Introduction to the C programming language

Lists and Trees

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March 10, 2010

Outline

Searching

- Suppose we have an address list.
  - For each person name, we have the address and the telephone number.
  - All entries are stored in an array.

Class Entry

- The following class represents an entry

address.hpp

```
class Entry {
    char name[50];
    char address[100];
    char telephone[20];
public:
    Entry();
    Entry(char *s, char *a, char *t);
    char *get_name();
    char *get_address();
    char *get_telephone();
    void print();
};
```

Class AddressBook

- The following class represents an address book with maximum 100 entries

address.hpp

```
class AddressBook {
    Entry array[100];
    int num;
public:
    AddressBook();
    void insert(Entry e);
    Entry search(char *name);
    void printall();
};
```

Implementation of Entry

address.cpp

```
Entry::Entry()
{
    strcpy(name, "");
    strcpy(address, "");
    strcpy(telephone, "");
}

Entry::Entry(char *s, char *a, char *t)
{
    strncpy(name, s, 50);
    strncpy(address, a, 100);
    strncpy(telephone, t, 20);
}
```

Implementation of AddressBook

address.cpp

```
AddressBook::AddressBook() : num(0)
{}

void AddressBook::insert(Entry e)
{
    array[num++] = e;
}

Entry AddressBook::search(char *name)
{
    int i;
    Entry null_entry;
    for (i=0; i<num; i++) {
        if (strcmp(name, array[i].get_name()) == 0)
            return array[i];
    }
    return null_entry;
}
```

- Notice that we must go through the entire list if we want to search for an element

Main

- Reading from file

main1.cpp

```
int main(int argc, char *argv[])
{
    if (argc < 2) {
        cout << "Usage: " << argv[0] << " <filename> " << endl;
        exit(-1);
    }
    ifstream f(argv[1]);
    char s[50]; char a[100]; char t[20];

    while (!f.eof()) {
        f >> s;
        if (f.eof()) break;
        f.getline(a, 99);
        f.getline(t, 19);
        Entry e(s, a, t);
        abook.insert(e);
    }
    abook.printall();
```

Main - II

- Searching names:

main1.cpp

```
    bool quit = false;
    while (!quit) {
        cout << "Insert Name to search: ";
        cin >> s;
        if (strcmp(s, "quit") == 0) break;
        else {
            Entry e = abook.search(s);
            cout << "Result: " << endl;
            e.print();
        }
    }
}
```

Improving the data structures

- We have two problems here:
  - Fixed size: we can allow only 100 entries. It would be better to dynamically change the size of the array depending on the needs of the program
  - Searching takes linear time with the number of entries. Can we do better than that?
- Let's first solve the second problem

Improving search time

- The idea is to sort the array first
- Then, start looking in the middle
  - If we have found the entry, finish with success
  - If the entry is “greater” than the one we look for, continue looking in the first half
  - If the entry is “less” than the one we look for, continue looking in the second half
- This is a recursive algorithm!
- Exercise:
  - Implement a `sort()` function for the AddressBook class
  - modify the previous “search()” function to implement the algorithm described above (hint: may need an intermediate function)

Lists

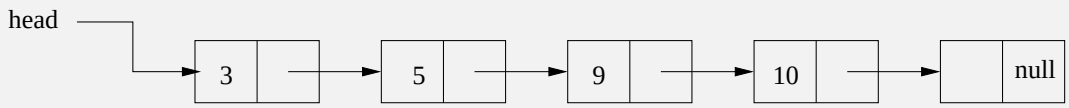
- One important data structure is the linked list
- The nice and important property of a list is the possibility to insert elements at any point without requiring any complex operation

Ordered Insertion

- **Problem:** suppose we have an ordered array of integers, from smalles to largest
- Suppose that we need to insert another number, and that after insertion the array must still be ordered
  - **Solution 1:** Insert at the end, then run a sorting algorithm (i.e. insert sort or bubble sort)
  - **Solution 2:** Identify where the number has to be inserted, and move all successive numbers one position forth
- Both solutions require additional effort to maintain the data structured ordered
- Another solution is to have completely different data structure

Lists

- A list is a chain of linked elements



- Every element of the list contains the data (in this case an integer), and a pointer to the following element in the list

List of Addresses

- We now see how we can use a list to implement an address book
- First of all we define a list element

list.hpp

```
#include "address.hpp"

class ListEntry {
    Entry entry;
    ListEntry *next;
public:
    ListEntry(Entry e);
    void link(ListEntry *next);
    Entry get_data();
    ListEntry *get_next();
};
```

- From `address.hpp`, we reuse the Entry class

List definition

- Now the class AddressList class

list.hpp

```
class AddressList {
    ListEntry *head;
public:
    AddressList();
    void insert(Entry e);
    Entry search(char *s);
    void printall();
};
```

- Notice how similar is the interface with AddressBook

Implementation of ListEntry

list.cpp

```
ListEntry::ListEntry(Entry e): entry(e), next(0)
{}

void ListEntry::link(ListEntry *n)
{
    next = n;
}

Entry ListEntry::get_data()
{
    return entry;
}

ListEntry *ListEntry::get_next()
{
    return next;
}
```

Implementation of AddressList

- The `insert()` operation requires to go through the list until we find the correct position

list.cpp

```
AddressList::AddressList() : head(0)
{}

void AddressList::insert(Entry e)
{
    ListEntry *le = new ListEntry(e);
    ListEntry *p = head;
    ListEntry *q = 0;
    while (p != 0) {
        if (strcmp(p->get_data().get_name(), e.get_name()) > 0) {
            q = p;
            p = p->get_next();
        }
        else break;
    }
    if (q == 0)  // Insertion at the head
        head = le;
    else q->link(le);
    le->link(p);
}
```

Implementation of AddressList

- Searching and printing

list.cpp

```
Entry AddressList::search(char *s)
{
    ListEntry *p = head;
    Entry null_entry;
    while (p != 0) {
        if (strcmp(p->get_data().get_name(), s) == 0)
            return p->get_data();
        else p = p->get_next();
    }
    return null_entry;
}

void AddressList::printall()
{
    ListEntry *p=head;
    while (p != 0) {
        p->get_data().print();
        p=p->get_next();
    }
}
```

Main

- Almost the same as in AddressBook, except for the type of the variable abook, and the includes.

main2.cpp

```
#include "list.hpp"

using namespace std;

AddressList abook;
```

main2.cpp

```
    bool quit = false;
    while (!quit) {
        cout << "Insert Name to search: ";
        cin >> s;
        if (strcmp(s, "quit") == 0) break;
        else {
            Entry e = abook.search(s);
            cout << "Result: " << endl;
            e.print();
        }
    }
```

Problems with lists

- One of the problems with the list is that searching is a O(n) operation
  - while the previous algorithm on the array was O(log(n))
- The list is useful if we frequently insert and extract from the head
  - For example, inside an operating system, the list of processes (executing programs) may be implemented as a list ordered by process priority
  - In general, when most of the operations are inserting/estracting from the headm the list is the simplest and most effective solution

Data structures so far

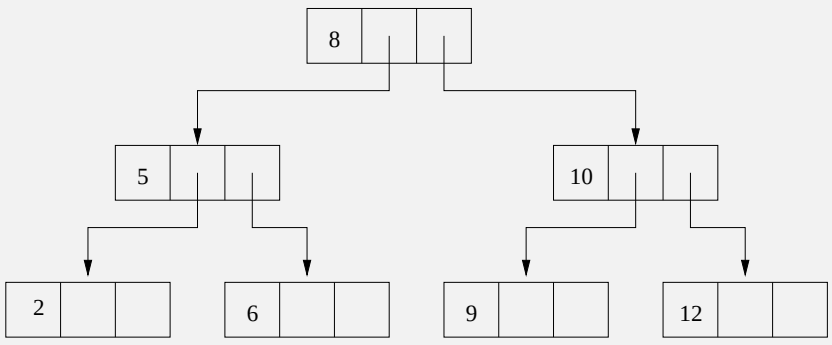
- **Stack**
  - Insertion/extraction only at/from the top (LIFO)
  - All operations are *O(1)*
- **Queue** (Circular Array)
  - Insertion at tail, extraction from head (FIFO)
  - All operations are *O(1)*
- **Array** (random access)
  - Insertion at any point requires *O(n)*
  - Extraction from any point requires *O(n)*
  - Sorting requires *O(n log(n))*
  - Searching (in sorted array) requires *O(log(n))*
- **List** (ordered)
  - Insertion at any point requires *O(n)*
  - Extraction from any point requires *O(1)*
  - Searching requires *O(n)*

More powerful data structures

- No data structure so far allows:
  - Insertion in *O(log(n))*
  - Searching in *O(log(n))*
- It is important to implement efficienlty such data structures, because in most application you exactly need to seach the data structure very efficiently, and insert/remove efficiently
- On such data structure is the balanced binary tree

Trees

- A tree is a data structure where each element can have two *children*
- The parent element can be the *child* of another higher level element
- The topmost element is called *root*



Recursion

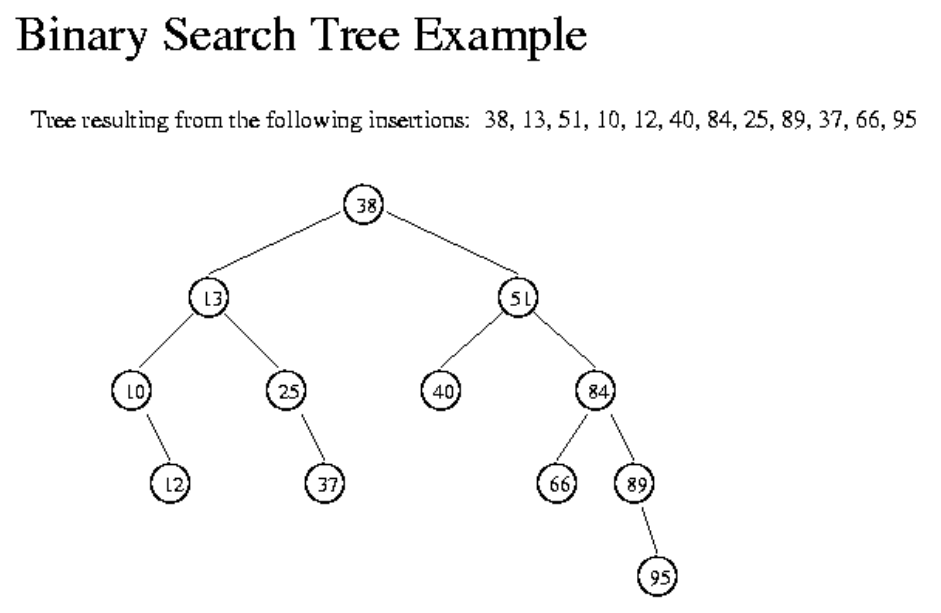
- The tree is a *recursive* data structure
  - The root node has two *subtrees*, one on the left and one on the right
  - Each node can be seen has root of its own subtree
- **Recursive definition**: a tree can be
  - empty (i.e. contains no nodes)
  - consisting of one *root* node, plus one left tree and one right tree
- The tree is defined by itself!

Searching in a tree

- Given a node that contains element *k*, the main idea is:
  - to put all elements that are *less than k* to the left
  - to put all elements that are *greater than k* to the right
- If the tree is balanced (i.e. it has approximately the same number of nodes in the left and in the right subtrees), searching takes O(log(n))
- Also, insertion takes O(log(n))
  - However, inserting elements make the tree *unbalanced*

Example of tree

- In the following figure we have a tree of integers



Tree interface

- Here is an example of class that implements a simple tree

simpletree.hpp

```
class AddressTree {
public:
    AddressTree();
    void insert(Entry e);
    Entry search(char *s);
    void print_all();
    void print_structure();
private:
    TreeEntry *root;

    TreeEntry * _insert(TreeEntry *r, Entry e);
    Entry _search(TreeEntry *r, char *s);
    int _get_level(TreeEntry *r);
    void _print_all(TreeEntry *r);
    void _print_level(TreeEntry *r, int l, int n);
};
```

Tree implementation - 1

- The functions insert and search call the internal recursive versions

simpletree.cpp

```
AddressTree::AddressTree() : root(0)
{}

void AddressTree::insert(Entry e)
{
    root = _insert(root, e);
}

Entry AddressTree::search(char *s)
{
    return _search(root, s);
}
```

Tree searching

- Simply looks in the current node, in the left one or in the right one

simpletree.cpp

```
Entry AddressTree::_search(TreeEntry *r, char *s)
{
    Entry null_entry;
    if (r == 0) return null_entry;
    else if (strcmp(r->get_data().get_name(), s) == 0)
        return r->get_data();
    else if (strcmp(r->get_data().get_name(), s) < 0)
        return _search(r->get_left(), s);
    else if (strcmp(r->get_data().get_name(), s) > 0)
        return _search(r->get_right(), s);
    else return null_entry;
}
```

Tree insertion

- Interts to the right or to the left, depending on the ordering

simpletree.cpp

```
TreeEntry *AddressTree::_insert(TreeEntry *r, Entry e)
{
    if (r == 0)
        r = new TreeEntry(e);
    else if (strcmp(r->get_data().get_name(), e.get_name()) < 0)
        r->link_left(_insert(r->get_left(), e));
    else if (strcmp(r->get_data().get_name(), e.get_name()) > 0)
        r->link_right(_insert(r->get_right(), e));
    else if (strcmp(r->get_data().get_name(), e.get_name()) == 0)
        cout << "Element already present" << endl;
    return r;
}
```

The main

- The same as before

maintree.cpp

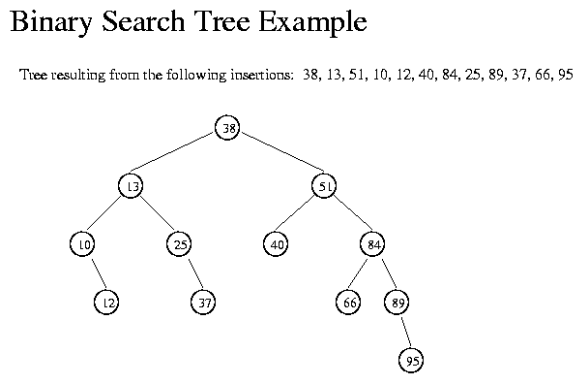
```
AddressTree abook;

int main(int argc, char *argv[])
{
    if (argc < 2) {
        cout << "Usage: " << argv[0] << " <filename> " << endl;
        exit(-1);
    }
    ifstream f(argv[1]);
    char s[50]; char a[100]; char t[20];

    while (!f.eof()) {
        f >> s;
        if (f.eof()) break;
        f.getline(a, 99);
        f.getline(t, 19);
        Entry e(s, a, t);
        abook.insert(e);
    }
    abook.print_all();

    abook.print_structure();

    bool quit = false;
```

Balance

- Unfortunately, the tree is not balances
  - (see output of maintree on example2.txt)
- This means that the insertion and search operation do not necessarily take O(log(n))
  - It is necessary to constantly keep the tree balanced to achieve good performance

Height

- The *height* of a tree is how may pointers I have to follow in the worst case before reaching a leaves
- It can be defined recursively;
  - The height of an empty tree is 0
  - The height of a tree is equal to the maximum between the heights of the left and right subtrees plus 1
- Example: what is the height of this subtree?



Balance

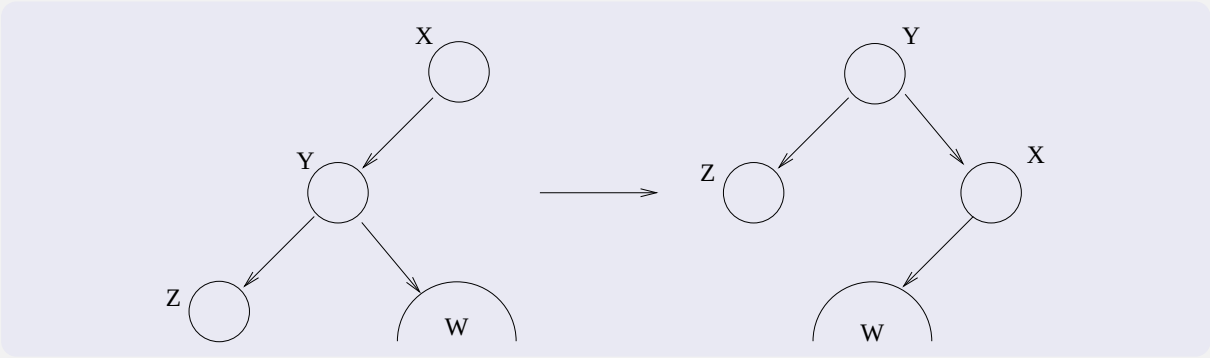
- The difference between the height of the left subtree and the height of the right subtree is called *balance*.
- A tree is said to be *balanced* if
  - the balance is -1, 0 or 1
  - Both the left and the right subtrees are balanced
- (again a recursive definition!)
- Is the tree in the previous slide balanced?
- What is the balance of the tree obtained by example2.txt?

Rotation

- When we insert a new element, the tree can become unbalanced
- Therefore, we have to re-balance it
- The operation that we use to balance the tree must preserve the ordering!
- The balance can be obtained by *rotating a tree*
  - A rotate operation charges the structure of the tree so that the tree becomes balanced after the operation, and the order is preserved
- There are many different implementation of the rotation operation, that produce different types of balanced tree
  - Red-black trees
  - AVL trees
  - etc.
- We will analyze the AVL tree

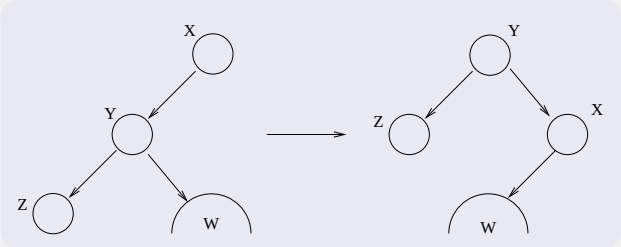
Left-left rotation

- Suppose the tree with root X is unbalanced to the left (i.e. balance $= -2$)
  - In this case, the height of the left subtree (with root Y) is larger than the height of the right subtree by 2 levels
- Also, suppose that the left subtree of Y (which has root Z) is higher than its right subtree
- We apply a *left rotation*:



Left-left rotation

- What happened?
  - Before the rotation,
    - suppose that the right subtree of X had height h,
    - Y had height $h + 2$
    - Z had height $h + 1$
    - W had height h

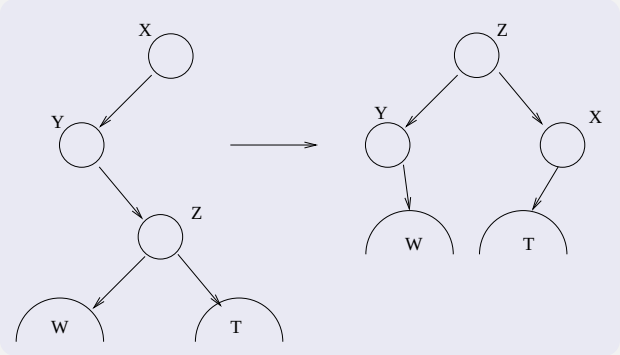


  - After the rotation, Y is the new root
    - X has height $h + 1$,
    - Z has height $h + 1$
  - Also, notice that the order is preserved:
    - Before the rotation, $Z < Y < W < X$
    - After the rotation, $Z < Y < W < X$

Left-right

- A different case is when the left subtree has balance +1
- In such a case we need to perform a left-right rotation

- Before the rotation,
  - suppose that the right subtree of X had height h,
  - Y had height $h + 2$
  - Z had height $h + 1$
  - W had height h



  - After the rotation, Y is the new root
    - X has height $h + 1$,
    - Z has height $h + 1$
  - The order is still preserved

Rotations

- There are 4 possible rotations
  - **left-left**: when the tree is unbalanced to the left and the left subtree has balance -1
  - **left-right**: when the tree is unbalanced to the left, and the left subtree has balance +1
  - **right-left**: when the tree is unbalanced to the right, and the right subtree has balance -1
  - **right-left**: when the tree is unbalanced to the right, and the right subtree has balance +1

Rotations

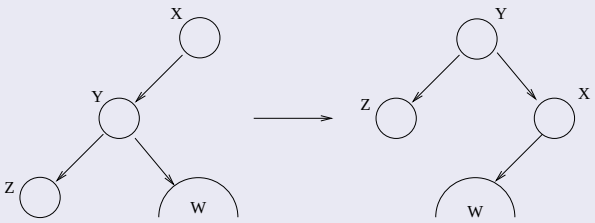


Figure: left-left

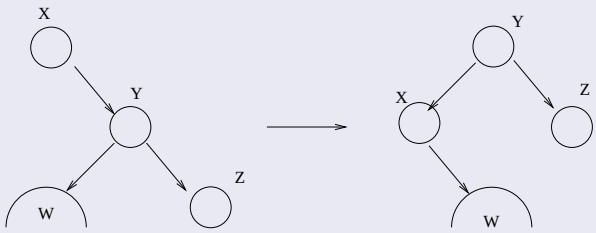


Figure: right-right

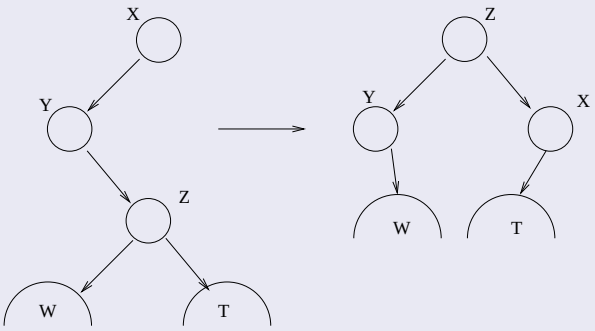


Figure: left-right

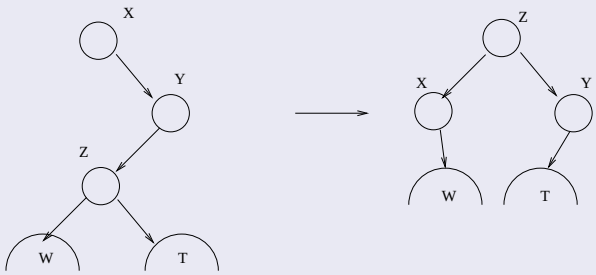


Figure: right-left

Implementation

- Now we look at the implementation

avltree.hpp

```
class AddressTree {
public:
    AddressTree();
    void insert(Entry e);
    Entry search(char *s);
    void print_all();
    void print_structure();
private:
    TreeEntry *root;

    TreeEntry * _insert(TreeEntry *r, Entry e);
    Entry _search(TreeEntry *r, char *s);
    int _get_level(TreeEntry *r);
    void _print_all(TreeEntry *r);
    void _print_level(TreeEntry *r, int l, int n);

    TreeEntry * _rotate_ll(TreeEntry *r);
    TreeEntry * _rotate_lr(TreeEntry *r);
    TreeEntry * _rotate_rl(TreeEntry *r);
    TreeEntry * _rotate_rr(TreeEntry *r);
};
```

Rotations (right)

avltree.cpp

```
TreeEntry * AddressTree::_rotate_rr(TreeEntry *x)
{
    TreeEntry *y = x->get_right();

    x->link_right(y->get_left());
    y->link_left(x);

    return y;
}

TreeEntry * AddressTree::_rotate_rl(TreeEntry *x)
{
    TreeEntry *y = x->get_right();
    TreeEntry *z = y->get_left();

    x->link_right(z->get_left());
    y->link_left(z->get_right());
    z->link_left(x);
    z->link_right(y);

    return z;
}
```

Rotations (left)

avltree.cpp

```
TreeEntry * AddressTree::_rotate_ll(TreeEntry *x)
{
    TreeEntry *y = x->get_left();

    x->link_left(y->get_right());
    y->link_right(x);

    return y;
}

TreeEntry * AddressTree::_rotate_lr(TreeEntry *x)
{
    TreeEntry *y = x->get_left();
    TreeEntry *z = y->get_right();

    x->link_left(z->get_right());
    y->link_right(z->get_left());
    z->link_right(x);
    z->link_left(y);

    return z;
}
```

Height

- The following function returns the tree level:

avltree.cpp

```
int AddressTree::_get_level(TreeEntry *r)
{
    if (r == 0) return 0;
    else return (1 + max(_get_level(r->get_left()),
                         _get_level(r->get_right())));
}
```

- The search remains the same
- Now we look at the insert

Insertion to the left

avltree.cpp

```
TreeEntry *AddressTree::_insert(TreeEntry *r, Entry e)
{
    if (r == 0)
        r = new TreeEntry(e);
    else if (strcmp(r->get_data().get_name(), e.get_name()) < 0) {
        // insert
        r->link_left(_insert(r->get_left(), e));

        // check balance since I inserted to the left, it can be
        // balanced, or in LL or in LR
        int ll = _get_level(r->get_left());
        int rl = _get_level(r->get_right());
        if (ll > (rl + 1)) {
            int lll = _get_level(r->get_left()->get_left());
            int lrl = _get_level(r->get_left()->get_right());

            if (lll > lrl)
                r = _rotate_ll(r);
            else r = _rotate_lr(r);
        }
    }
```

Insertion to the right

avltree.cpp

```
    else if (strcmp(r->get_data().get_name(), e.get_name()) > 0) {
        r->link_right(_insert(r->get_right(), e));

        int ll = _get_level(r->get_left());
        int rl = _get_level(r->get_right());
        if (rl > (ll + 1)) {
            int rrl = _get_level(r->get_right()->get_right());
            int rll = _get_level(r->get_right()->get_left());
            if (rrl > rll) r = _rotate_rr(r);
            else r = _rotate_rl(r);
        }
    }
    else if (strcmp(r->get_data().get_name(), e.get_name()) == 0)
        cout << "Element already present" << endl;

    return r;
}
```