
Example



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Bounding complexity

- Since $g(0,L)$ is a step function, we can check feasibility only at deadline points.
- If tasks are synchronous and $U_p < 1$, we can check feasibility up to the hyperperiod H:

$$H = lcm(T_1, \dots, T_n)$$

Bounding complexity

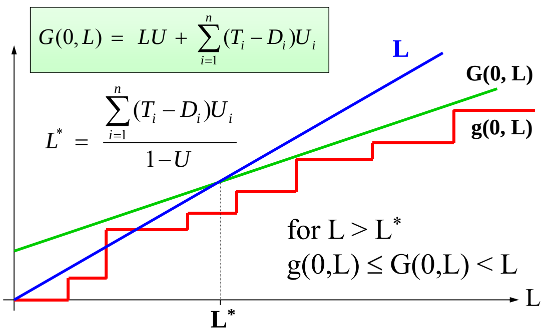
- Moreover we note that: $g(0, L) \leq G(0, L)$

$$G(0,L) = \sum_{i=1}^{n} \left(\frac{L + T_i - D_i}{T_i} \right) C_i$$

$$= \sum_{i=1}^{n} L \frac{C_i}{T_i} + \sum_{i=1}^{n} (T_i - D_i) \frac{C_i}{T_i}$$

$$= LU + \sum_{i=1}^{n} (T_i - D_i) U_i$$

Limiting L



Processor Demand Test

$$\forall L \in D, \quad g(0,L) \leq L$$

$$D = \{d_k \mid d_k \leq \min(H, L^*)\}$$

$$\begin{cases} H = lcm(T_1, \dots, T_n) \\ L^* = \dfrac{\sum_{i=1}^{n} (T_i - D_i) U_i}{1 - U} \end{cases}$$

Summary

- **Three scheduling approaches:**
  - Off-line construction (Timeline)
  - Fixed priority (RM, DM)
  - Dynamic priority (EDF)
- **Three analysis techniques:**
  - Processor Utilization Bound $U \leq U_{lub}$
  - Response Time Analysis $\forall i \;\; R_i \leq D_i$
  - Processor Demand Criterion $\forall L \;\; g(0,L) \leq L$

Schedulability Analysis

	$D_i = T_i$	$D_i \leq T_i$
RM	***Suff.:*** *polynomial* $O(n)$ LL: $\Sigma U_i \leq n(2^{1/n} - 1)$ HB: $\Pi(U_i+1) \leq 2$ ***Exact*** *pseudo-polynomial* RTA	***pseudo-polynomial*** Response Time Analysis $\forall i \;\; R_i \leq D_i$ $R_i = C_i + \sum_{k=1}^{i-1} \left\lceil \frac{R_i}{T_k} \right\rceil C_k$
EDF	***polynomial:*** $O(n)$ $\Sigma U_i \leq 1$	***pseudo-polynomial*** Processor Demand Analysis $\forall L > 0, \quad g(0,L) \leq L$