Introduction to the C programming language
Lists and Trees

Giuseppe Lipari
http://retis.sssup.it/~lipari

Scuola Superiore Sant’Anna – Pisa

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Outline

1. Searching
2. Lists
3. Balanced Binary Trees
4. AVL tree
5. Heap
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1. Searching
2. Lists
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Searching

Suppose we have an address list.

- For each person name, we have the address and the telephone number.
- All entries are stored in an array.
The following class represents an entry

```
address.hpp

class Entry {
    char name[50];
    char address[100];
    char telephone[20];

public:
    Entry();
    Entry(char *s, char *a, char *t);
    char *get_name();
    char *get_address();
    char *get_telephone();
    void print();
};
```
The following class represents an address book with maximum 100 entries

class AddressBook {
    Entry array[100];
    int num;
public:
    AddressBook();
    void insert(Entry e);
    Entry search(char *name);
    void printall();
};
address.cpp

Entry::Entry()
{
    strcpy(name, "") ;
    strcpy(address, ":") ;
    strcpy(telephone, "") ;
}

Entry::Entry(char *s, char *a, char *t)
{
    strncpy(name, s, 50) ;
    strncpy(address, a, 100) ;
    strncpy(telephone, t, 20) ;
}
Implementation of AddressBook

address.cpp

```cpp
AddressBook::AddressBook() : num(0) {}

void AddressBook::insert(Entry e) {
    array[num++] = e;
}

Entry AddressBook::search(char *name) {
    int i;
    Entry null_entry;
    for (i=0; i<num; i++) {
        if (strcmp(name, array[i].get_name()) == 0) {
            return array[i];
        }
    }
    return null_entry;
}
```

Notice that we must go through the entire list if we want to search for an element.
Reading from file

```cpp
int main(int argc, char *argv[]) {
    if (argc < 2) {
        cout << "Usage: " << argv[0] << " <filename> " << endl;
        exit(-1);
    }
    ifstream f(argv[1]);
    char s[50]; char a[100]; char t[20];

    while (!f.eof()) {
        f >> s;
        if (f.eof()) break;
        f.getline(a, 99);
        f.getline(t, 19);
        Entry e(s, a, t);
        abook.insert(e);
    }
    abook.printall();
}
```
bool quit = false;

while (!quit) {
    cout << "Insert Name to search: ";
    cin >> s;
    if (strcmp(s, "quit") == 0) break;
    else {
        Entry e = abook.search(s);
        cout << "Result: " << endl;
        e.print();
    }
}
Improving the data structures

We have two problems here:

- Fixed size: we can allow only 100 entries. It would be better to dynamically change the size of the array depending on the needs of the program.
- Searching takes linear time with the number of entries. Can we do better than that?

Let’s first solve the second problem
Improving search time

- The idea is to sort the array first
- Then, start looking in the middle
  - If we have found the entry, finish with success
  - If the entry is “greater” than the one we look for, continue looking in the first half
  - If the entry is “less” than the one we look for, continue looking in the second half
- This is a recursive algorithm!
- Exercise:
  - Implement a `sort()` function for the AddressBook class
  - modify the previous “search()” function to implement the algorithm described above (hint: may need an intermediate function)
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One important data structure is the linked list.

The nice and important property of a list is the possibility to insert elements at any point without requiring any complex operation.
Ordered Insertion

- **Problem:** suppose we have an ordered array of integers, from smallest to largest
- Suppose that we need to insert another number, and that after insertion the array must still be ordered
  - **Solution 1:** Insert at the end, then run a sorting algorithm (i.e. insert sort or bubble sort)
  - **Solution 2:** Identify where the number has to be inserted, and move all successive numbers one position forth
- Both solutions require additional effort to maintain the data structured ordered
- Another solution is to have completely different data structure
A list is a chain of linked elements

Every element of the list contains the data (in this case an integer), and a pointer to the following element in the list.
List of Addresses

- We now see how we can use a list to implement an address book
- First of all we define a list element

```cpp
#include "address.hpp"

class ListEntry {
    Entry entry;
    ListEntry *next;
public:
    ListEntry(Entry e);
    void link(ListEntry *next);
    Entry get_data();
    ListEntry *get_next();
};
```

- From `address.hpp`, we reuse the Entry class
List definition

Now the class AddressList class

list.hpp

class AddressList {
    ListEntry *head;
public:
    AddressList();
    void insert(Entry e);
    Entry search(char *s);
    void printall();
};

Notice how similar is the interface with AddressBook
Implementation of ListEntry

list.cpp

ListEntry::ListEntry(Entry e): entry(e), next(0)
{
}

void ListEntry::link(ListEntry * n)
{
    next = n;
}

Entry ListEntry::get_data()
{
    return entry;
}

ListEntry * ListEntry::get_next()
{
    return next;
}
Implementation of AddressList

- The `insert()` operation requires to go through the list until we find the correct position

```cpp
AddressList::AddressList() : head(0) {}

void AddressList::insert(Entry e) {
    ListEntry *le = new ListEntry(e);
    ListEntry *p = head;
    ListEntry *q = 0;
    while (p != 0) {
        if (strcmp(p->get_data().get_name(), e.get_name()) > 0) {
            q = p;
            p = p->get_next();
        } else break;
    }
    if (q == 0) // Insertion at the head
        head = le;
    else q->link(le);
    le->link(p);
}
```
Implementation of AddressList

- Searching and printing

```
list.cpp

Entry AddressList::search(char *s)
{
    ListEntry *p = head;
    Entry null_entry;
    while (p != 0) {
        if (strcmp(p->get_data().get_name(), s) == 0)
            return p->get_data();
        else
            p = p->get_next();
    }
    return null_entry;
}

void AddressList::printall()
{
    ListEntry *p=head;
    while (p != 0) {
        p->get_data().print();
        p=p->get_next();
    }
}
```
Main

Almost the same as in AddressBook, except for the type of the variable abook, and the includes.

```cpp
#include "list.hpp"

using namespace std;

AddressList abook;

bool quit = false;
while (!quit) {
    cout << "Insert Name to search: ";
    cin >> s;
    if (strcmp(s, "quit") == 0) break;
    else {
        Entry e = abook.search(s);
        cout << "Result: 
" << endl;
        e.print();
    }
}
```
Problems with lists

- One of the problems with the list is that searching is a $O(n)$ operation
  - while the previous algorithm on the array was $O(\log(n))$
- The list is useful if we frequently insert and extract from the head
  - For example, inside an operating system, the list of processes (executing programs) may be implemented as a list ordered by process priority
  - In general, when most of the operations are inserting/extracting from the head, the list is the simplest and most effective solution
Data structures so far

- **Stack**
  - Insertion/extraction only at/from the top (LIFO)
  - All operations are $O(1)$

- **Queue** (Circular Array)
  - Insertion at tail, extraction from head (FIFO)
  - All operations are $O(1)$

- **Array** (random access)
  - Insertion at any point requires $O(n)$
  - Extraction from any point requires $O(n)$
  - Sorting requires $O(n \log(n))$
  - Searching (in sorted array) requires $O(\log(n))$

- **List** (ordered)
  - Insertion at any point requires $O(n)$
  - Extraction from any point requires $O(1)$
  - Searching requires $O(n)$
More powerful data structures

- No data structure so far allows:
  - Insertion in $O(\log(n))$
  - Searching in $O(\log(n))$

- It is important to implement efficiently such data structures, because in most applications you exactly need to search the data structure very efficiently, and insert/remove efficiently.

- On such data structure is the balanced binary tree
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Trees

- A tree is a data structure where each element can have two *children*.
- The parent element can be the *child* of another higher level element.
- The topmost element is called *root*.

```
  8
 / \
5   10
|    |
2 6 9 12
```
The tree is a *recursive* data structure

- The root node has two *subtrees*, one on the left and one on the right
- Each node can be seen has root of its own subtree

**Recursive definition**: a tree can be

- empty (i.e. contains no nodes)
- consisting of one *root* node, plus one left tree and one right tree

- The tree is defined by itself!
Searching in a tree

- Given a node that contains element $k$, the main idea is:
  - to put all elements that are less than $k$ to the left
  - to put all elements that are greater than $k$ to the right
- If the tree is balanced (i.e. it has approximately the same number of nodes in the left and in the right subtrees), searching takes $O(\log(n))$
- Also, insertion takes $O(\log(n))$
  - However, inserting elements make the tree unbalanced
Example of tree

In the following figure we have a tree of integers

Binary Search Tree Example

Tree resulting from the following insertions: 38, 13, 51, 10, 12, 40, 84, 25, 89, 37, 66, 95
Here is an example of class that implements a simple tree

```cpp
class AddressTree {
public:
    AddressTree();
    void insert(Entry e);
    Entry search(char *s);
    void print_all();
    void print_structure();
private:
    TreeEntry *root;

    TreeEntry * _insert(TreeEntry *r, Entry e);
    Entry _search(TreeEntry *r, char *s);
    int _get_level(TreeEntry *r);
    void _print_all(TreeEntry *r);
    void _print_level(TreeEntry *r, int l, int n);
};
```
The functions insert and search call the internal recursive versions

```cpp
AddressTree::AddressTree() : root(0)
{
}

void AddressTree::insert(Entry e)
{
    root = _insert(root, e);
}

Entry AddressTree::search(char *s)
{
    return _search(root, s);
}
```
Tree searching

- Simply looks in the current node, in the left one or in the right one

```cpp
Entry AddressTree::_search(TreeEntry *r, char *s)
{
    Entry null_entry;
    if (r == 0) return null_entry;
    else if (strcmp(r->get_data().get_name(), s) == 0)
        return r->get_data();
    else if (strcmp(r->get_data().get_name(), s) < 0)
        return _search(r->get_left(), s);
    else if (strcmp(r->get_data().get_name(), s) > 0)
        return _search(r->get_right(), s);
    else return null_entry;
}
```
Tree insertion

● Interts to the right or to the left, depending on the ordering

simpletree.cpp

```cpp
TreeEntry *AddressTree::_insert(TreeEntry *r, Entry e)
{
    if (r == 0)
        r = new TreeEntry(e);
    else if (strcmp(r->get_data().get_name(), e.get_name()) < 0)
        r->link_left(_insert(r->get_left(), e));
    else if (strcmp(r->get_data().get_name(), e.get_name()) > 0)
        r->link_right(_insert(r->get_right(), e));
    else if (strcmp(r->get_data().get_name(), e.get_name()) == 0)
        cout << "Element already present" << endl;
    return r;
}
```
The main

The same as before

maintree.cpp

AddressTree abook;

int main(int argc, char *argv[])
{
    if (argc < 2) {
        cout << "Usage: " << argv[0] << " <filename> " << endl;
        exit(-1);
    }
    ifstream f(argv[1]);
    char s[50]; char a[100]; char t[20];

    while (!f.eof()) {
        f >> s;
        if (f.eof()) break;
        f.getline(a, 99);
        f.getline(t, 19);
        Entry e(s, a, t);
        abook.insert(e);
    }
    abook.print_all();
    abook.print_structure();

    bool quit = false;
Unfortunately, the tree is not balanced
   (see output of maintree on example2.txt)
This means that the insertion and search operation do not necessarily take $O(\log(n))$
   It is necessary to constantly keep the tree balanced to achieve good performance
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The *height* of a tree is how many pointers I have to follow in the worst case before reaching a leaf.

It can be defined recursively:
- The height of an empty tree is 0.
- The height of a tree is equal to the maximum between the heights of the left and right subtrees plus 1.

Example: what is the height of this subtree?

Binary Search Tree Example

Tree resulting from the following insertions: 38, 13, 51, 10, 12, 40, 84, 25, 89, 37, 66, 95
Balance

- The difference between the height of the left subtree and the height of the right subtree is called *balance*.
- A tree is said to be *balanced* if
  - the balance is -1, 0 or 1
  - Both the left and the right subtrees are balanced
- (again a recursive definition!)
- Is the tree in the previous slide balanced?
- What is the balance of the tree obtained by example2.txt?
Rotation

- When we insert a new element, the tree can become unbalanced
- Therefore, we have to re-balance it
- The operation that we use to balance the tree must preserve the ordering!
- The balance can be obtained by *rotating a tree*
  - A rotate operation charges the structure of the tree so that the tree becomes balanced after the operation, and the order is preserved
- There are many different implementations of the rotation operation, that produce different types of balanced tree
  - Red-black trees
  - AVL trees
  - etc.
- We will analyze the AVL tree
Left-left rotation

- Suppose the tree with root X is unbalanced to the left (i.e. balance = −2)
  - In this case, the height of the left subtree (with root Y) is larger than the height of the right subtree by 2 levels
- Also, suppose that the left subtree of Y (which has root Z) is higher than its right subtree
- We apply a left rotation:

```
            X
           / 
          Y   Z
         /  
        W   W
```
Left-left rotation

- What happened?
  - Before the rotation,
    - suppose that the right subtree of X had height $h$,
    - Y had height $h + 2$
    - Z had height $h + 1$
    - W had height $h$

- After the rotation, Y is the new root
  - X has height $h + 1$,
  - Z has height $h + 1$

- Also, notice that the order is preserved:
  - Before the rotation, $Z < Y < W < X$
  - After the rotation, $Z < Y < W < X$
Left-right

- A different case is when the left subtree has balance +1
- In such a case we need to perform a left-right rotation

Before the rotation,
- suppose that the right subtree of X had height $h$,
- Y had height $h + 2$
- Z had height $h + 1$
- W had height $h$

After the rotation, Y is the new root
- X has height $h + 1$,
- Z has height $h + 1$

The order is still preserved
Rotations

There are 4 possible rotations

- **left-left**: when the tree is unbalanced to the left and the left subtree has balance -1
- **left-right**: when the tree is unbalanced to the left, and the left subtree has balance +1
- **right-left**: when the tree is unbalanced to the right, and the right subtree has balance -1
- **right-left**: when the tree is unbalanced to the right, and the right subtree has balance +1
Rotations

Figure: left-left

Figure: right-right

Figure: left-right

Figure: right-left
Now we look at the implementation

```cpp
class AddressTree {
public:
    AddressTree();
    void insert(Entry e);
    Entry search(char *s);
    void print_all();
    void print_structure();
private:
    TreeEntry *root;

    TreeEntry * _insert(TreeEntry *r, Entry e);
    Entry _search(TreeEntry *r, char *s);
    int _get_level(TreeEntry *r);
    void _print_all(TreeEntry *r);
    void _print_level(TreeEntry *r, int l, int n);

    TreeEntry * _rotate_ll(TreeEntry *r);
    TreeEntry * _rotate_lr(TreeEntry *r);
    TreeEntry * _rotate_rl(TreeEntry *r);
    TreeEntry * _rotate_rr(TreeEntry *r);
};
```
Rotations (right)

avltree.cpp

```cpp
TreeEntry * AddressTree::_rotate_rr(TreeEntry * x)
{
    TreeEntry * y = x->get_right();

    x->link_right(y->get_left());
    y->link_left(x);

    return y;
}

TreeEntry * AddressTree::_rotate_rl(TreeEntry * x)
{
    TreeEntry * y = x->get_right();
    TreeEntry * z = y->get_left();

    x->link_right(z->get_left());
    y->link_left(z->get_right());
    z->link_left(x);
    z->link_right(y);

    return z;
}
```
Rotations (left)

avltree.cpp

TreeEntry * AddressTree::_rotate_ll(TreeEntry * x)
{
    TreeEntry * y = x->get_left();

    x->link_left(y->get_right());
    y->link_right(x);

    return y;
}

TreeEntry * AddressTree::_rotate_lr(TreeEntry * x)
{
    TreeEntry * y = x->get_left();
    TreeEntry * z = y->get_right();

    x->link_left(z->get_right());
    y->link_right(z->get_left());
    z->link_right(x);
    z->link_left(y);

    return z;
}
The following function returns the tree level:

```cpp
int AddressTree::_get_level(TreeEntry * r)
{
    if (r == 0) return 0;
    else return (1 + max(_get_level(r->get_left()),
                         _get_level(r->get_right())));
}
```

The search remains the same

Now we look at the insert
Insertion to the left

avltree.cpp

TreeEntry *AddressTree::_insert(TreeEntry *r, Entry e) {
    if (r == 0)
        r = new TreeEntry(e);
    else if (strcmp(r->get_data().get_name(), e.get_name()) < 0) {
        // insert
        r->link_left(_insert(r->get_left(), e));

    // check balance since I inserted to the left, it can be
    // balanced, or in LL or in LR
    int ll = _get_level(r->get_left());
    int rl = _get_level(r->get_right());
    if (ll > (rl + 1)) {
        int lll = _get_level(r->get_left()->get_left());
        int lrl = _get_level(r->get_left()->get_right());

        if (lll > lrl)
            r = _rotate_ll(r);
        else
            r = _rotate_lr(r);
    }
}
}
else if (strcmp(r->get_data().get_name(), e.get_name()) > 0) {
    r->link_right(_insert(r->get_right(), e));

    int ll = _get_level(r->get_left());
    int rl = _get_level(r->get_right());
    if (rl > (ll + 1)) {
        int rrl = _get_level(r->get_right()->get_right());
        int rll = _get_level(r->get_right()->get_left());
        if (rrl > rll) r = _rotate_rr(r);
        else r = _rotate_rl(r);
    } }

else if (strcmp(r->get_data().get_name(), e.get_name()) == 0)
    cout << "Element already present" << endl;

return r;
A complete example

- A complete example can be found in program `examples/maintree.cpp`

**Exercise:** modify the code to change the order in which the elements are stored

**Exercise:** Modify the code so that:

1. All elements are stored in an array (i.e. a AddressBook data structure), and only the pointers to the data elements are stored in the tree
2. Write a different kind of tree that sorts elements by address.
3. In this way, you will have the same data structure ordered by name and by address at the same time
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Heap

- An heap is a data structure that is used mainly for implementing priority queues
- A heap is a binary tree in which, for each node A, the value stored in the node is always greater than the values stored in the children
- The data structure is also called max-heap (or min-heap if we require that the node be less than its children)

**Figure:** Example of max-heap
Properties

- Another property of max-heap is the fact that the heap is “full” in all its levels except maybe the last one.
- Also, on the last level, all nodes are present from left to right without holes.

Figure: All nodes are full from left to right.
Operations

- The most important operations you can do on a heap are:
  - Insert an element in a ordered fashion
  - Read the top element
  - Extract the top element

- An heap is used mainly for sorted data structures in which you need to quickly know the maximum element
To insert an element, we proceed in two steps:

- First the element is inserted in the first free position in the tree.
- Then, by using a procedure called heapify, the node is moved to its correct position by swapping elements.

Suppose we want to insert element 15 in the heap below:

**Insertion**

**Step 1**

**Step 2**

**Step 3**
Deleting

- For deleting an element, we proceed in a similar way
  - We first remove the top most element, and we substitute it with the last element in the heap
  - Then, we move down the element to its correct position by a sequence of swaps

- Suppose that we remove the top element in the heap below. We substitute it with the last element (4)
Heap implementation

- The heap can be efficiently implemented with an array
- The root node is stored at index 0 of the array
- Given a node at index $i$:
  - its left child can be stored at $2i + 1$
  - its right child can be stored at $2i + 2$
  - the parent of node $j$ is at $\left\lfloor \frac{j-1}{2} \right\rfloor$

Figure: Efficiently storing a heap in an array