Sistemi in tempo reale

Fixed Priority scheduling

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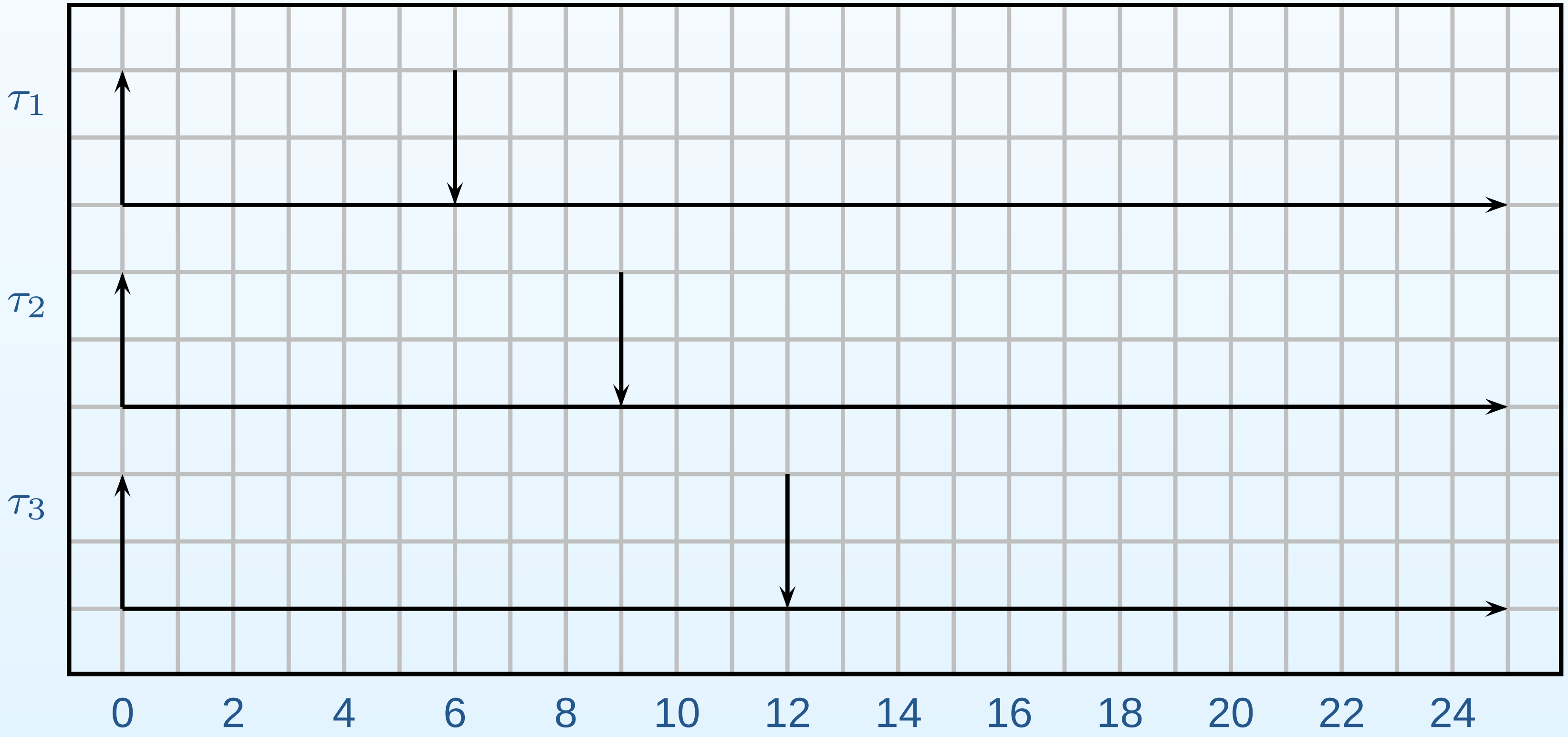
The fixed priority scheduling algorithm

- very simple scheduling algorithm;
  - every task τ_i is assigned a fixed priority p_i;
  - the active task with the highest priority is scheduled.
- Priorities are integer numbers: the higher the number, the higher the priority;
  - In the research literature, sometimes authors use the opposite convention: the lowest the number, the highest the priority.
- In the following we show some examples, considering periodic tasks, and constant execution time equal to the period.

Example of schedule

- Consider the following task set: $\tau_1 = (2, 6, 6)$, $\tau_2 = (2, 9, 9)$, $\tau_3 = (3, 12, 12)$. Task τ_1 has priority $p_1 = 3$ (highest), task τ_2 has priority $p_2 = 2$, task τ_3 has priority $p_3 = 1$ (lowest).



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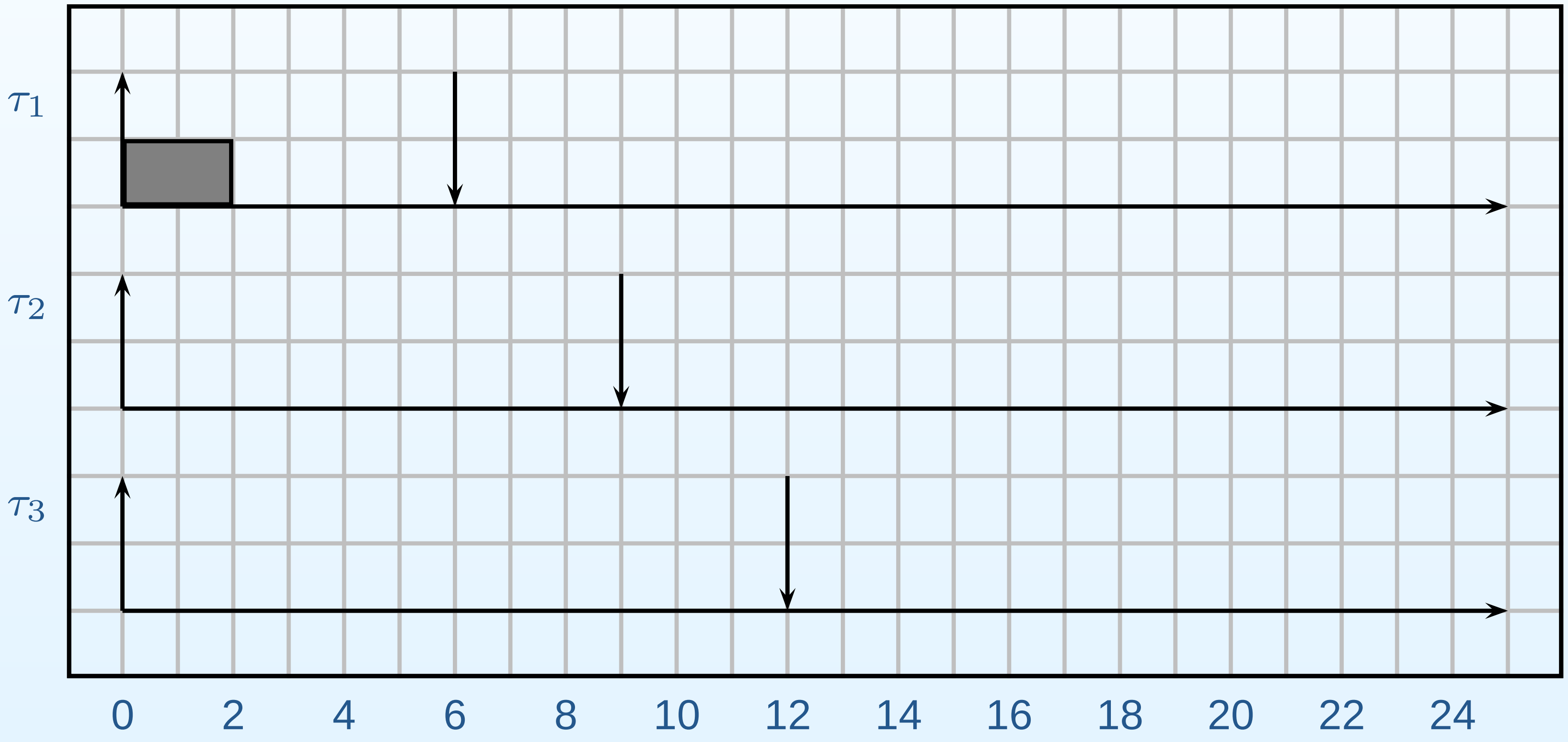

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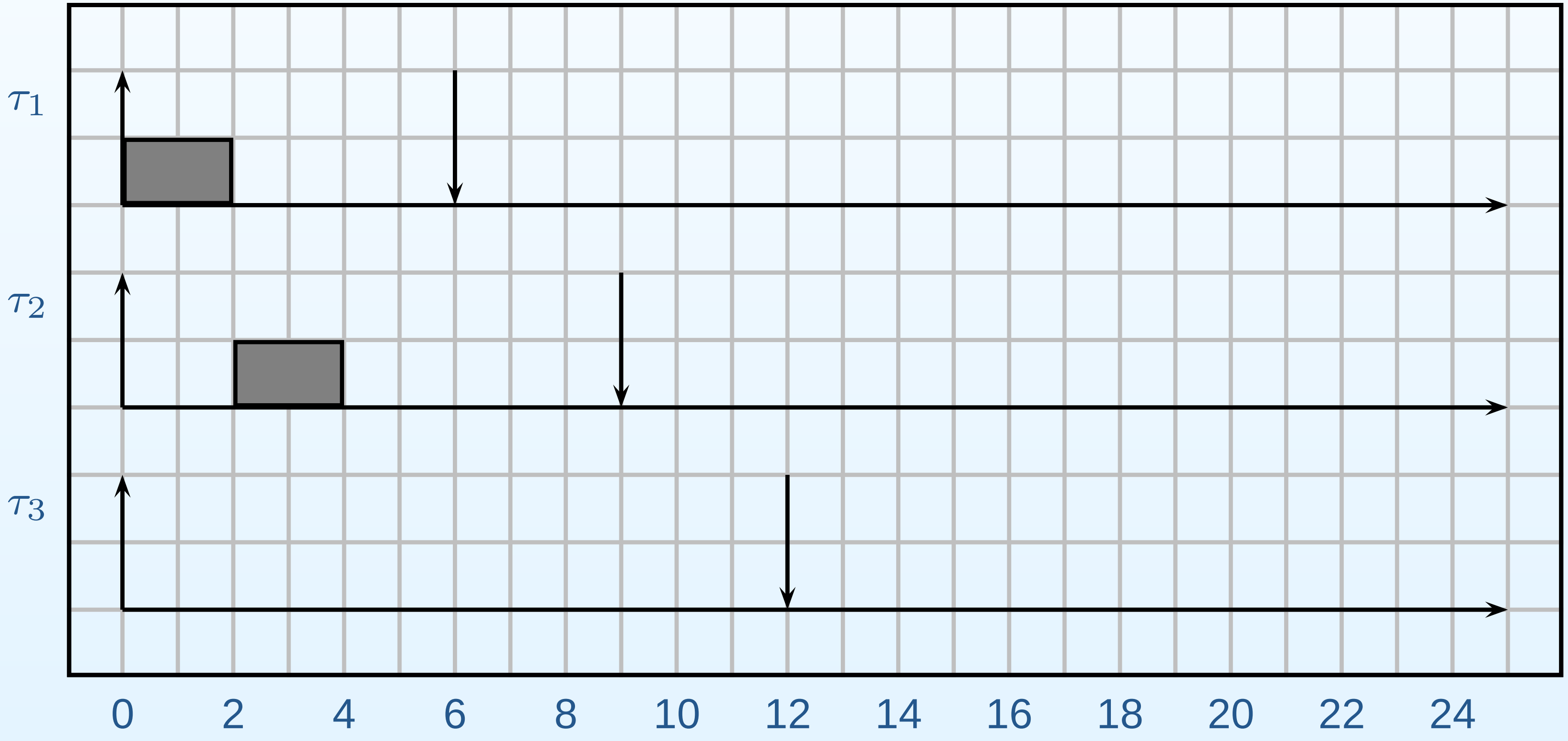

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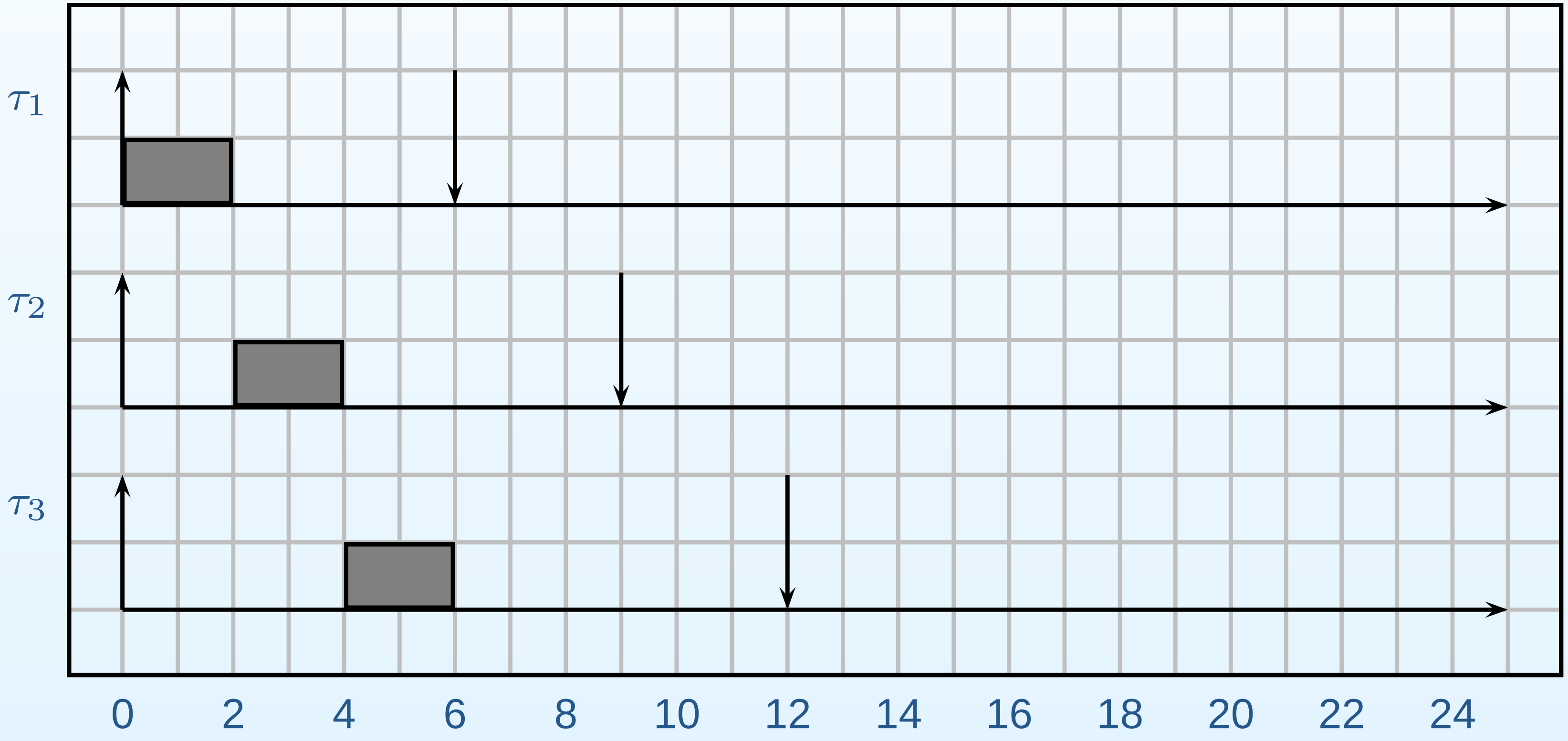

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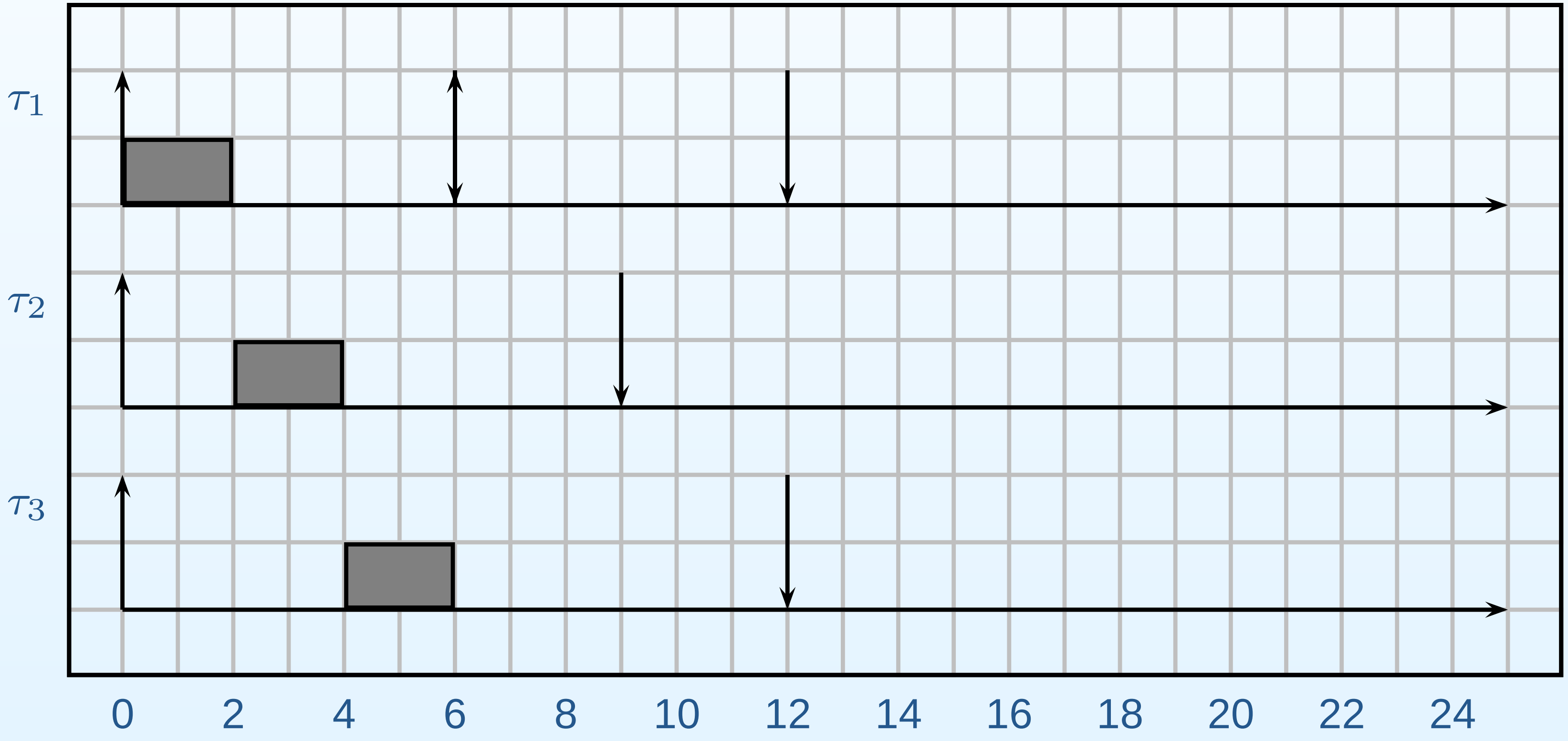

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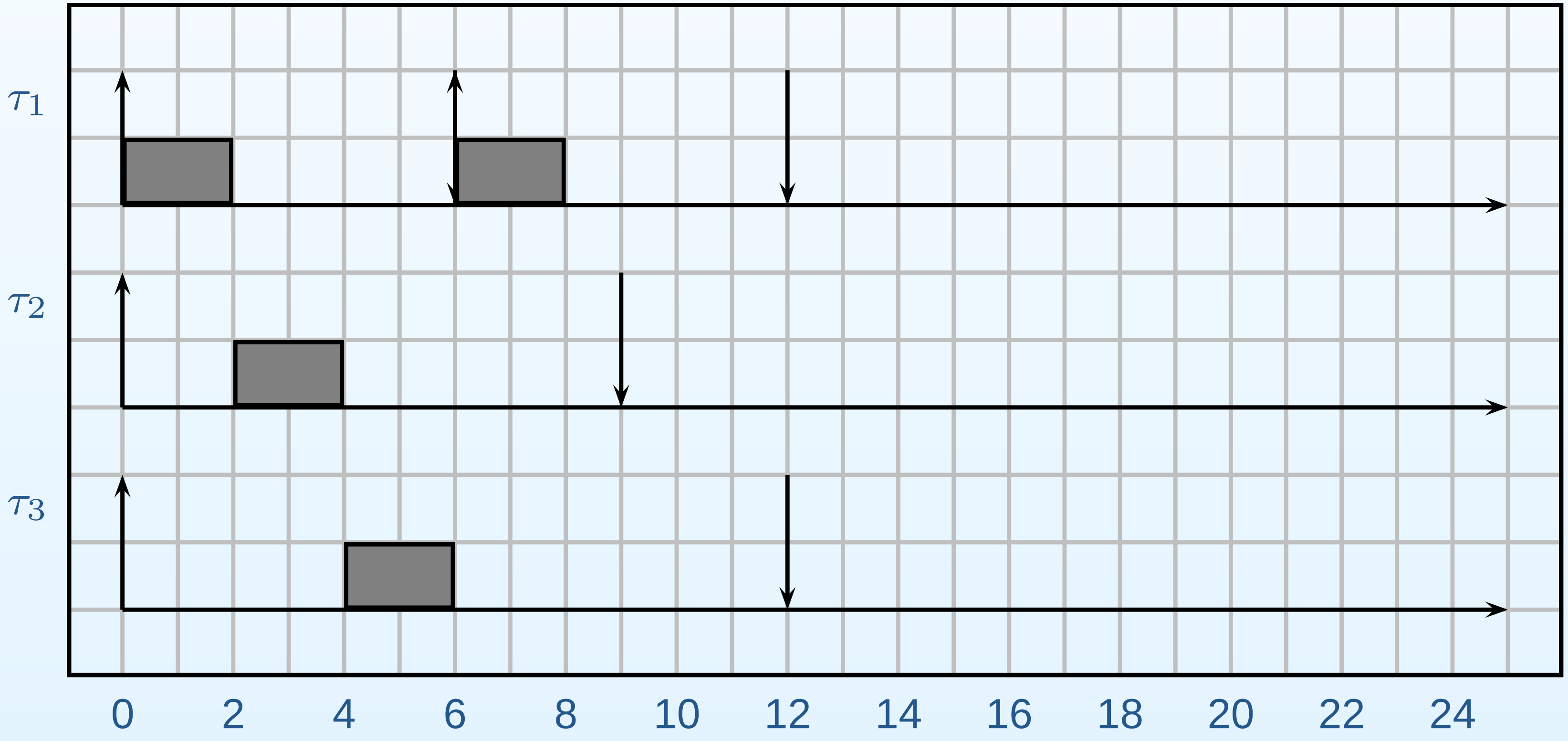

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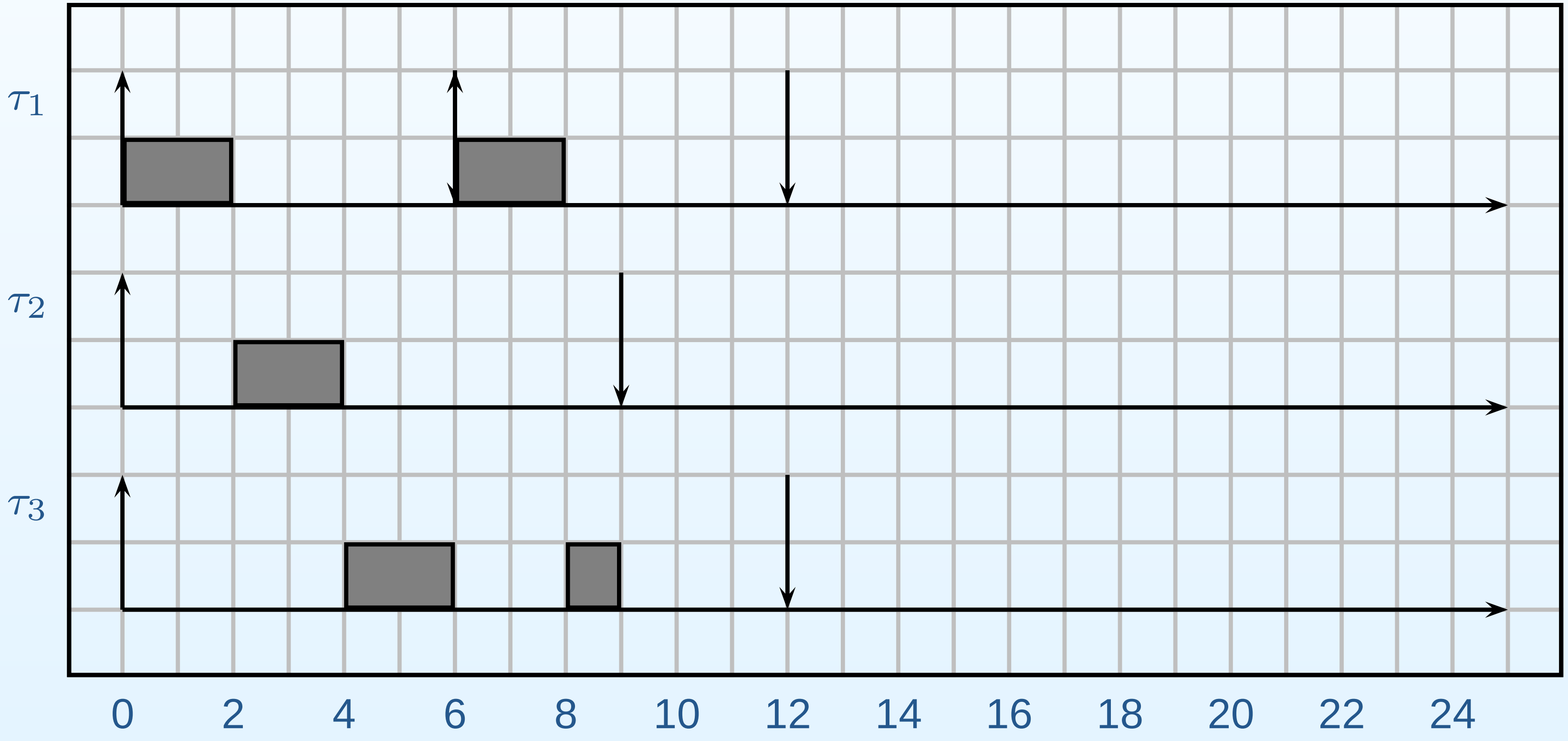

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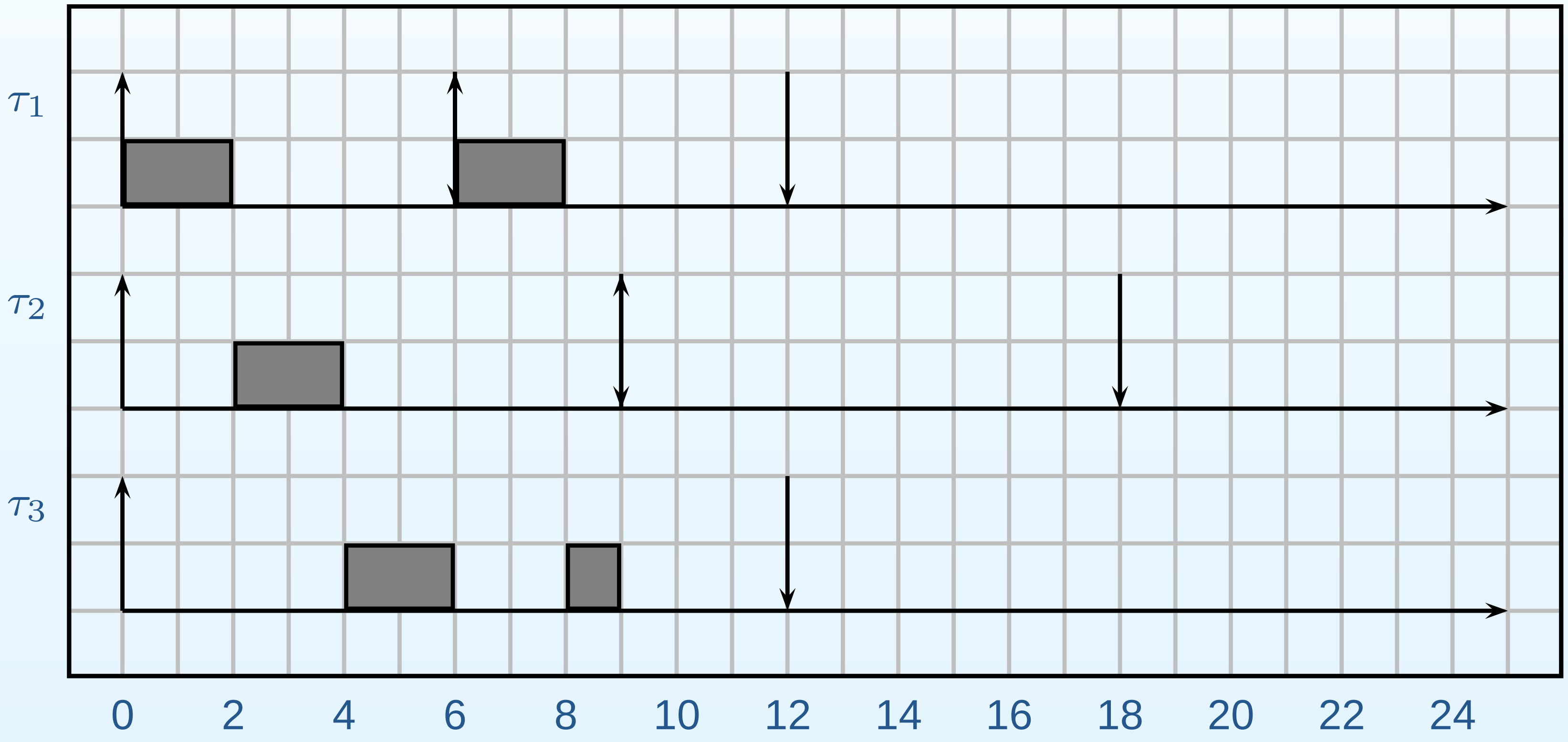

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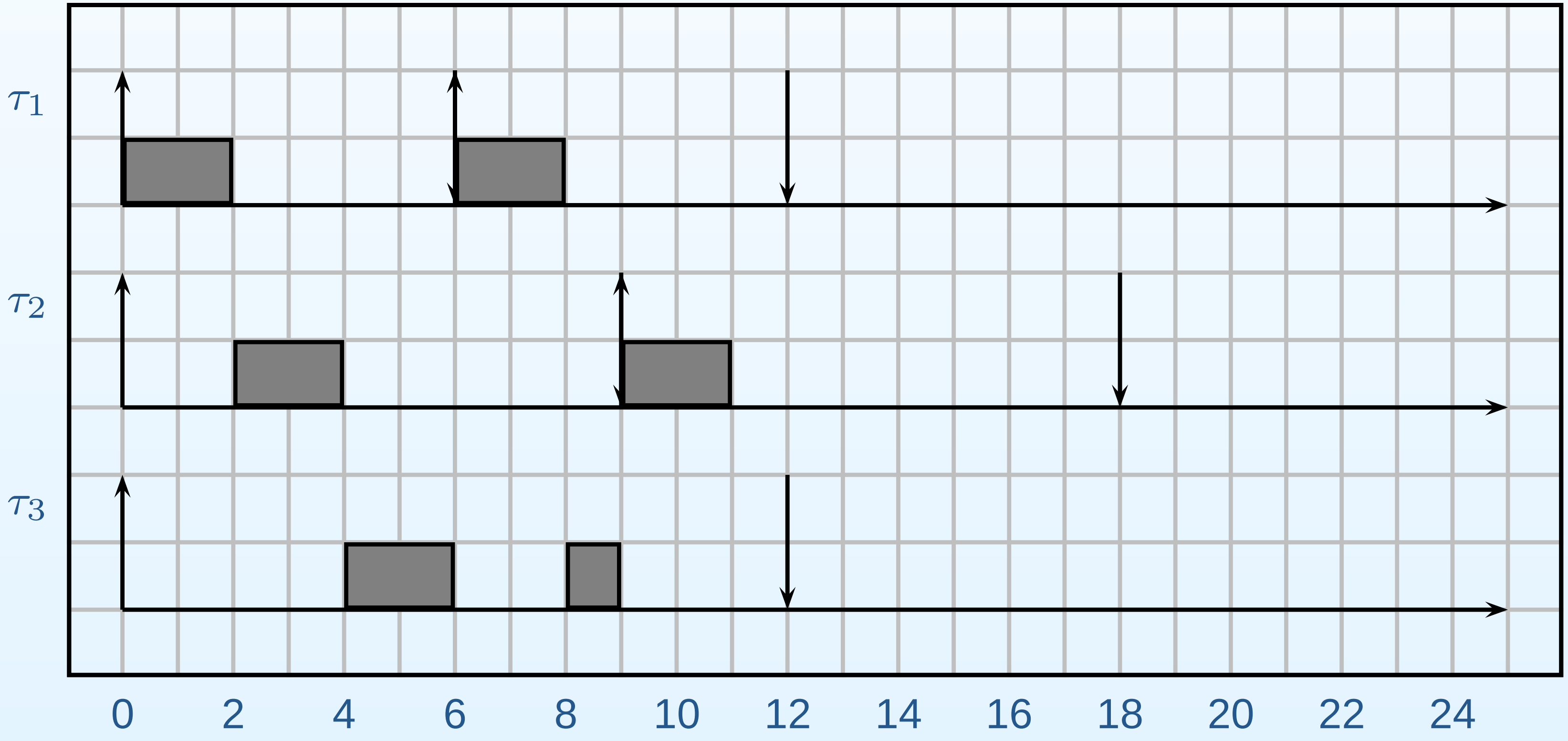

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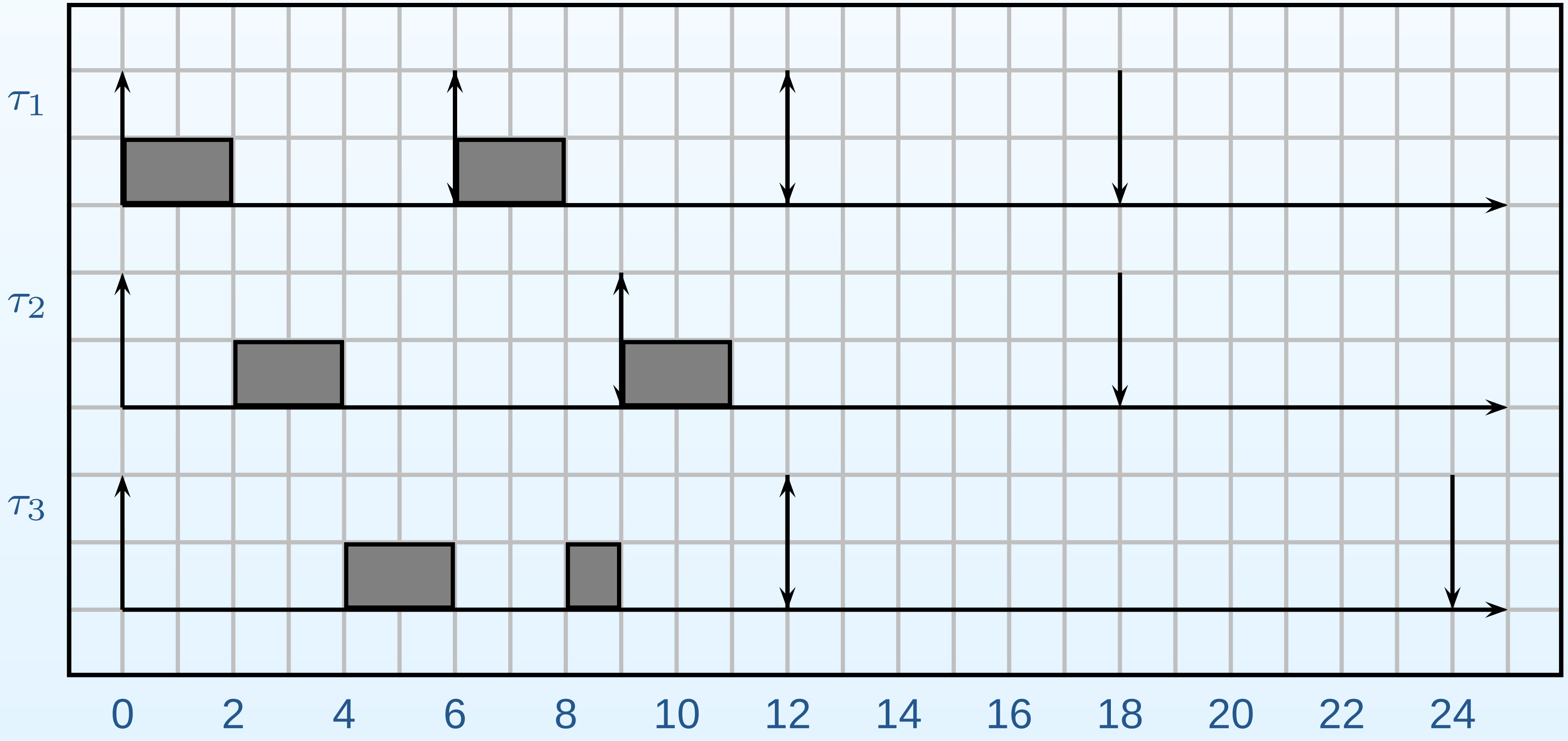

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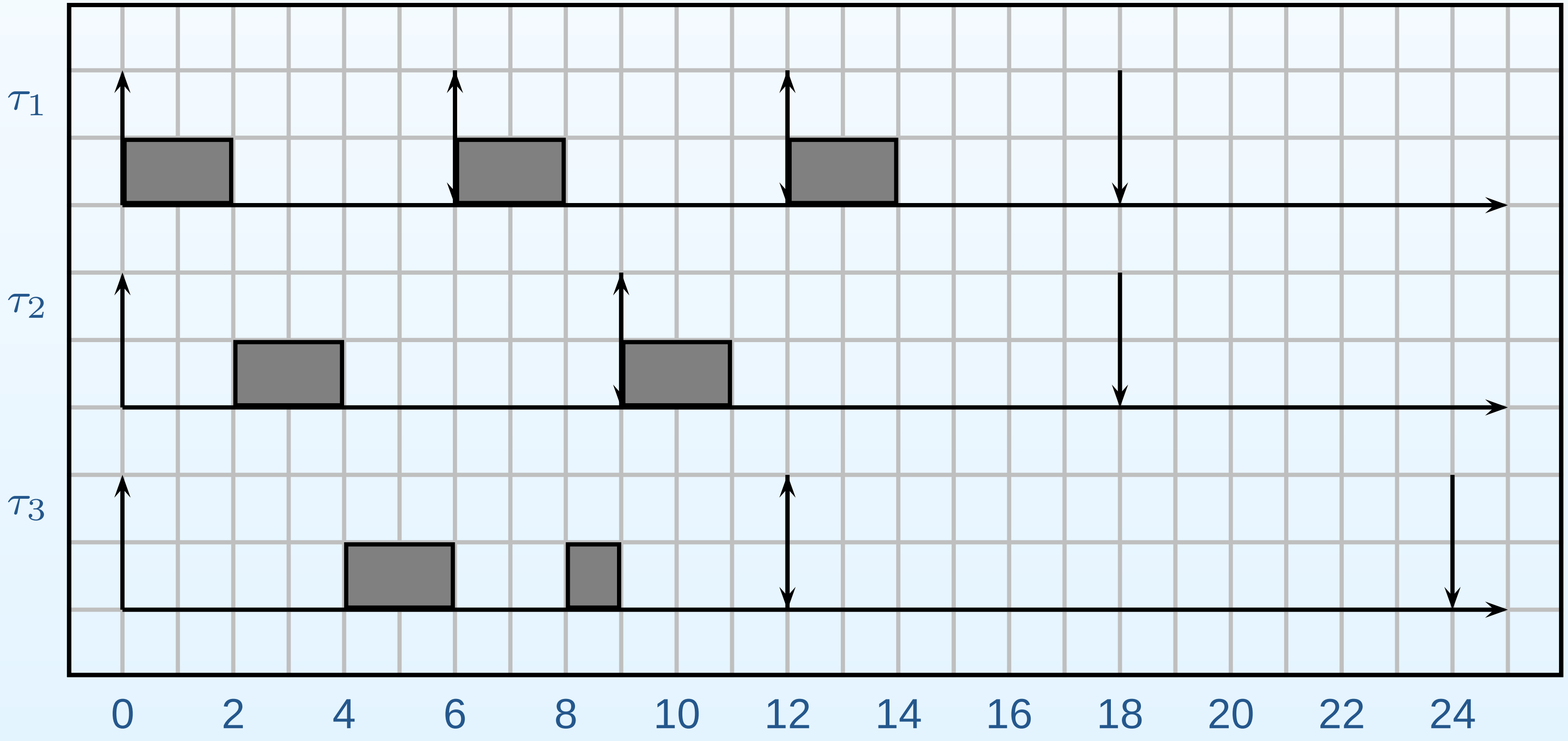

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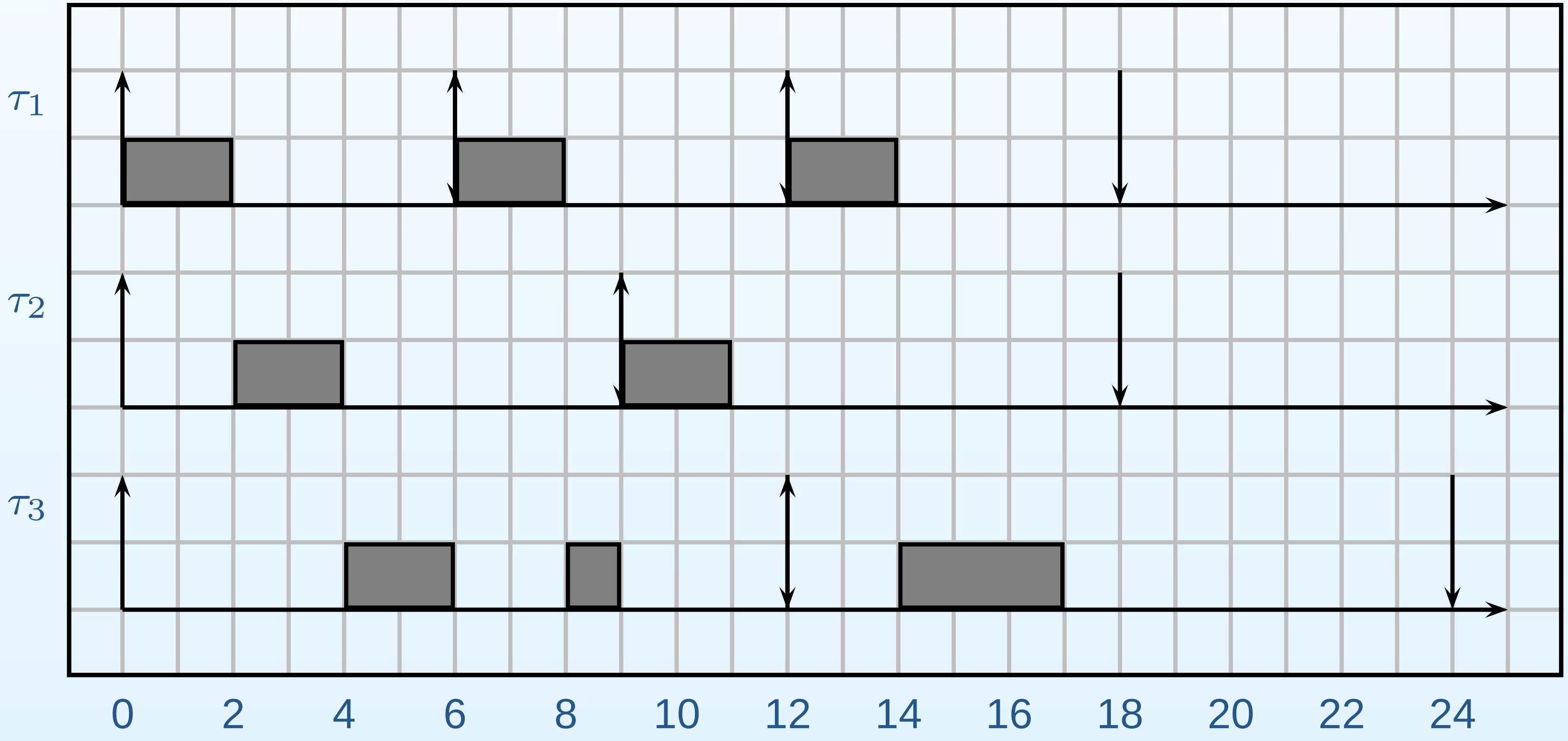

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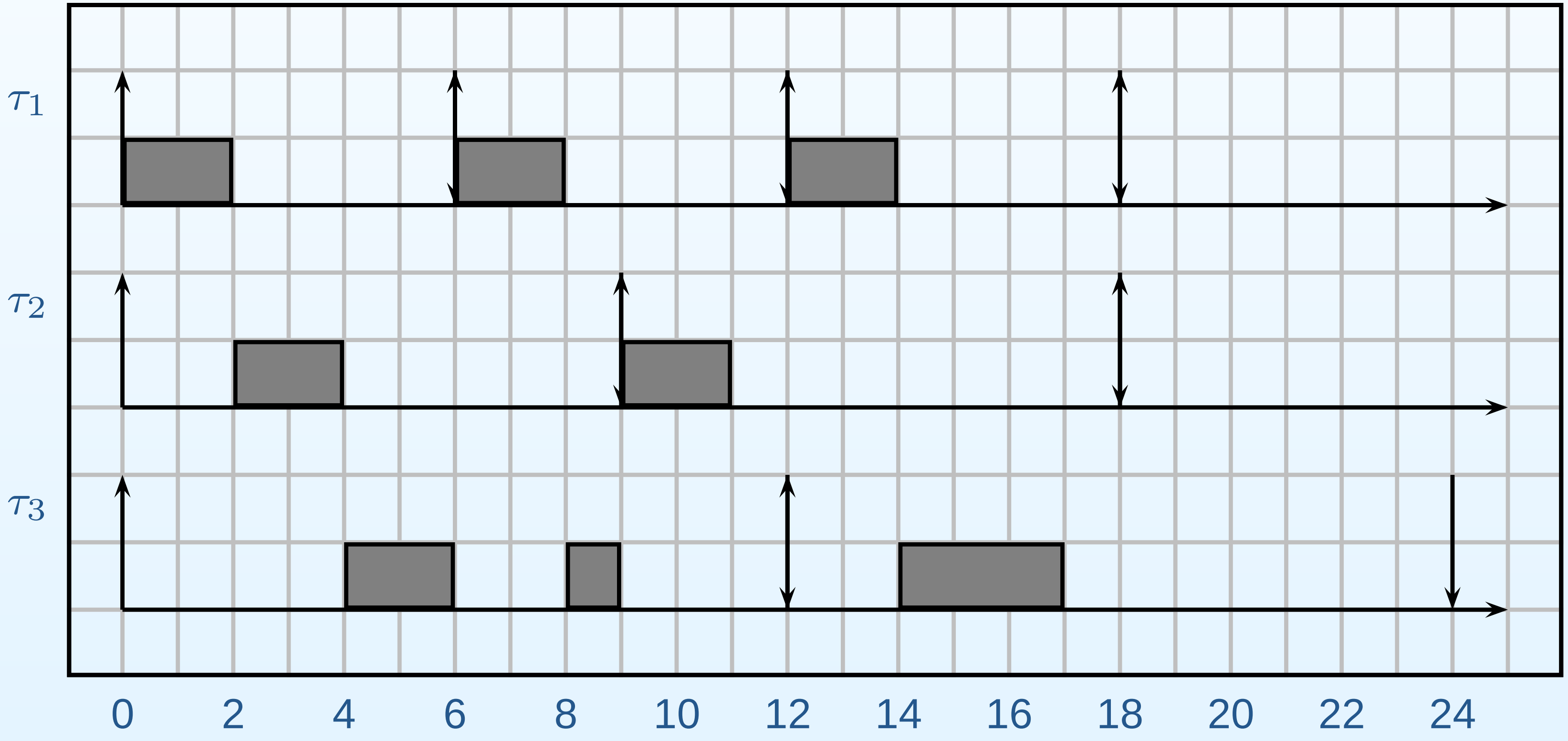

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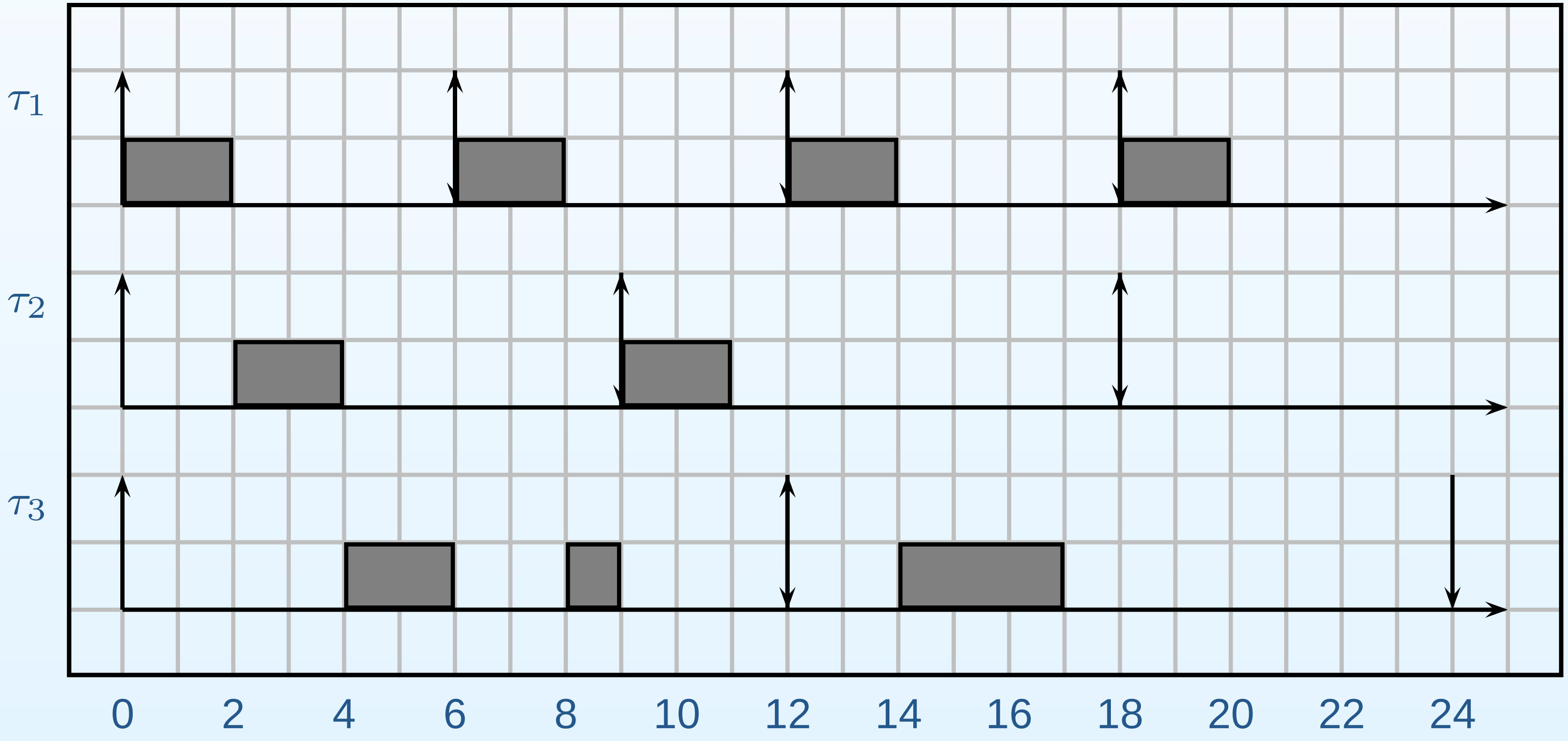

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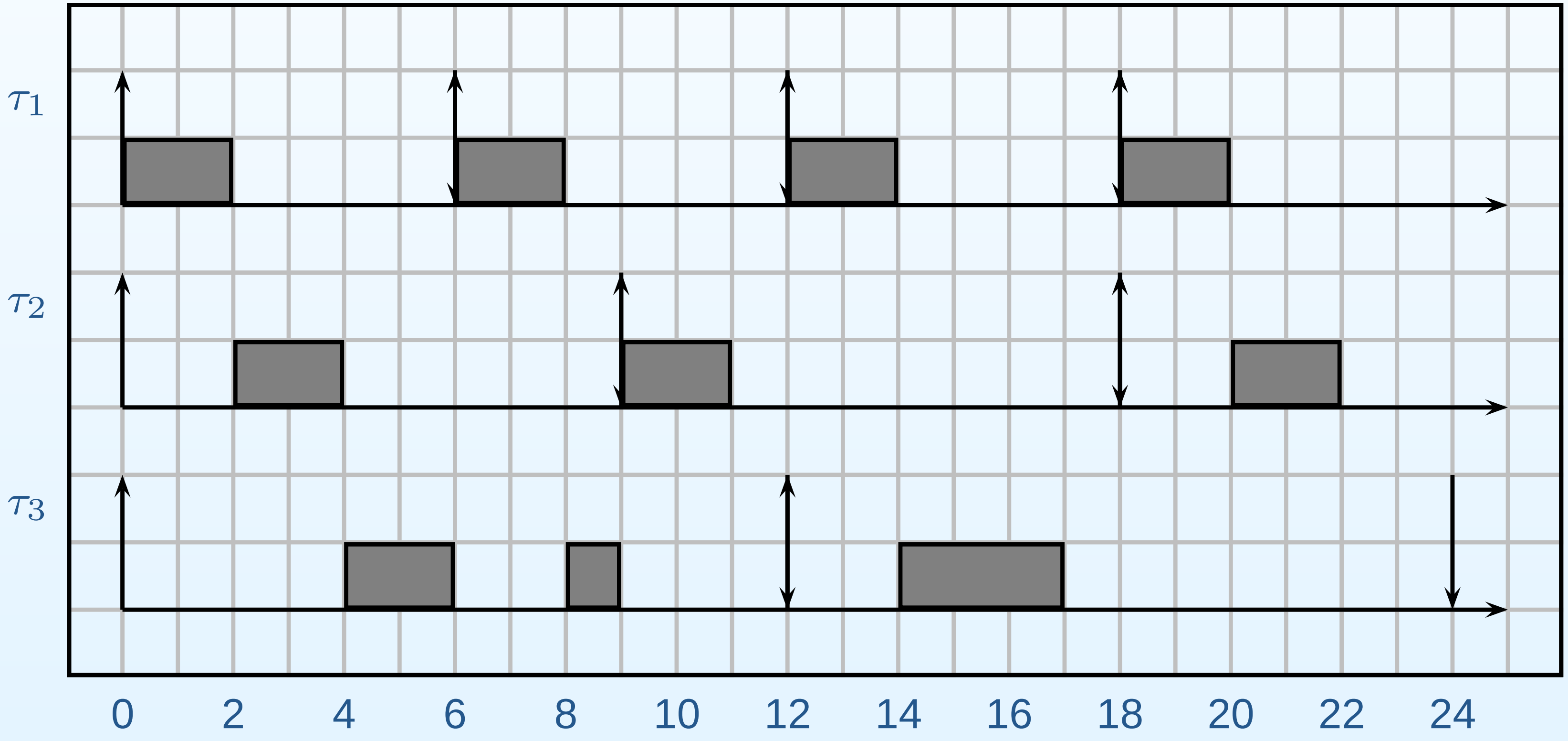

Another example (non-schedulable)

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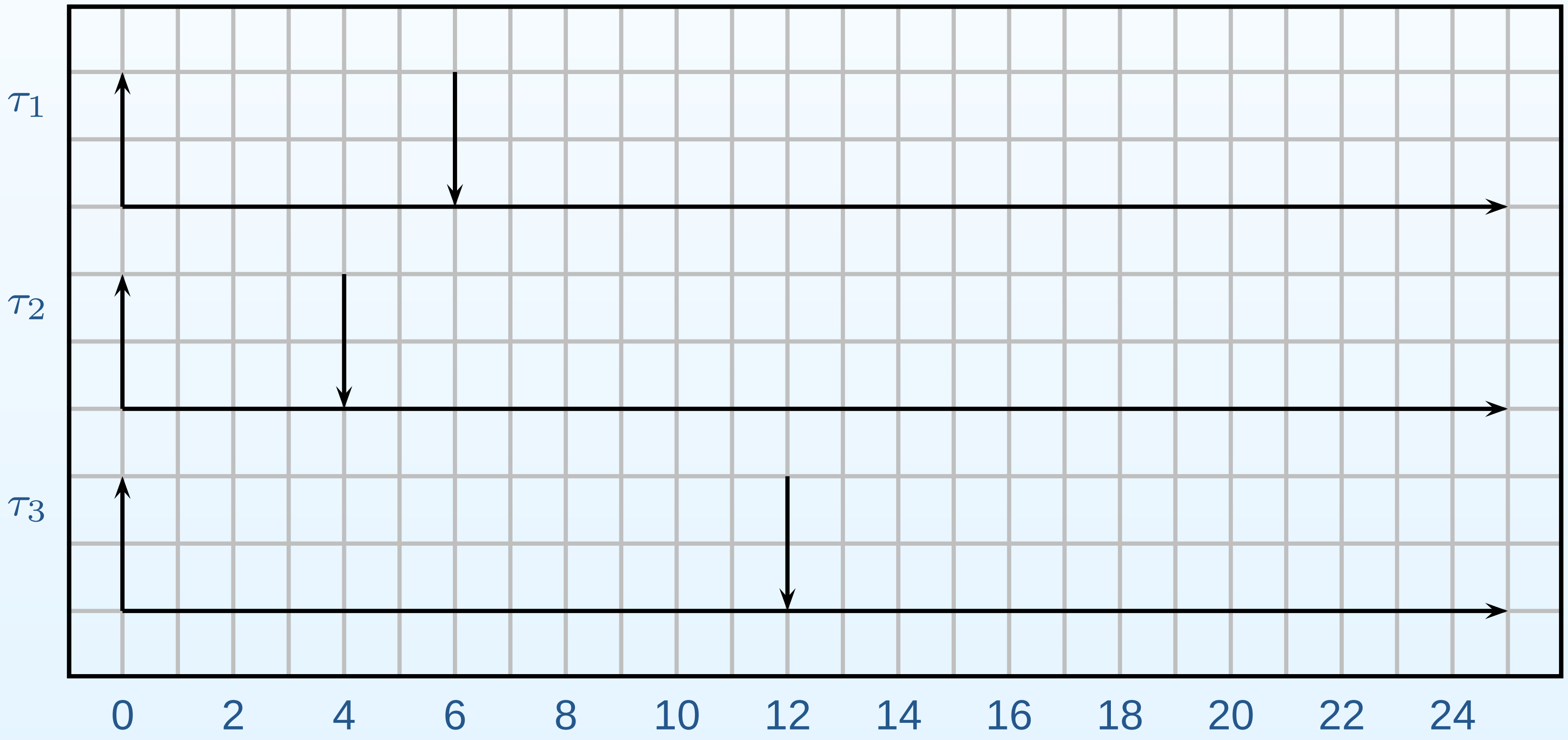

In this case, task τ_3 misses its deadline!

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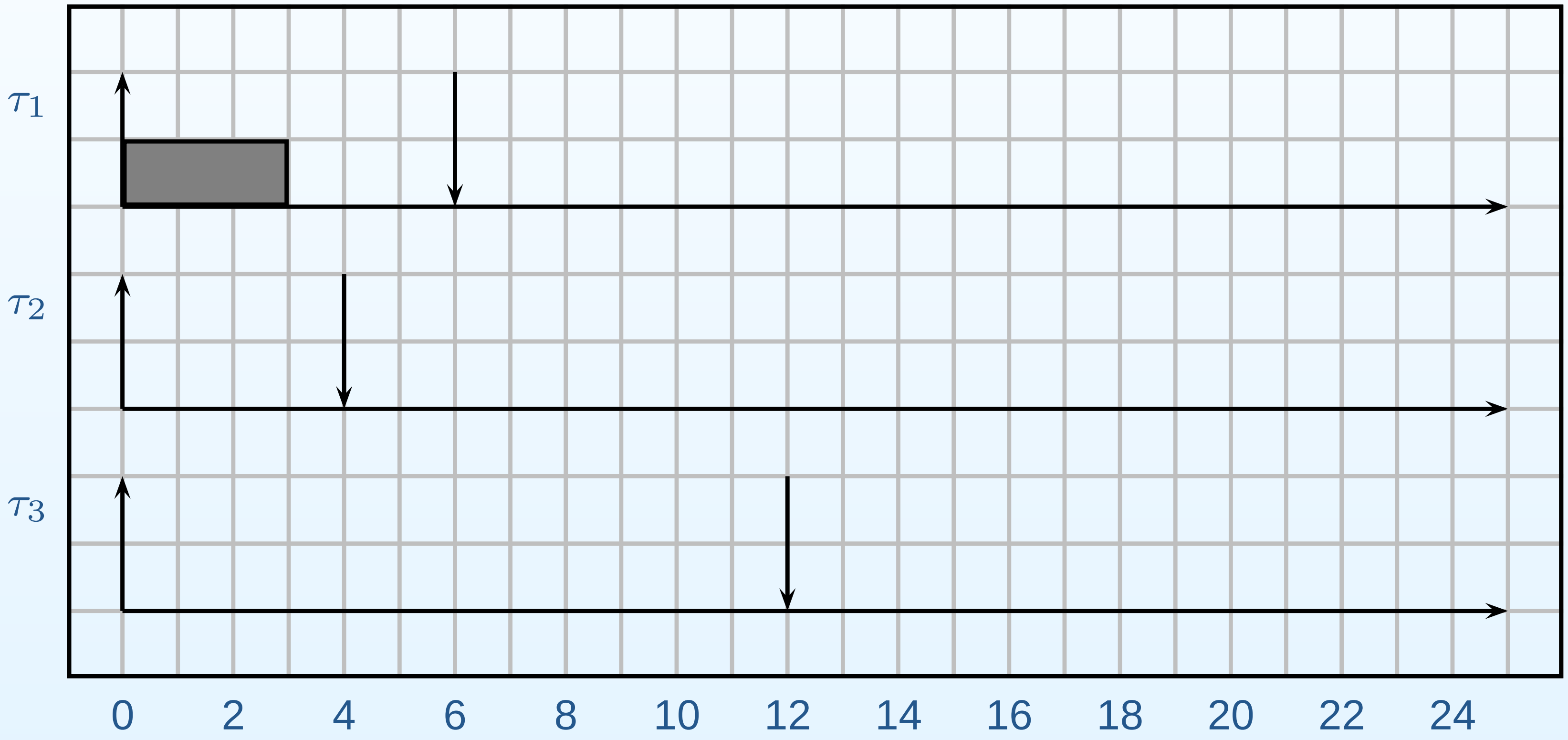

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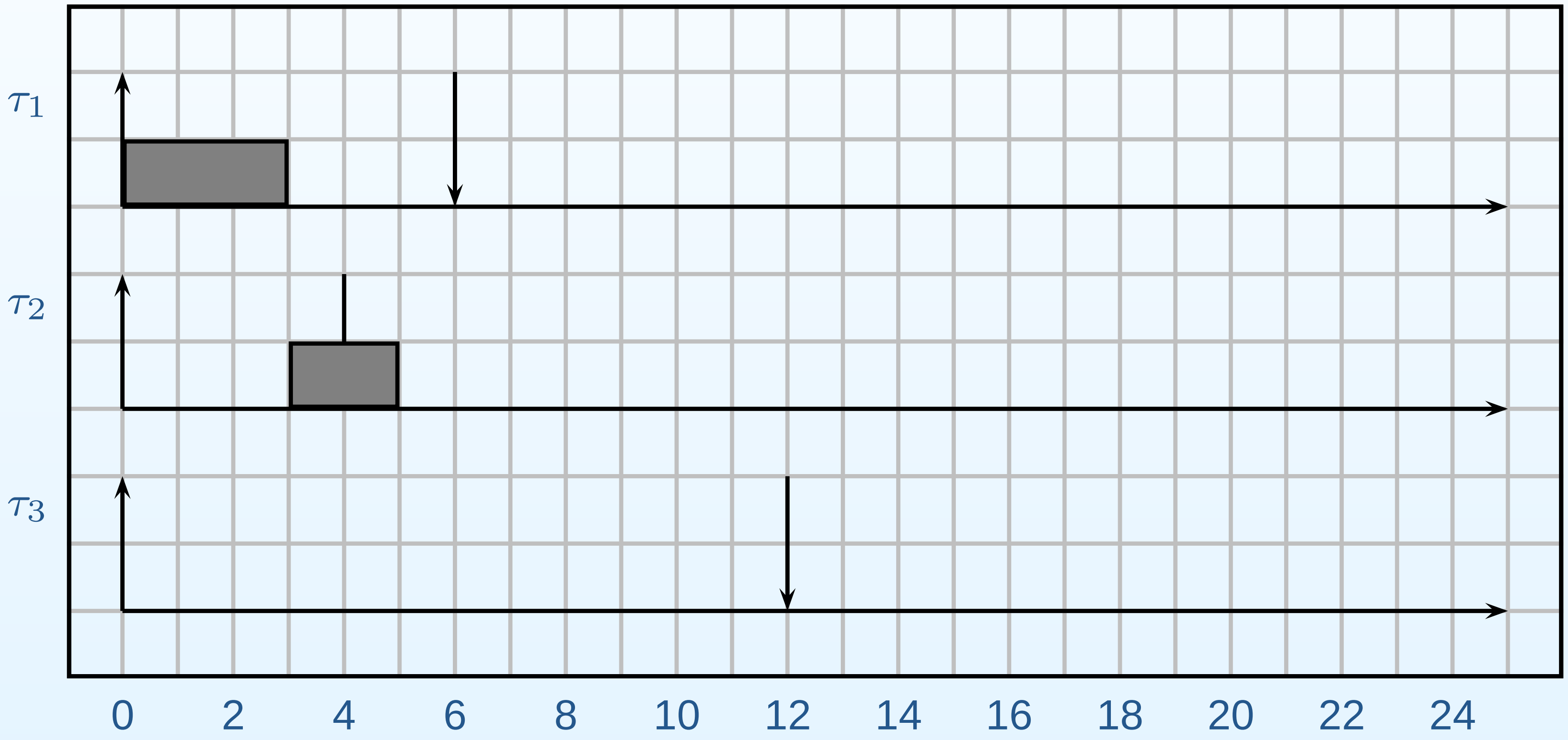

In this case, task τ_3 misses its deadline!

Note

- Some considerations about the schedule shown before:
  - The response time of the task with the highest priority is minimum and equal to its WCET.
  - The response time of the other tasks depends on the *interference* of the higher priority tasks;
  - The priority assignment may influence the schedulability of a task.

Priority assignment

Priority assignment

- Given a task set, how to assign priorities?
- There are two possible objectives:
  - Schedulability (i.e. find the priority assignment that makes all tasks schedulable)
  - Response time (i.e. find the priority assignment that minimize the response time of a subset of tasks).
- By now we consider the first objective only
- An *optimal* priority assignment *Opt* is such that:
  - If the task set is schedulable with another priority assignment, then it is schedulable with priority assignment *Opt*.
  - If the task set is not schedulable with *Opt*, then it is not schedulable by any other assignment.

Optimal priority assignment

- Given a periodic task set with all tasks having deadline equal to the period ($\forall i, \quad D_i = T_i$), and with all offsets equal to 0 ($\forall i, \quad \phi_i = 0$):
  - The best assignment is the *Rate Monotonic* assignment
  - Tasks with shorter period have higher priority
- Given a periodic task set with deadline different from periods, and with all offsets equal to 0 ($\forall i, \quad \phi_i = 0$):
  - The best assignement is the *Deadline Monotonic* assignment
  - Tasks with shorter relative deadline have higher priority
- For sporadic tasks, the same rules are valid as for periodic tasks with offsets equal to 0.

Example revised

- Consider the example shown before with deadline monotonic:
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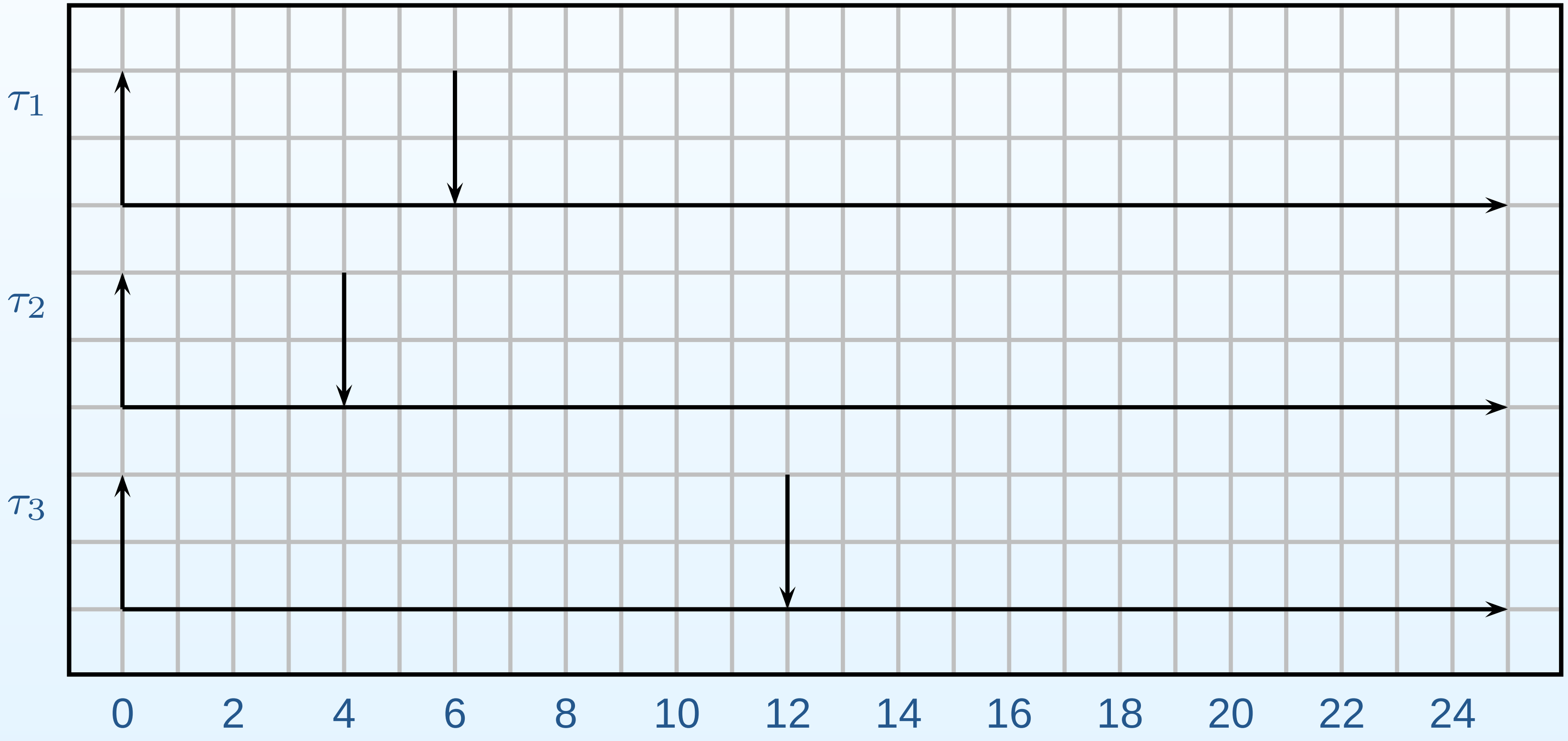

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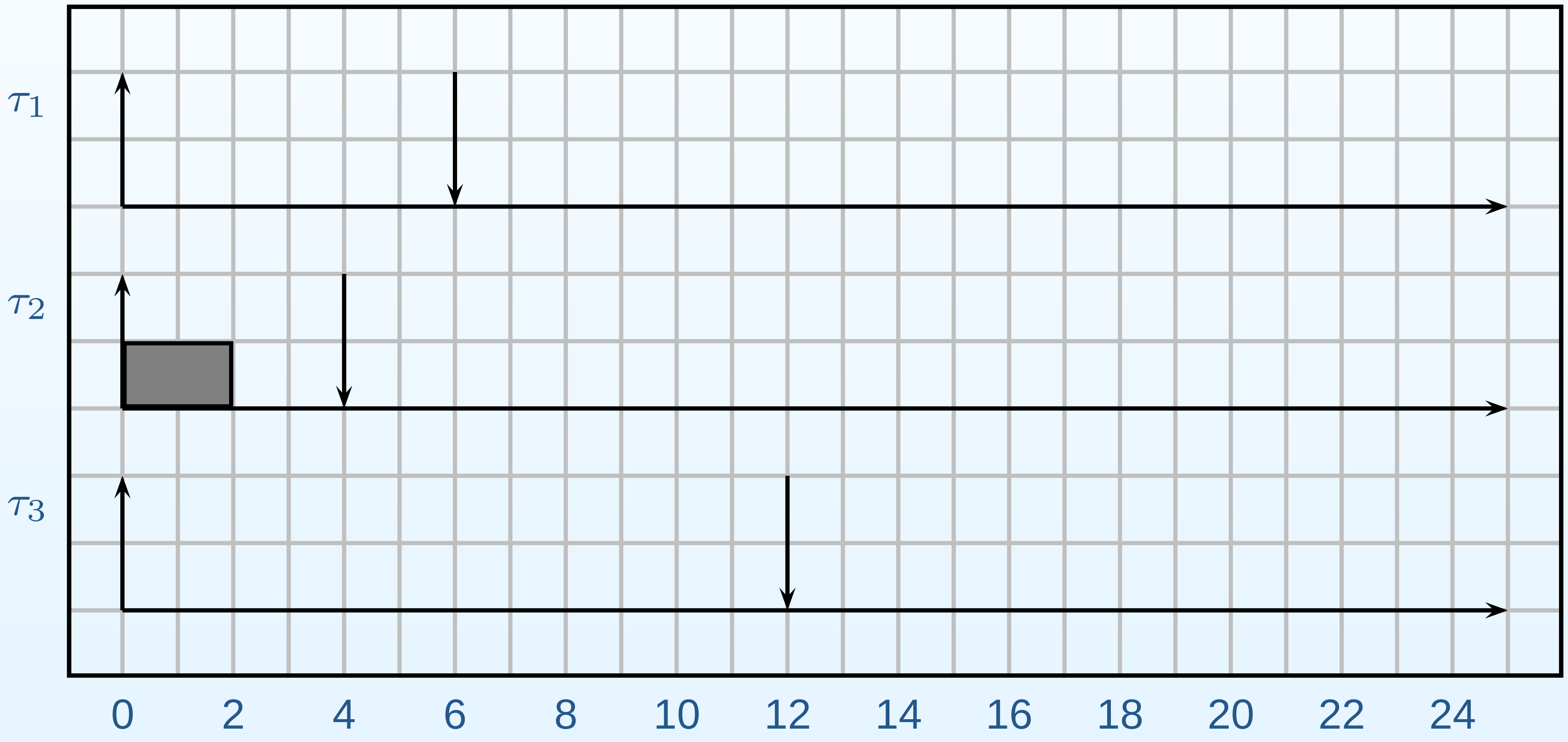

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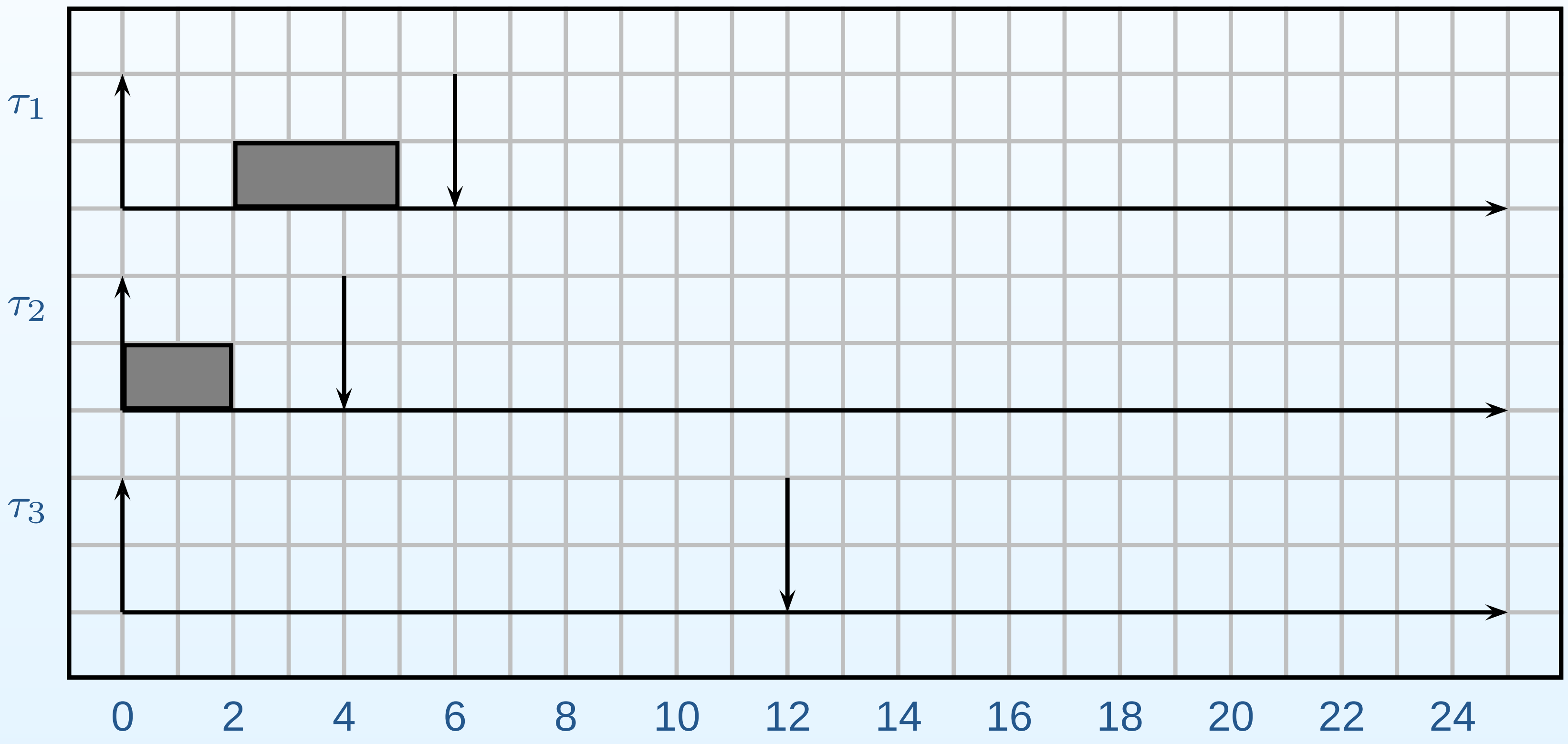

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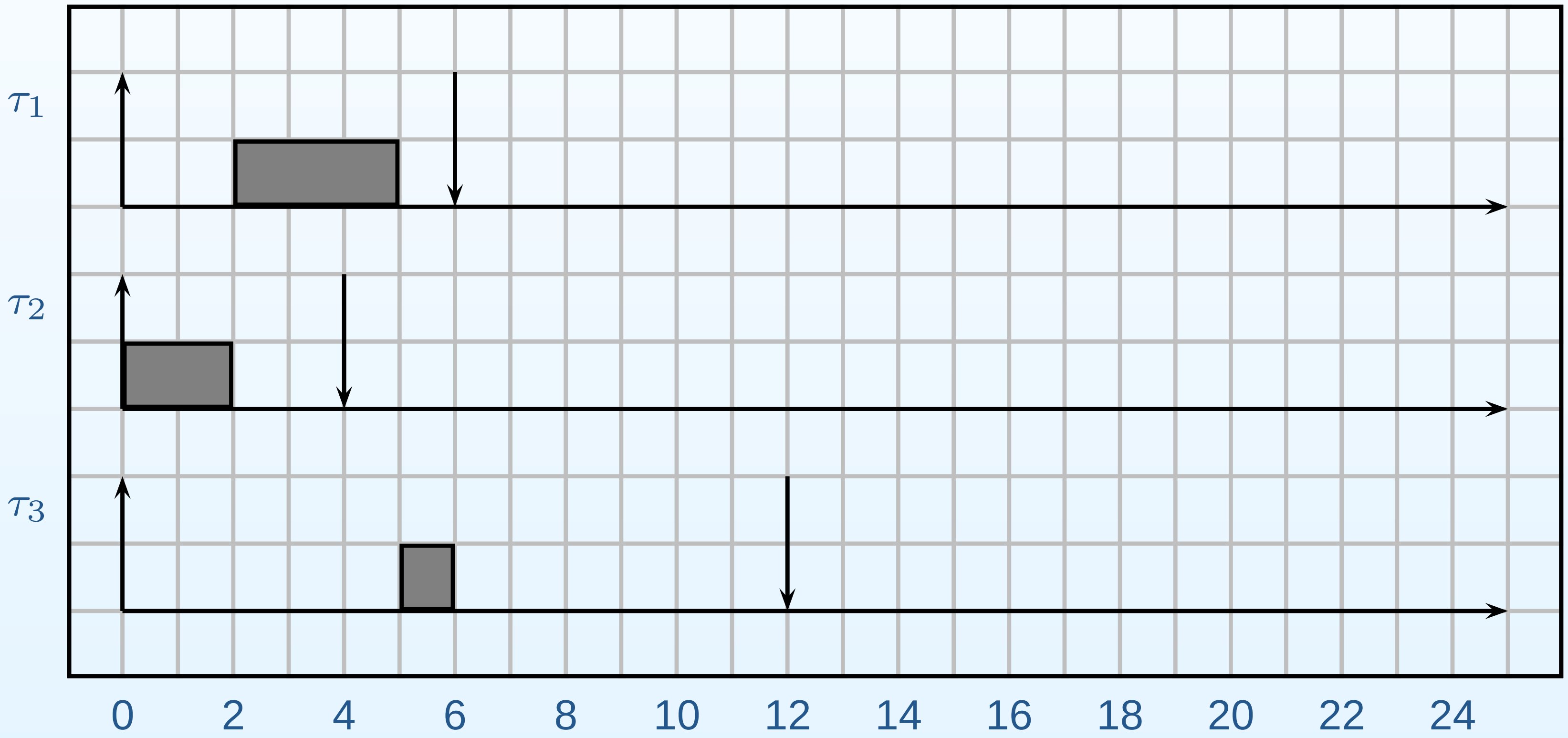

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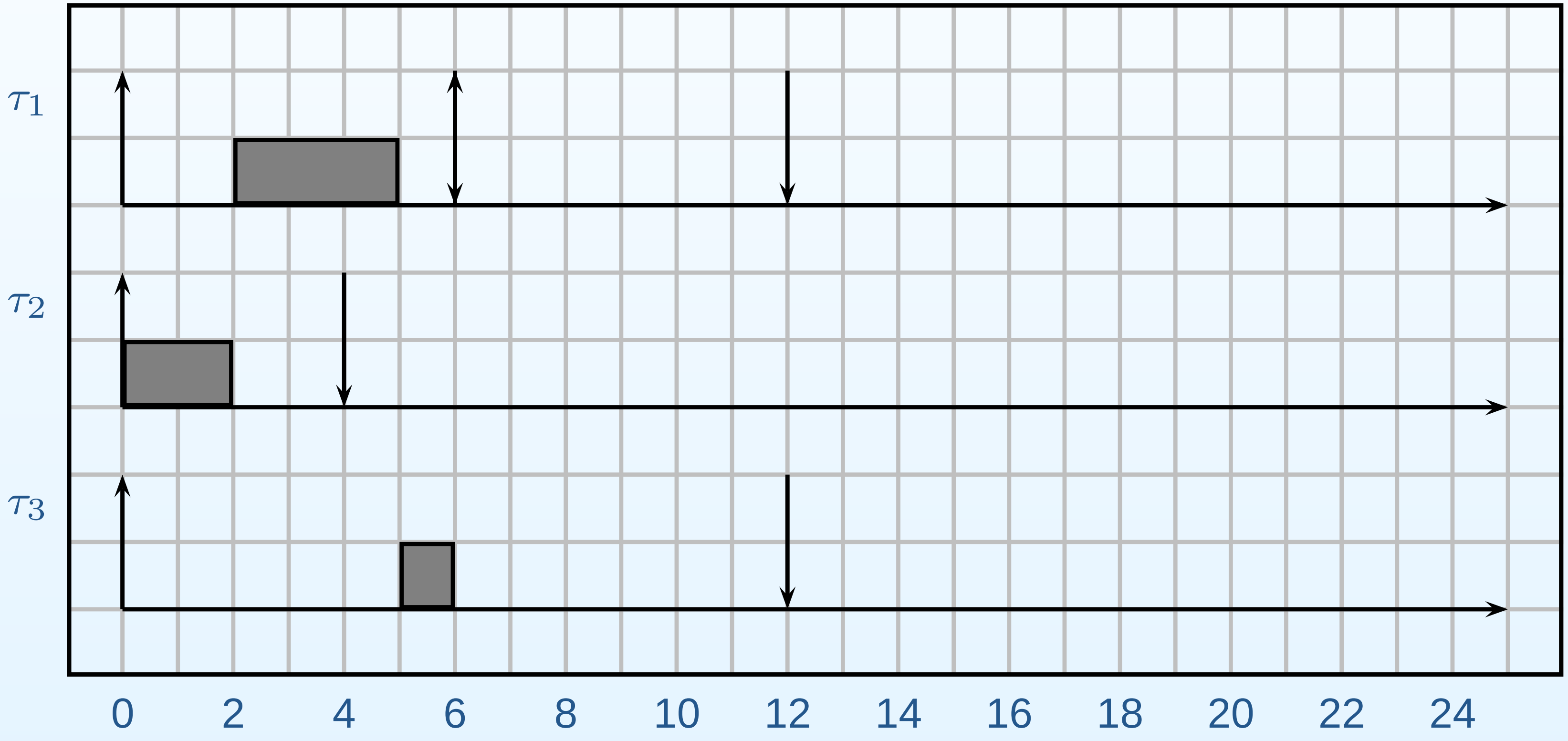

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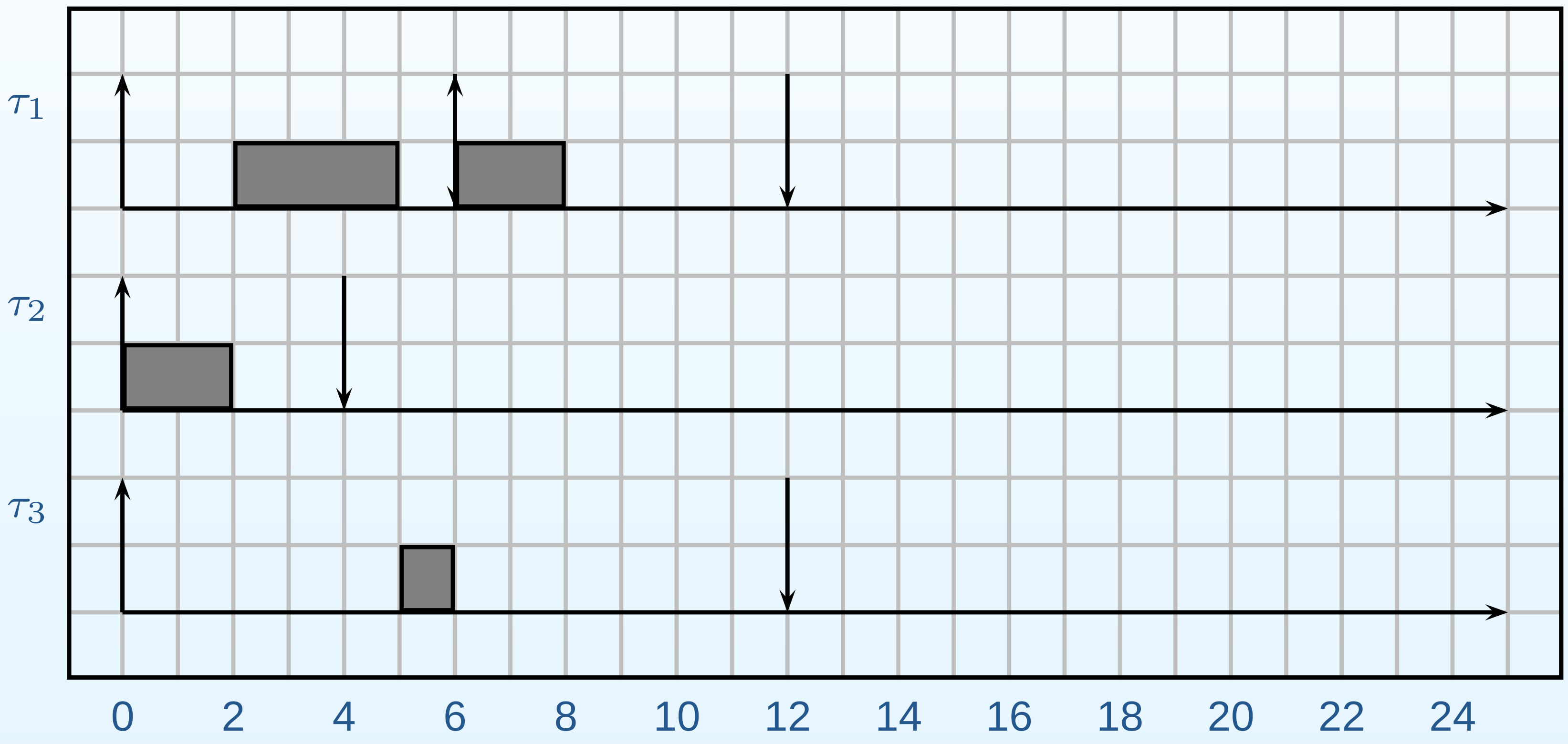

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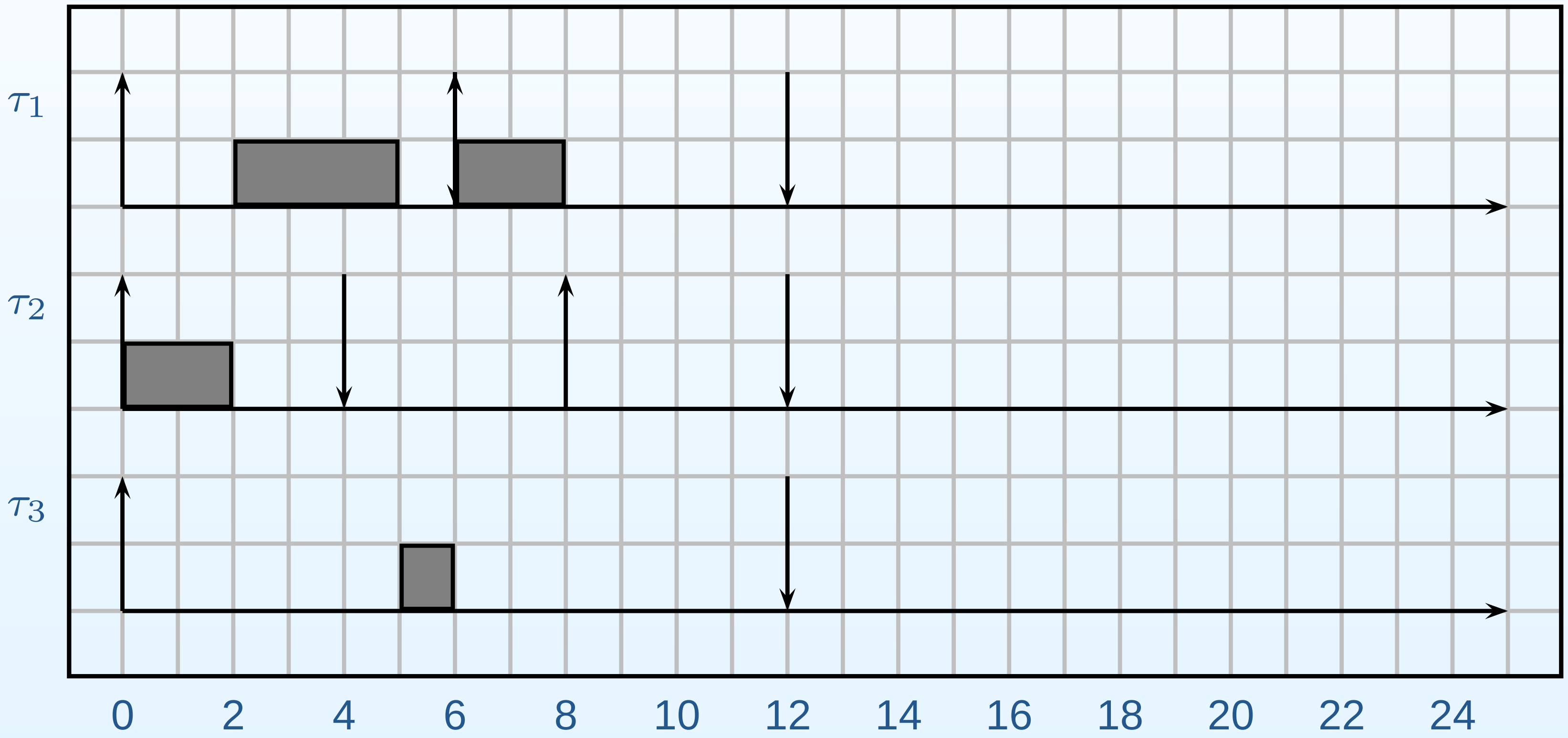

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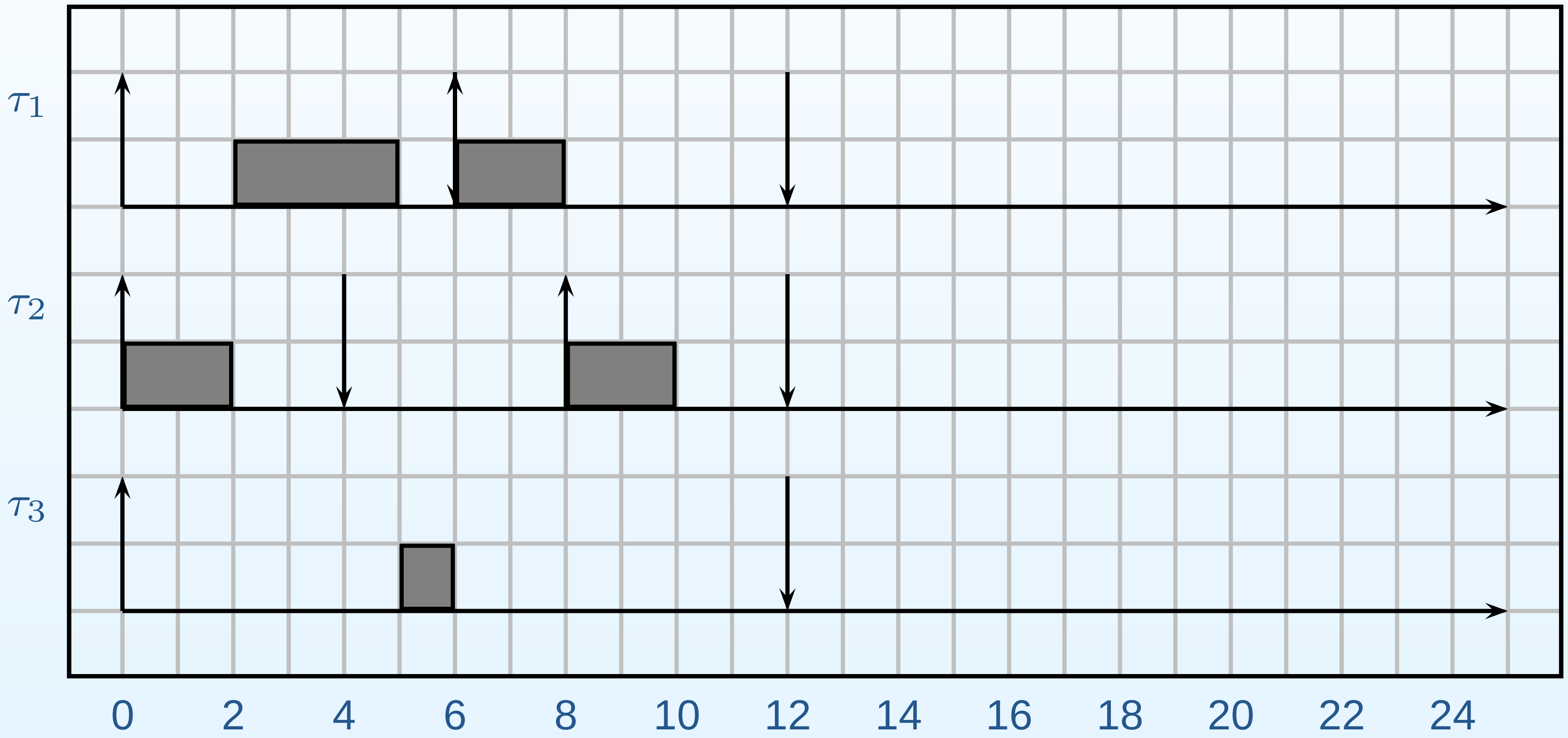

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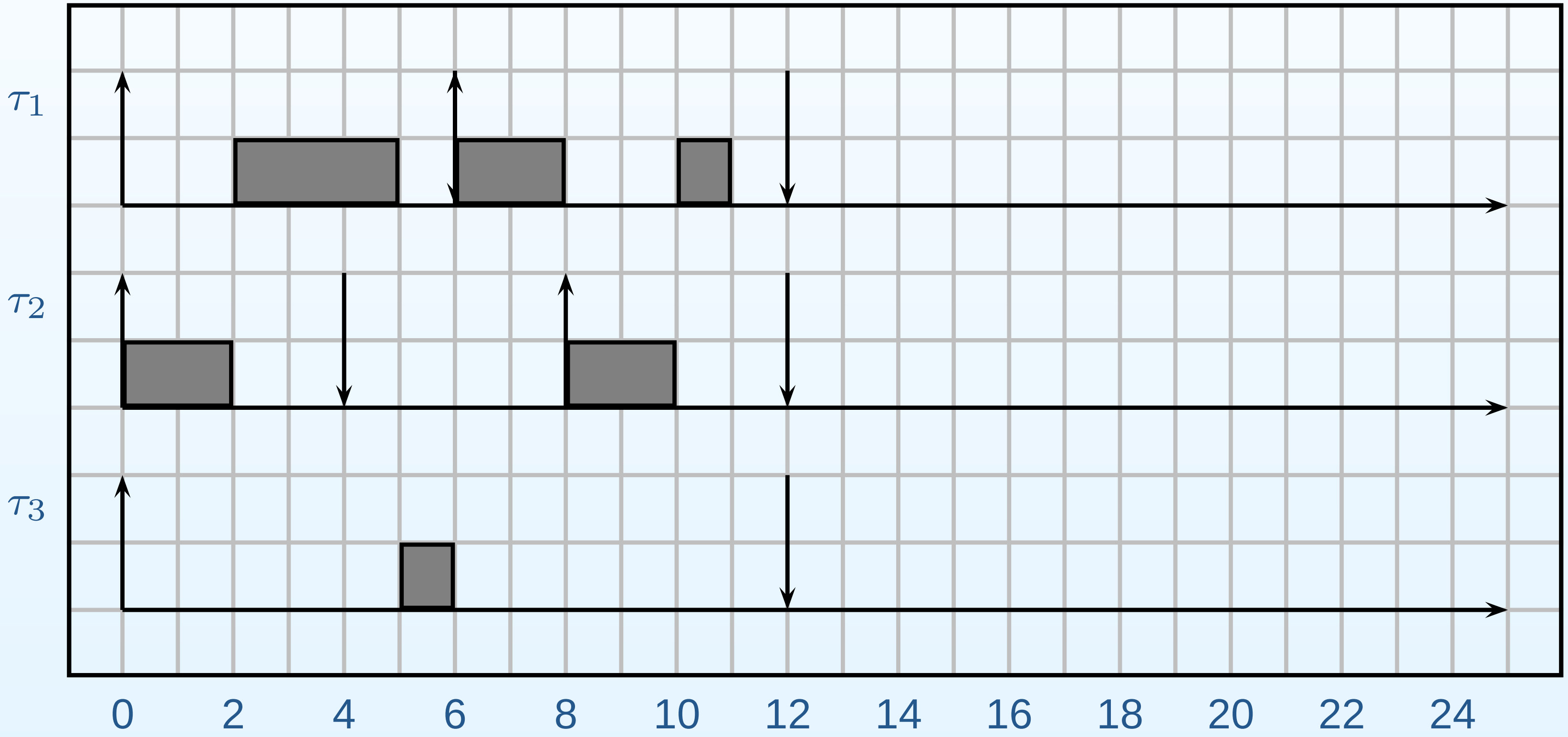

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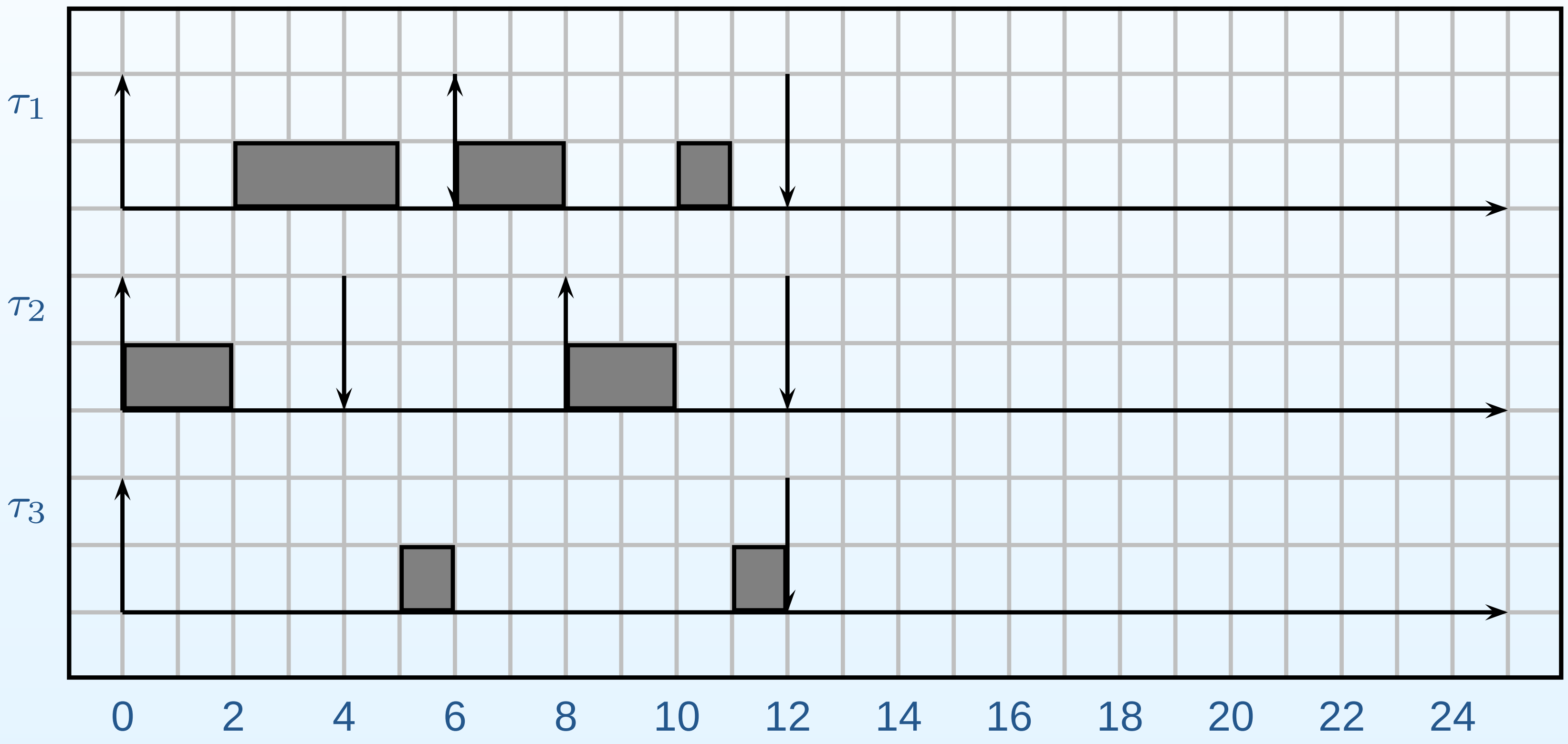

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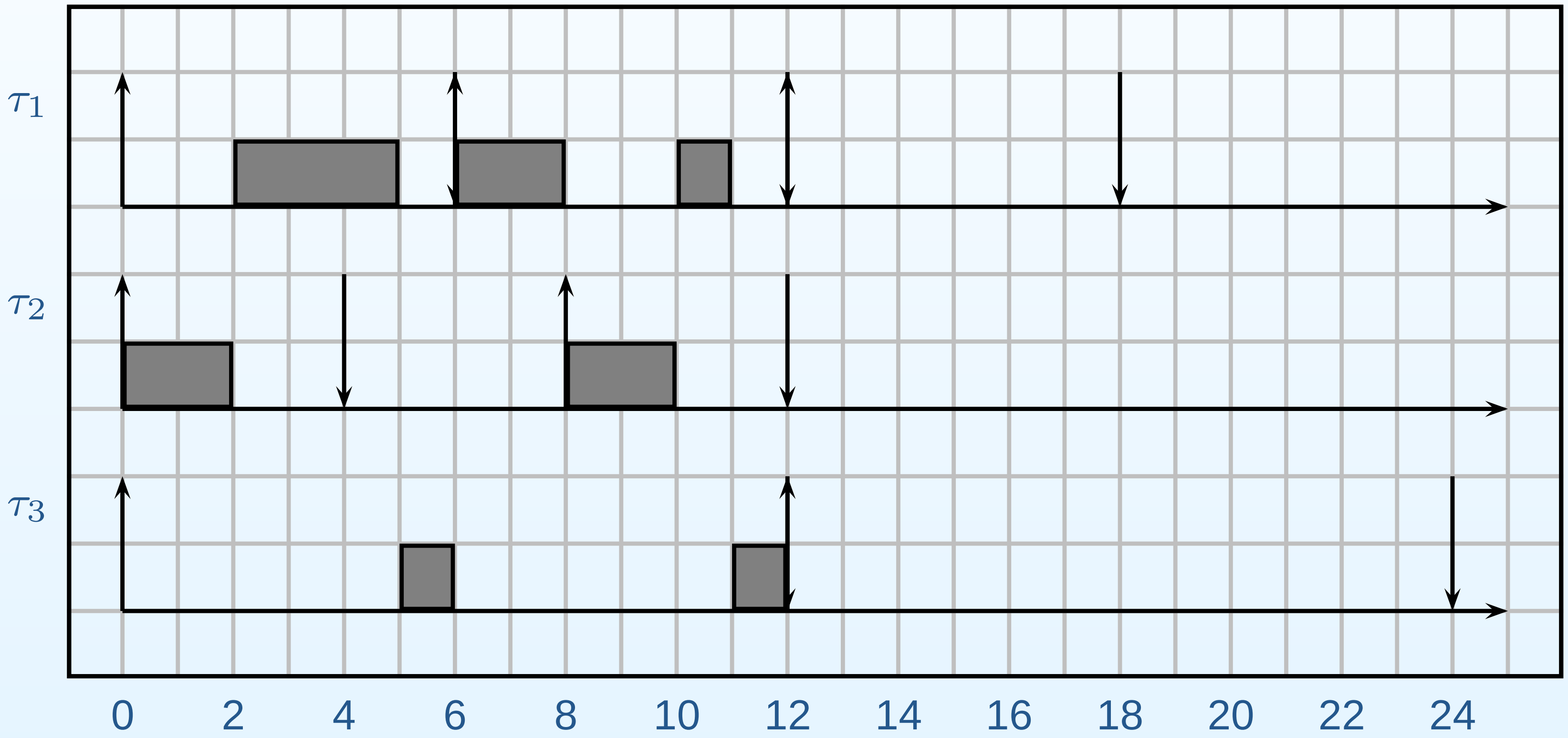

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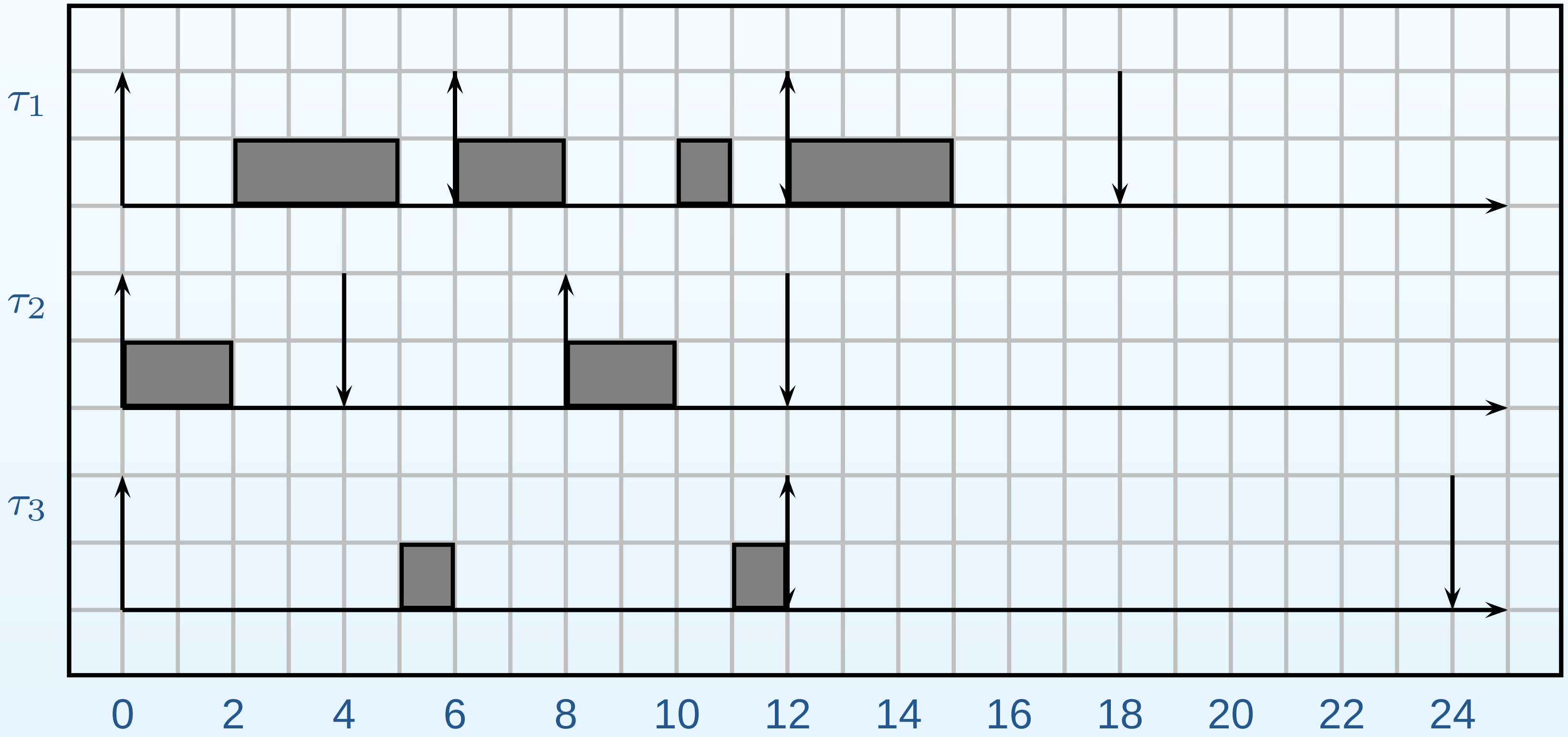

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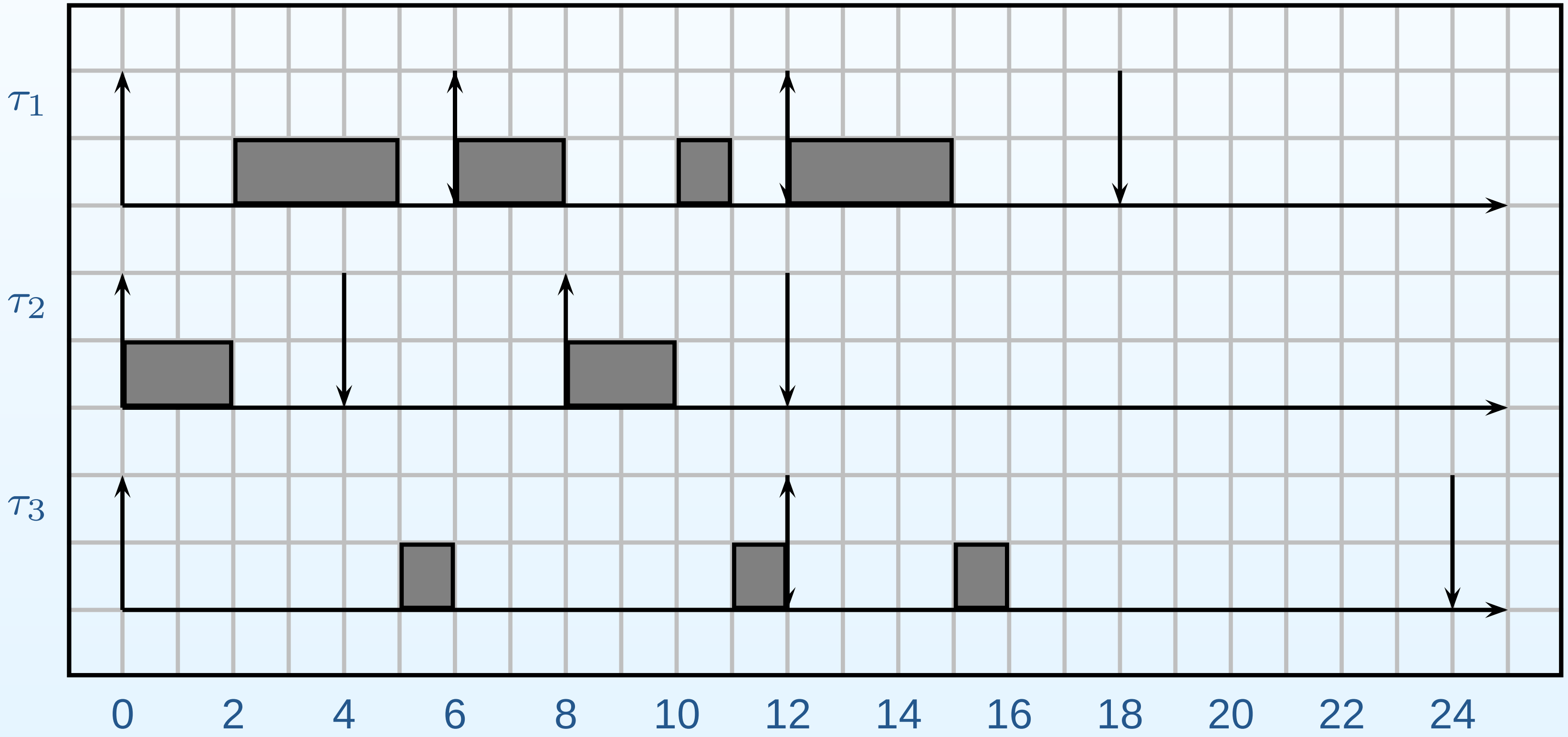

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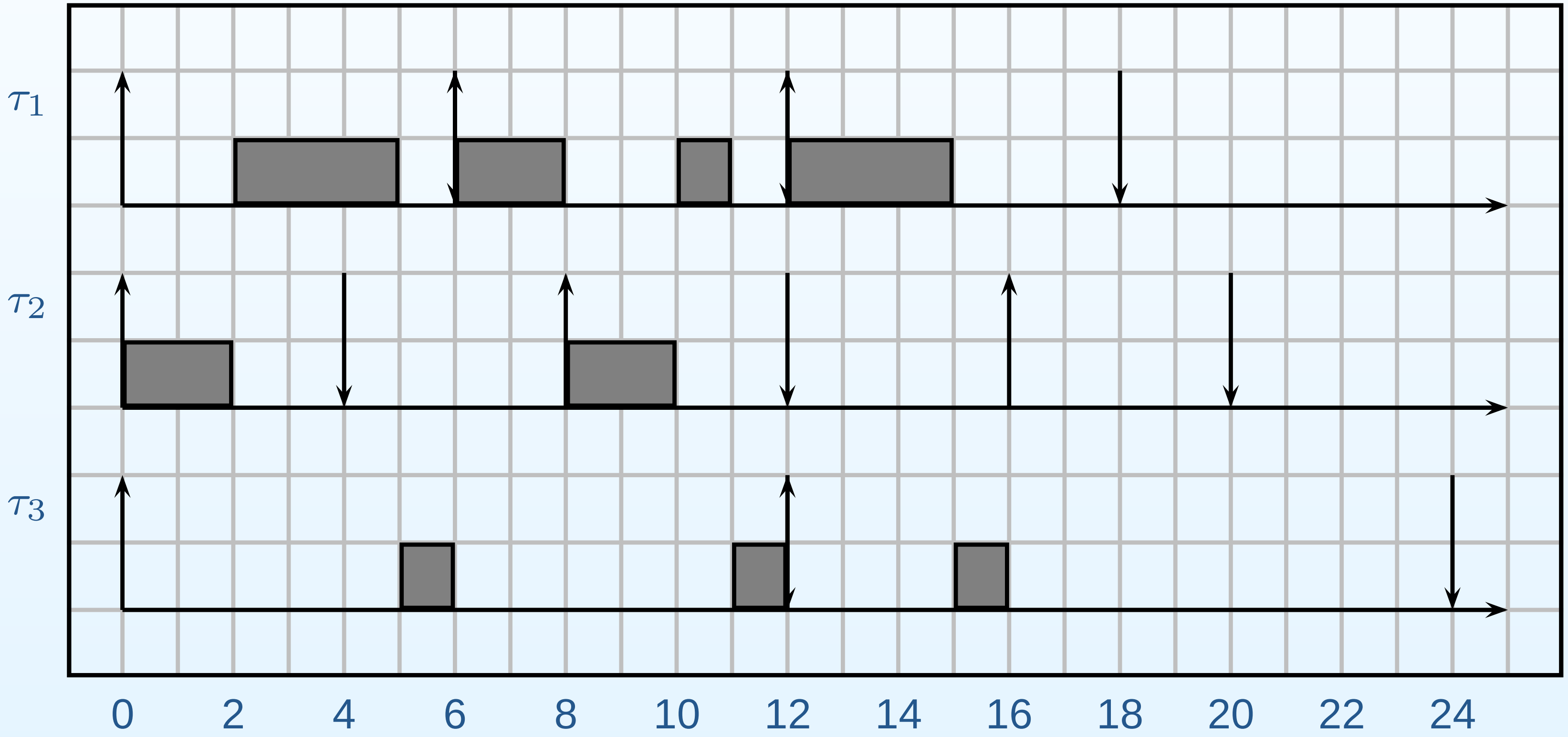

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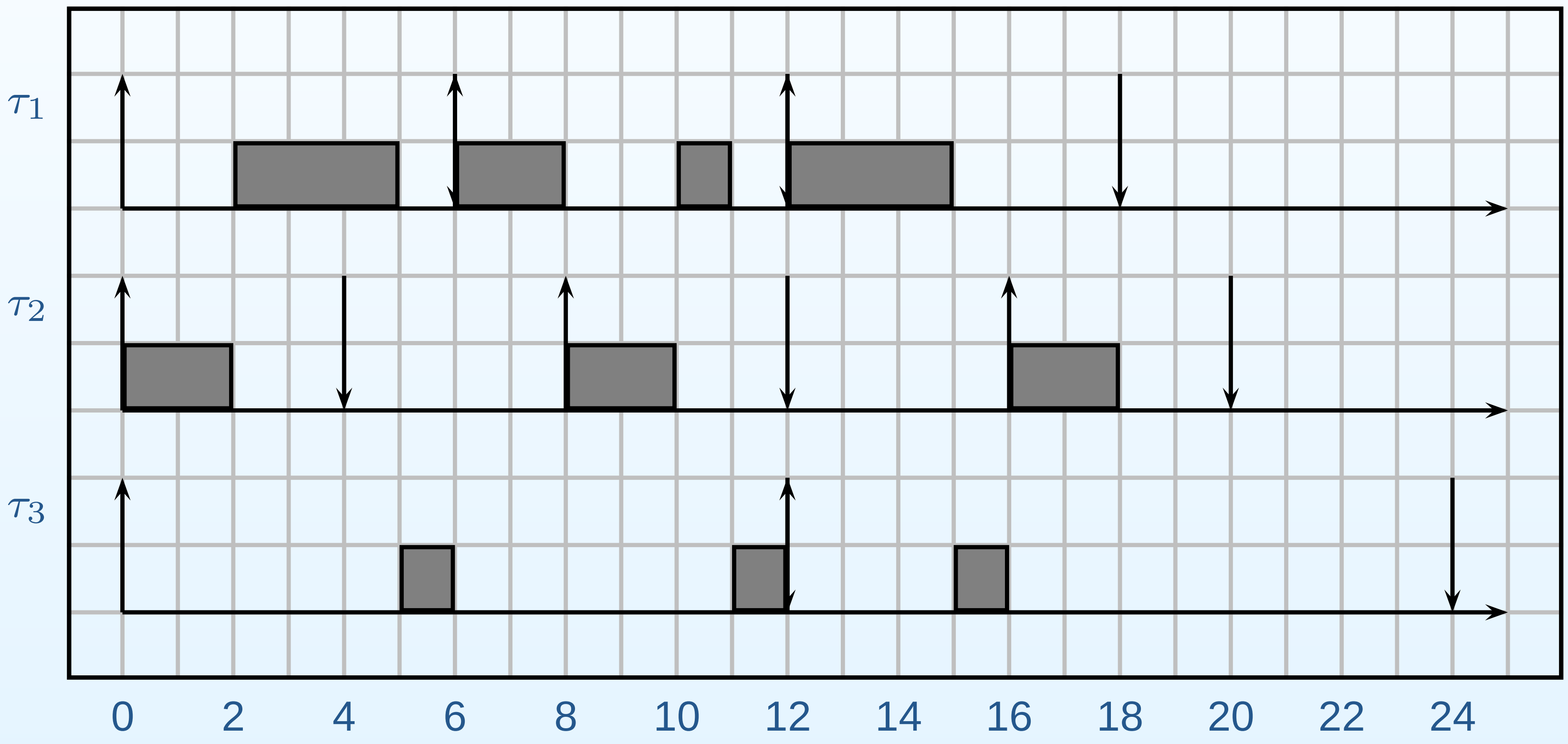

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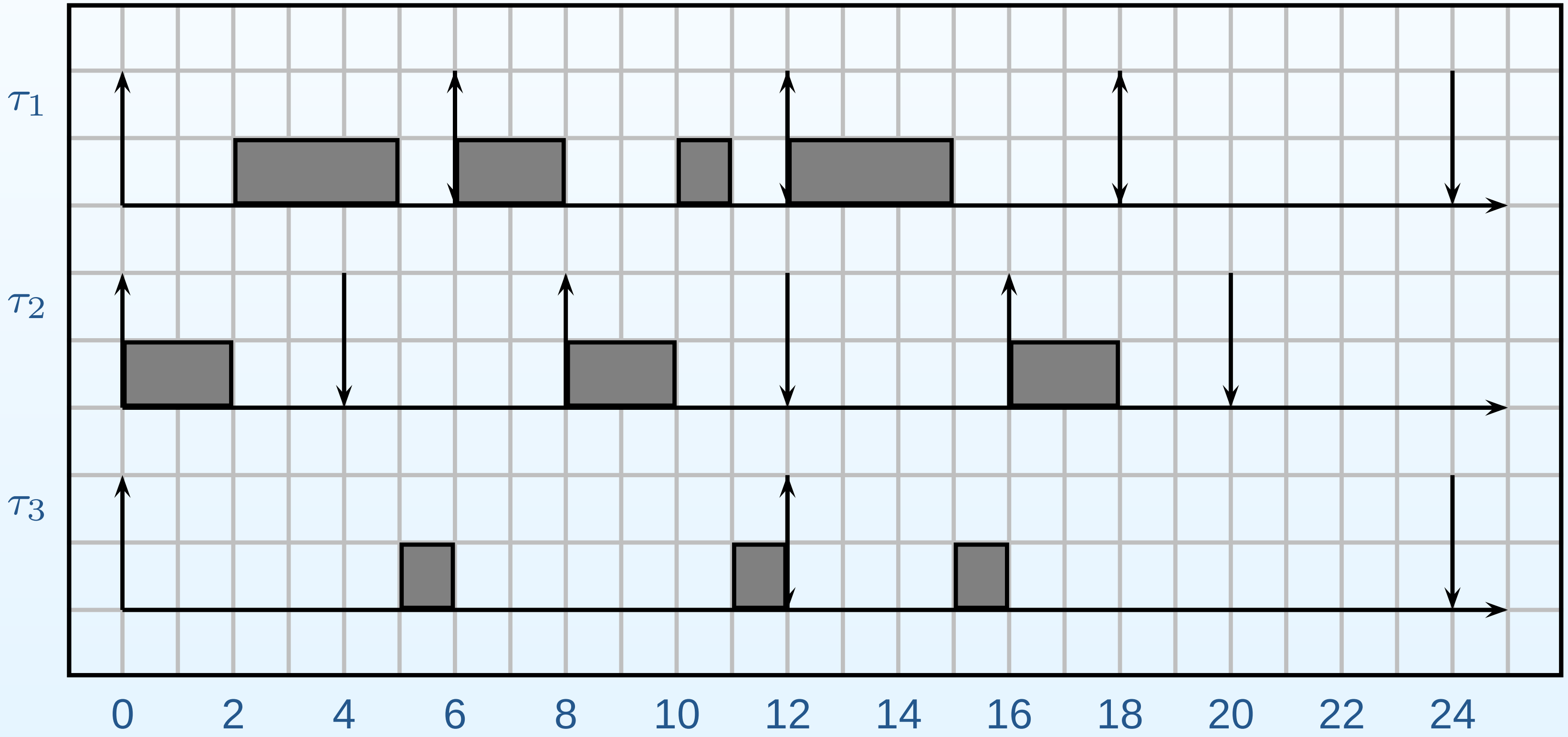

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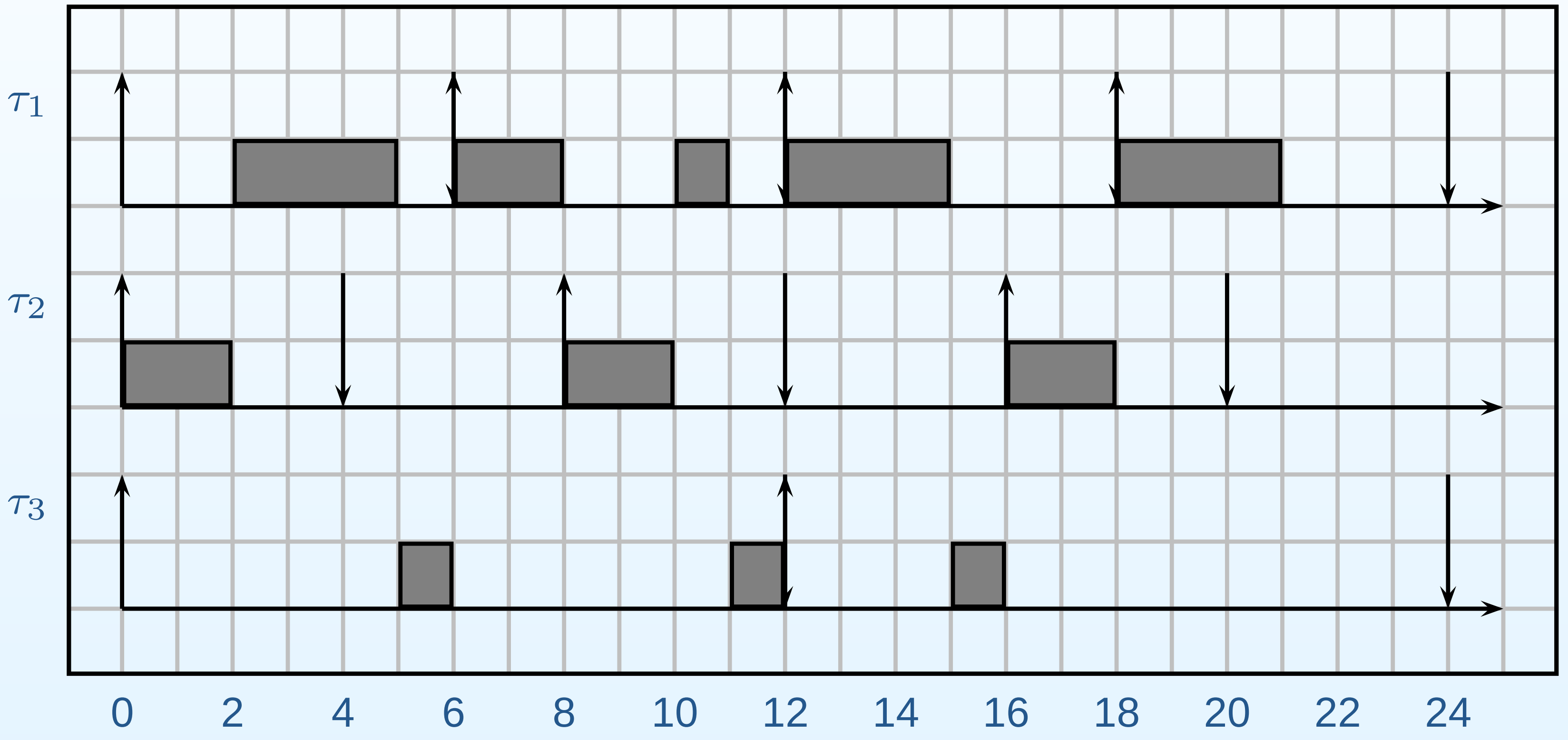

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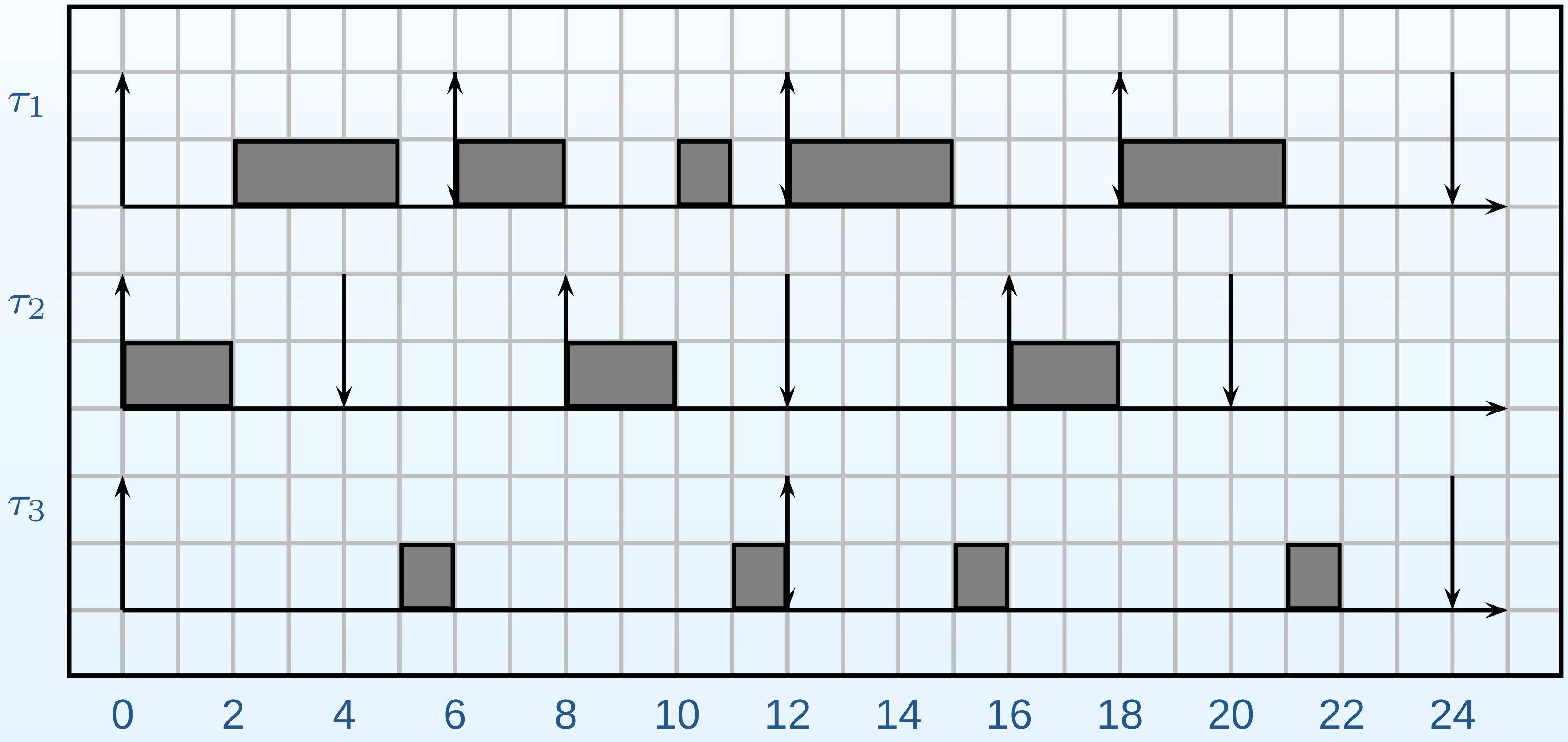
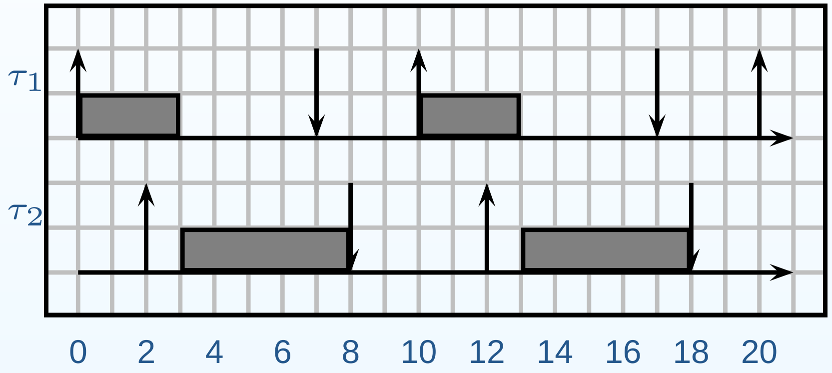


Presence of offsets

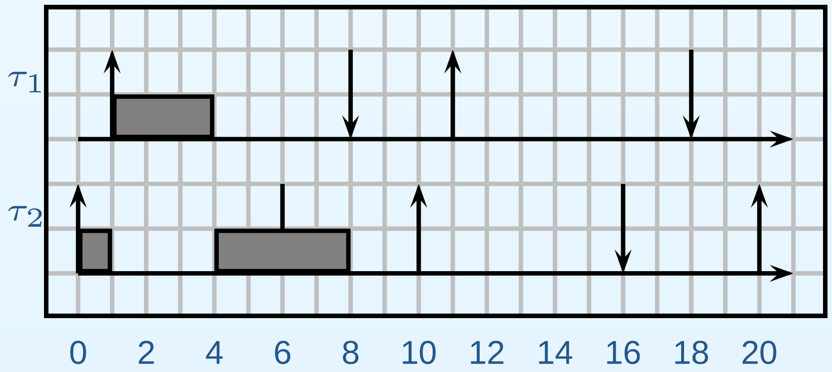
- If instead we consider periodic tasks with offsets, then *there is no optimal priority assignment*
  - In other words,
    - $\rightarrow$ if a task set $\mathcal{T}_1$ is schedulable by priority O_1 and not schedulable by priority assignment O_2,
    - $\rightarrow$ it may exist another task set $\mathcal{T}_2$ that is schedulable by O_2 and not schedulable by O_1.
  - For example, $\mathcal{T}_2$ may be obtained from $\mathcal{T}_1$ simply changing the offsets!

Example of non-optimality with offsets

Example: priority to τ_1:



Changing the offset:



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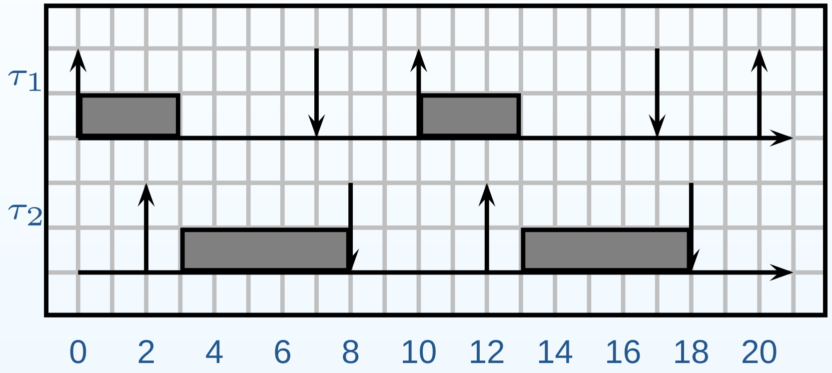
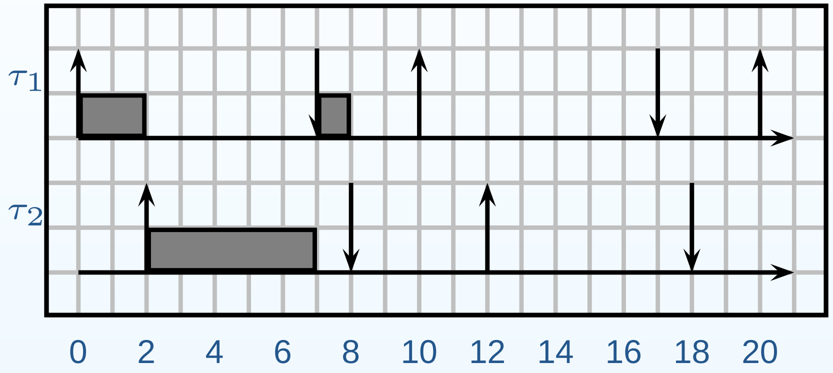


Example: priority to τ_2:



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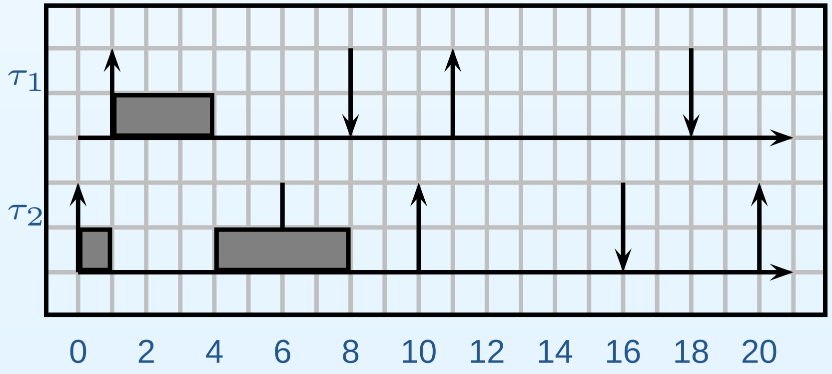

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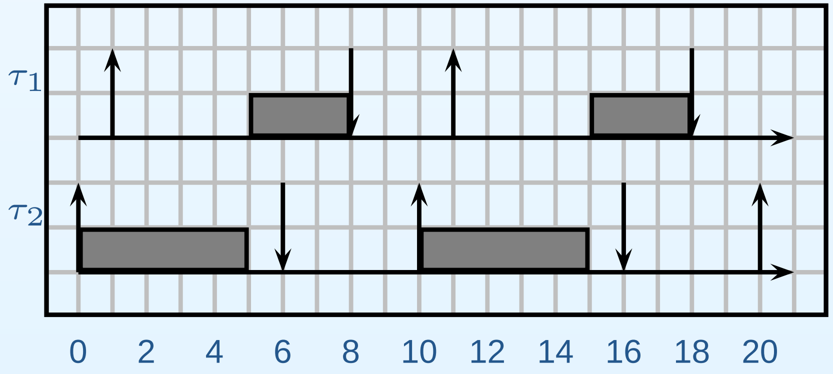


Scheduling analysis

Analysis

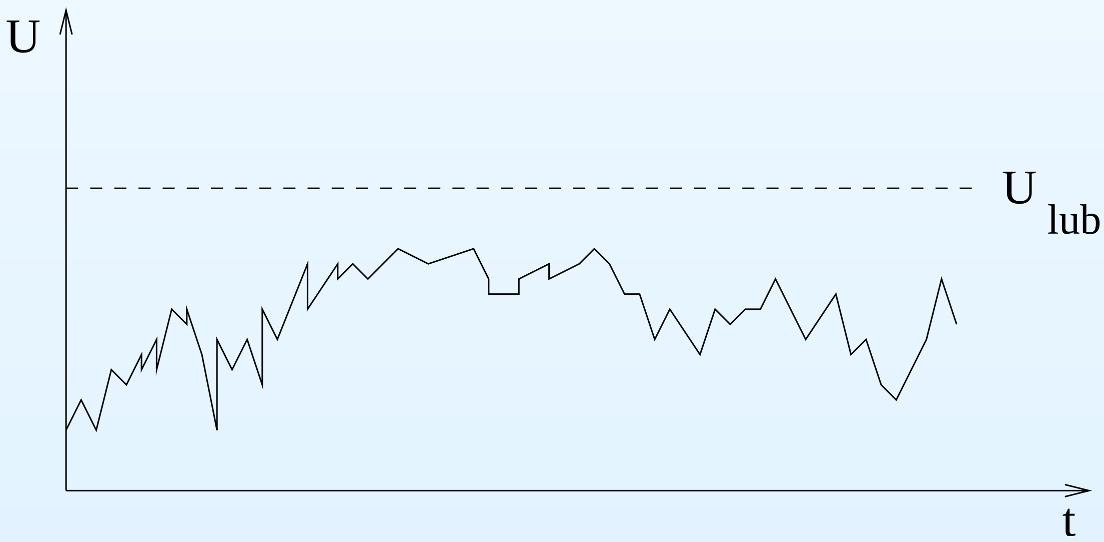
- Given a task set, how can we guarantee if it is schedulable of not?
- The first possibility is to *simulate* the system to check that no deadline is missed;
- The execution time of every job is set equal to the WCET of the corresponding task;
  - In case of periodic task with no offsets, it is sufficient to simulate the schedule until the *hyperperiod* ($H = lcm_i(T_i)$).
  - In case of offsets, it is sufficient to simulate until $2H + \phi_{\max}$.
  - If tasks periods are prime numbers the hyperperiod can be very large!

Example

- Exercise: Compare the hyperperiods of this two task sets:
  - $T_1 = 8, T_2 = 12, T_3 = 24;$
  - $T_1 = 7, T_2 = 12, T_3 = 25.$
- In case of sporadic tasks, we can assume them to arrive at the highest possible rate, so we fall back to the case of periodic tasks with no offsets!

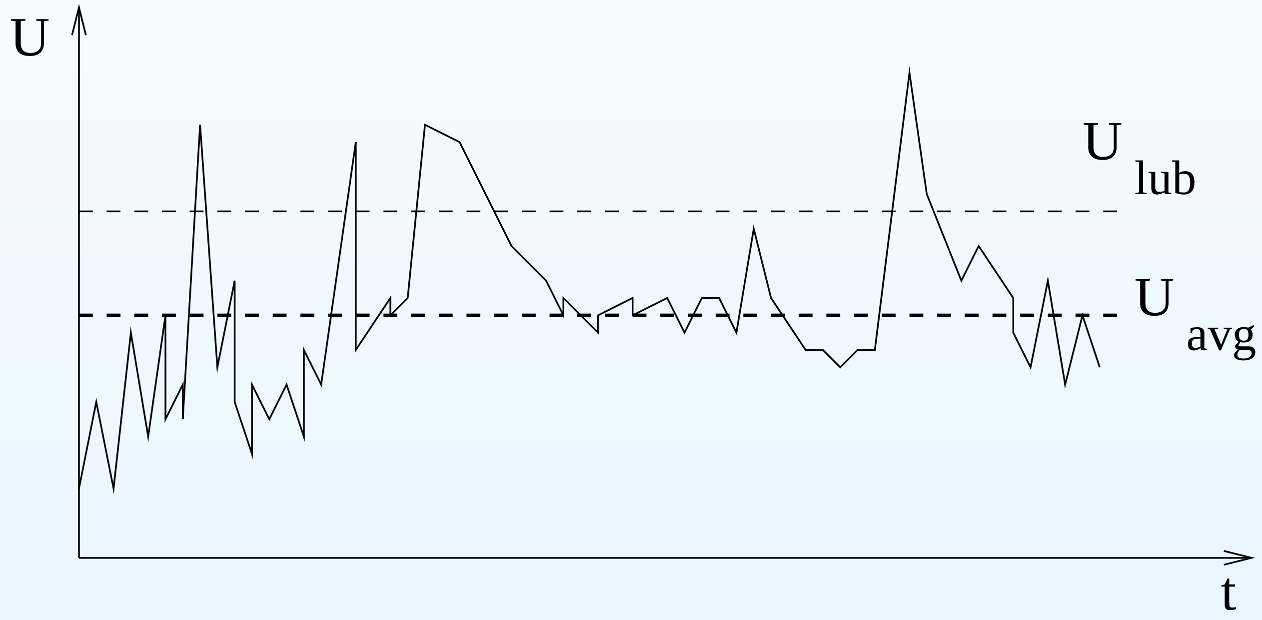
Utilization analysis

- In many cases it is useful to have a very simple test to see if the task set is schedulable.
- A sufficient test is based on the *Utilization bound*:
  - The *utilization least upper bound* for scheduling algorithm $\mathcal{A}$ is the smallest possible utilization U_{lub} such that, for any task set $\mathcal{T}$, if the task set's utilization U is not greater than U_{lub} ($U \leq U_{lub}$), then the task set is schedulable by algorithm $\mathcal{A}$.



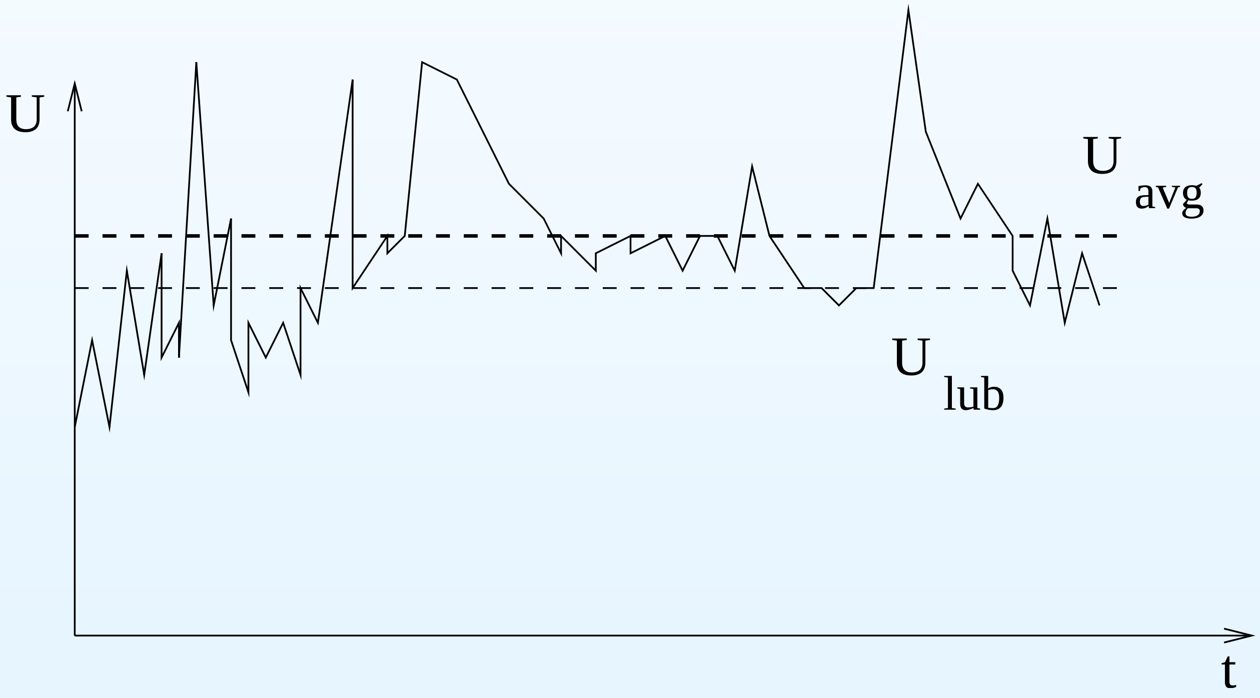
Maximum and average utilization

- If the average utilization is less than U_{lub}, the task set may or may not be schedulable:



Maximum and average utilization

- If the average utilization is greater than U_{lub}, the task set is "probably" not schedulable (depends on the scheduling algorithm).



Utilization bound for RM

- We consider n periodic (or sporadic) tasks with relative deadline equal to periods.
- Priorities are assigned with Rate Monotonic;
- $U_{lub} = n(2^{1/n} - 1)$
  - U_{lub} is a decreasing function of n;
  - For large n: $U_{lub} \approx 0.69$

n	**U**$_{lub}$	**n**	**U**$_{lub}$
2	0.828	7	0.728
3	0.779	8	0.724
4	0.756	9	0.720
5	0.743	10	0.717
6	0.734	11	...

Schedulability test

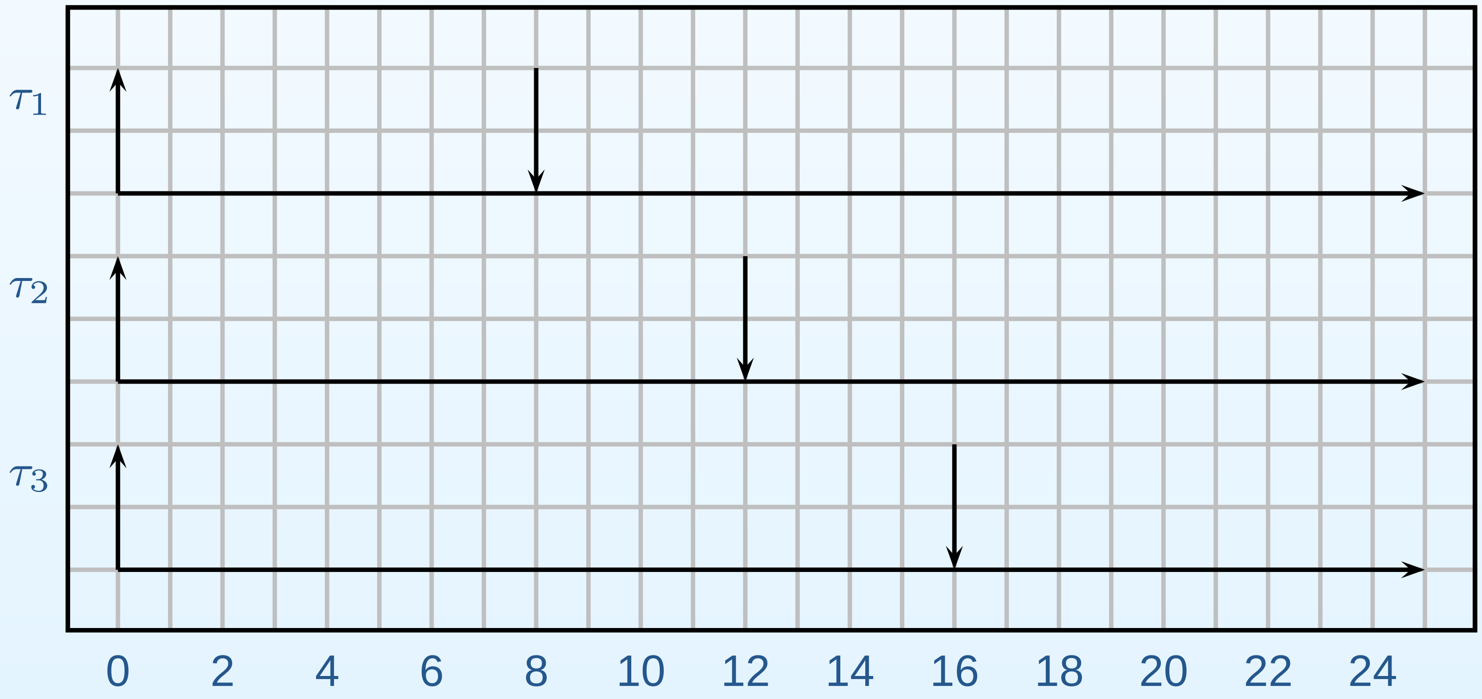
- Therefore the schedulability test consist in:
  - Compute $U = \sum_{i=1}^{n} \frac{C_i}{T_i}$;
  - if $U \leq U_{lub}$, the task set is schedulable;
  - if $U > 1$ the task set is not schedulable;
  - if $U_{lub} < U \leq 1$, the task set may or may not be schedulable;

Example

Example in which we show that for 3 tasks, if $U < U_{lub}$, the system is schedulable.

$\tau_1 = (2, 8), \tau_2 = (3, 12), \tau_3 = (4, 16);$

$$U = 0.75 < U_{lub} = 0.77$$



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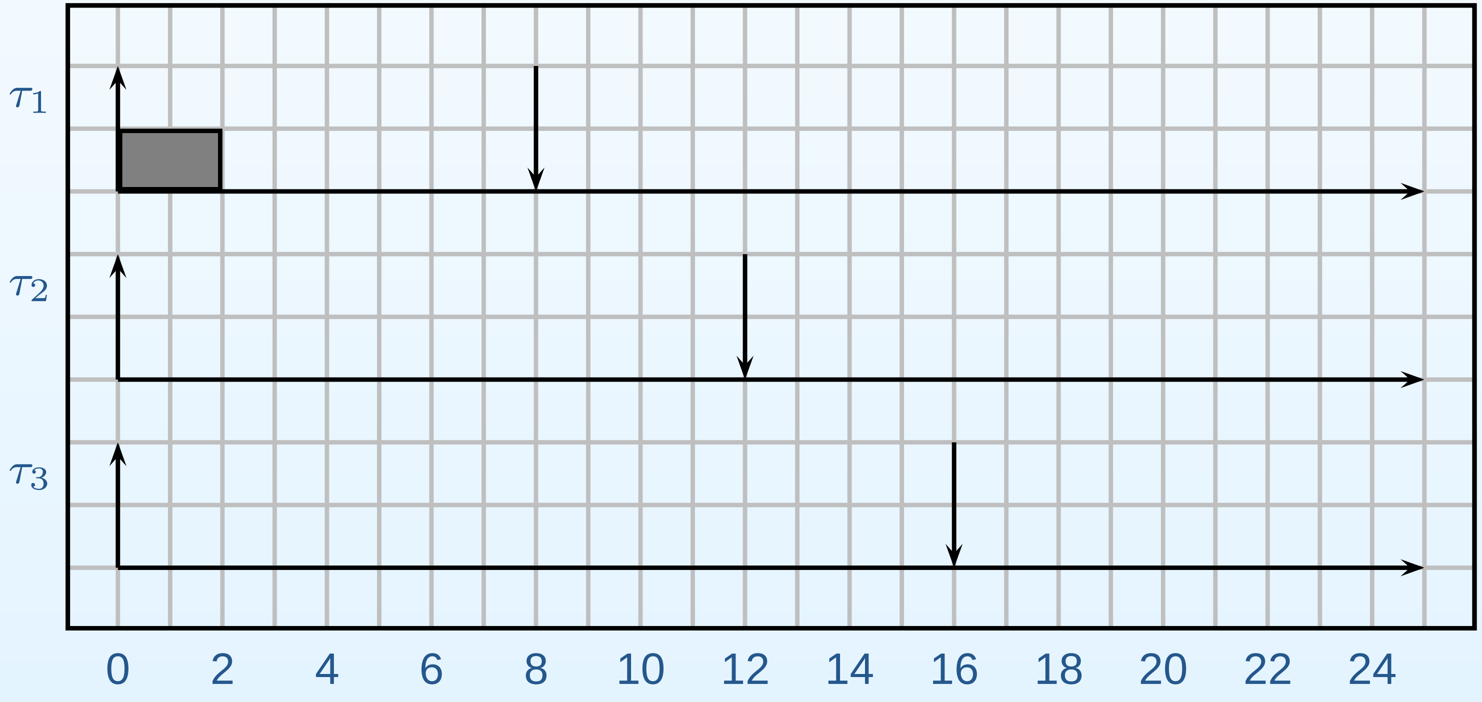

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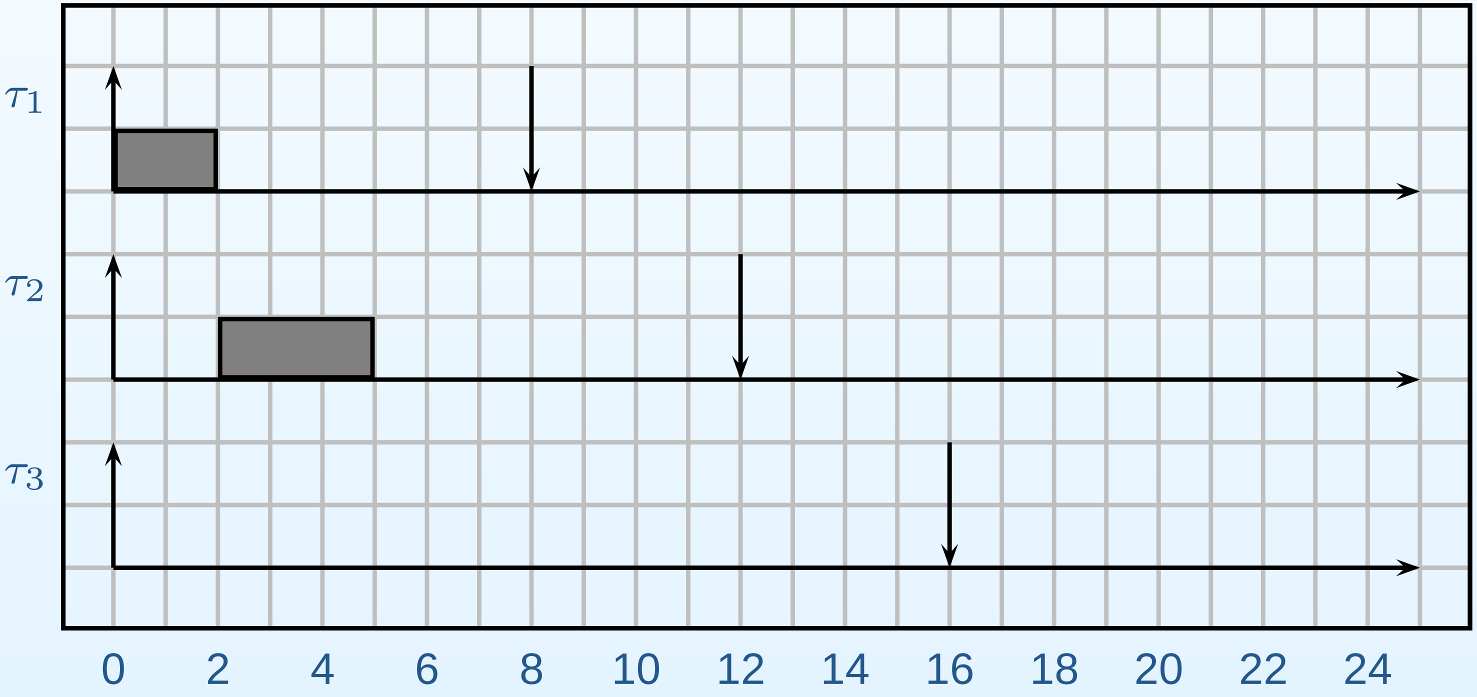

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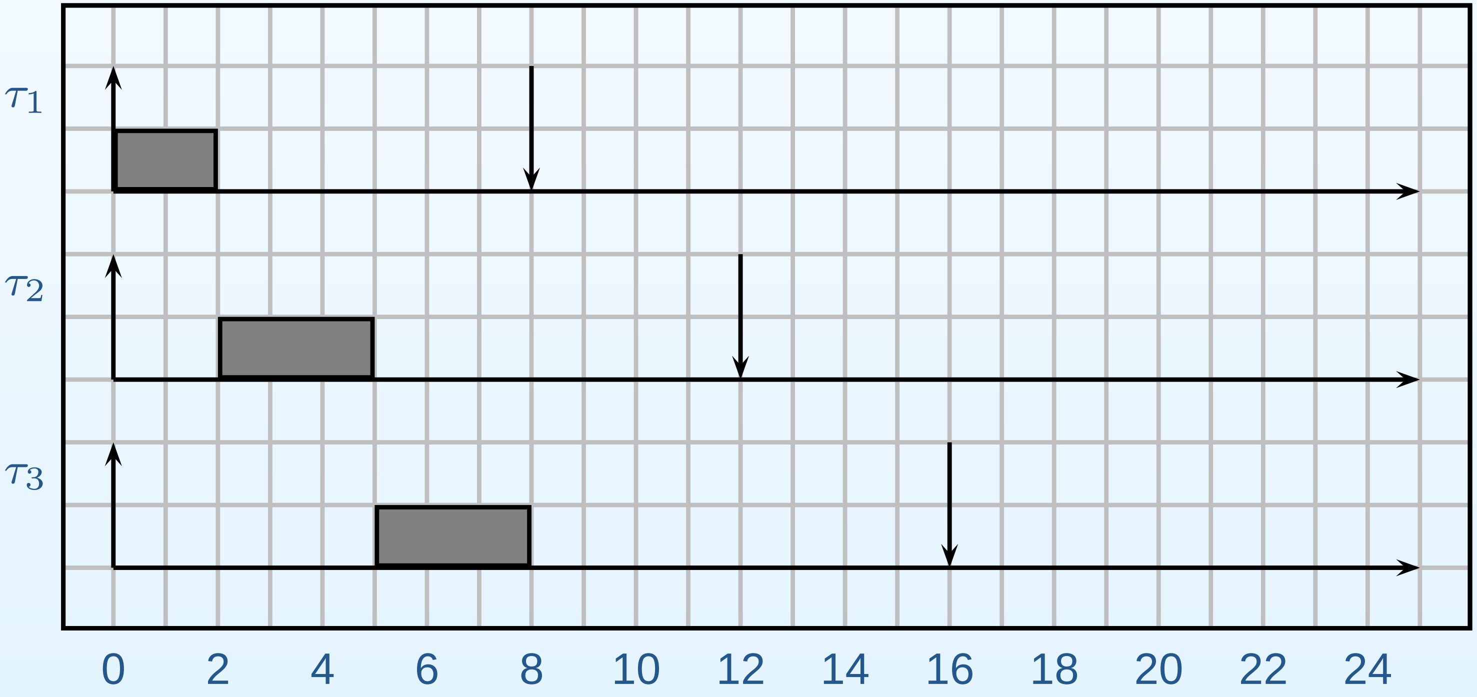

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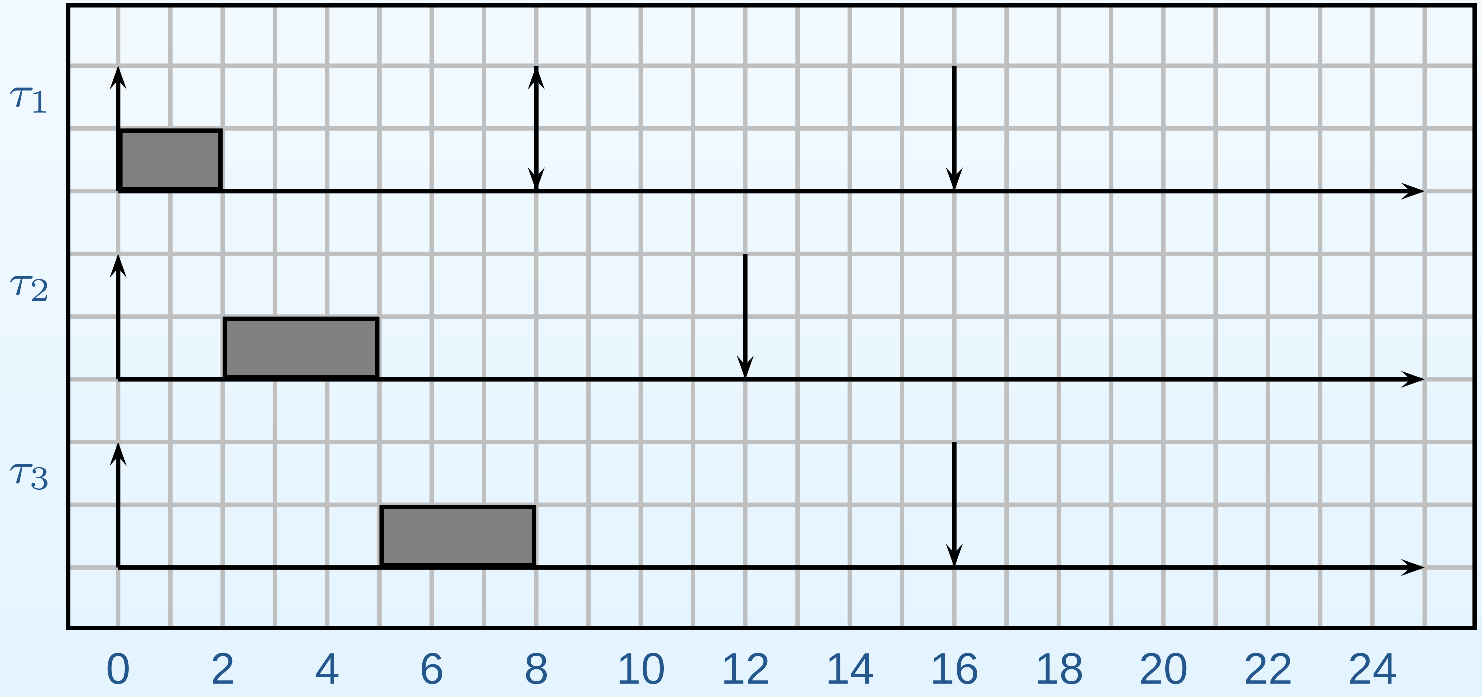

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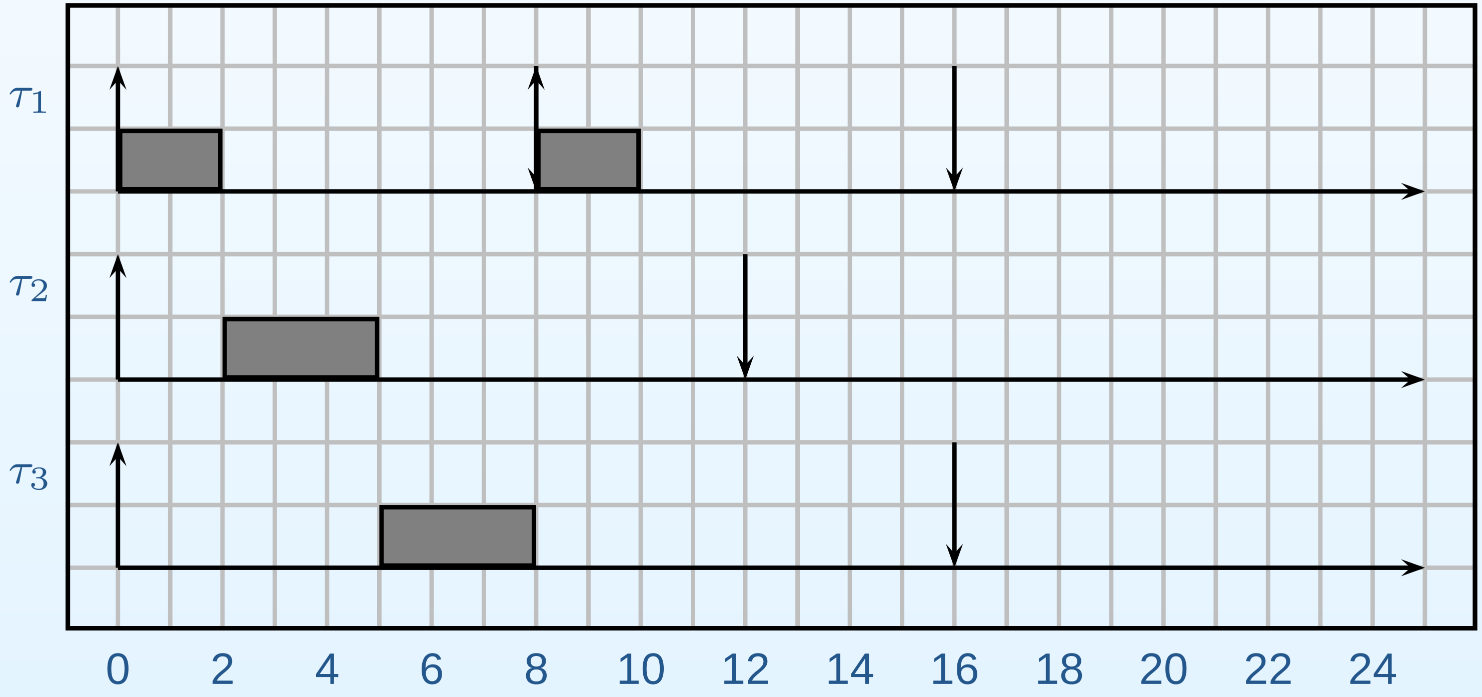

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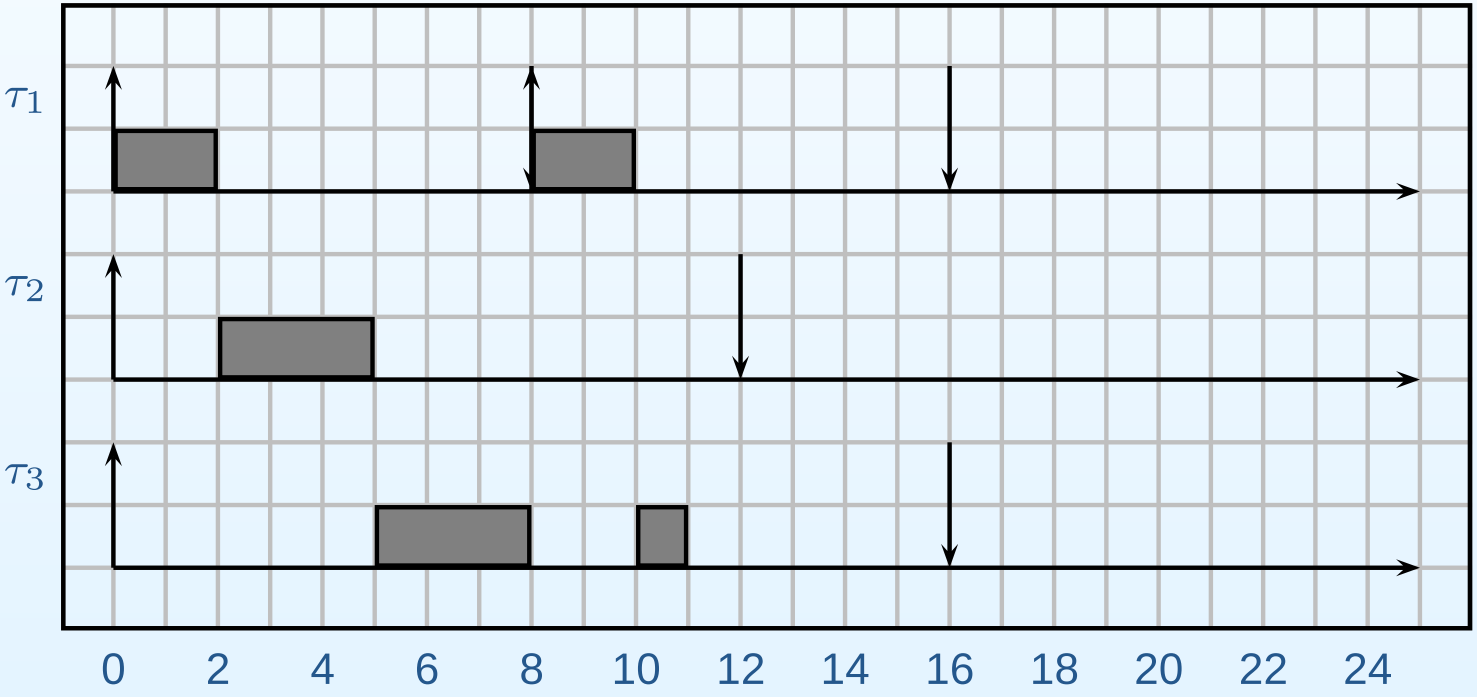

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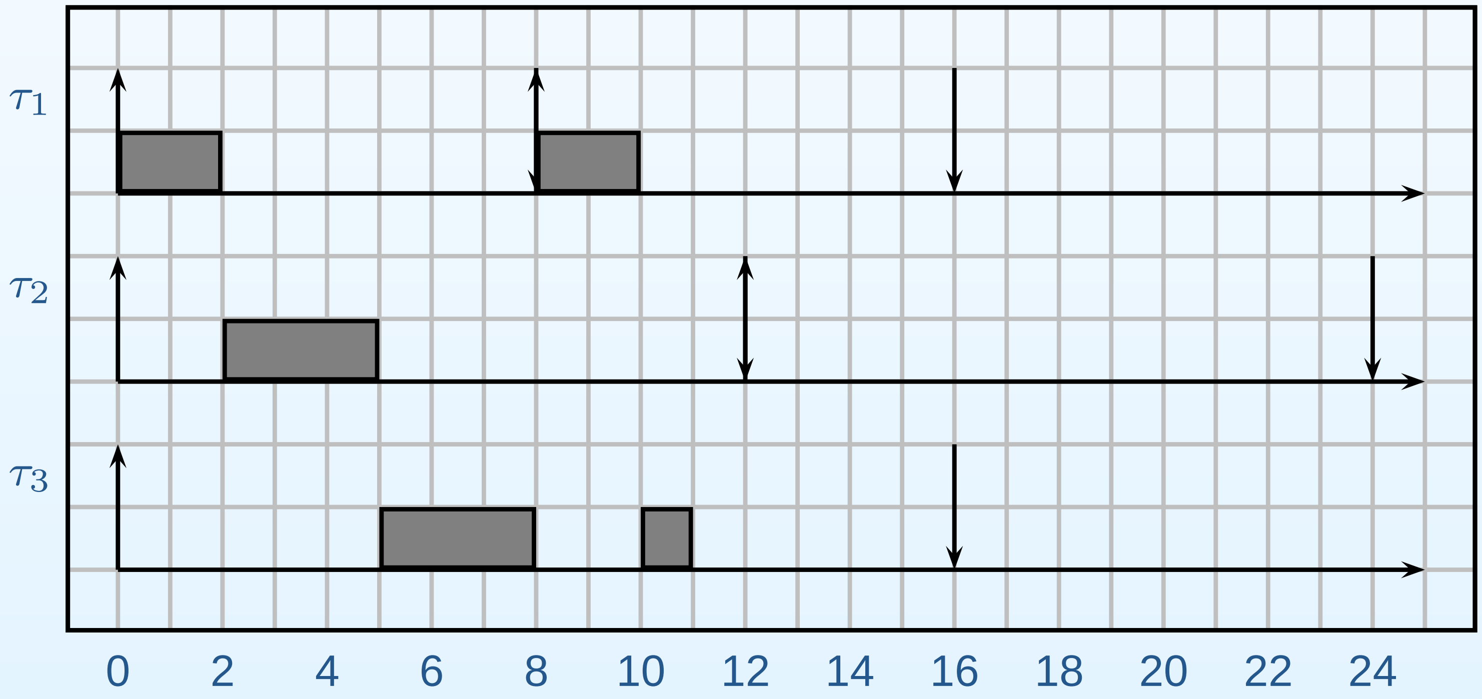

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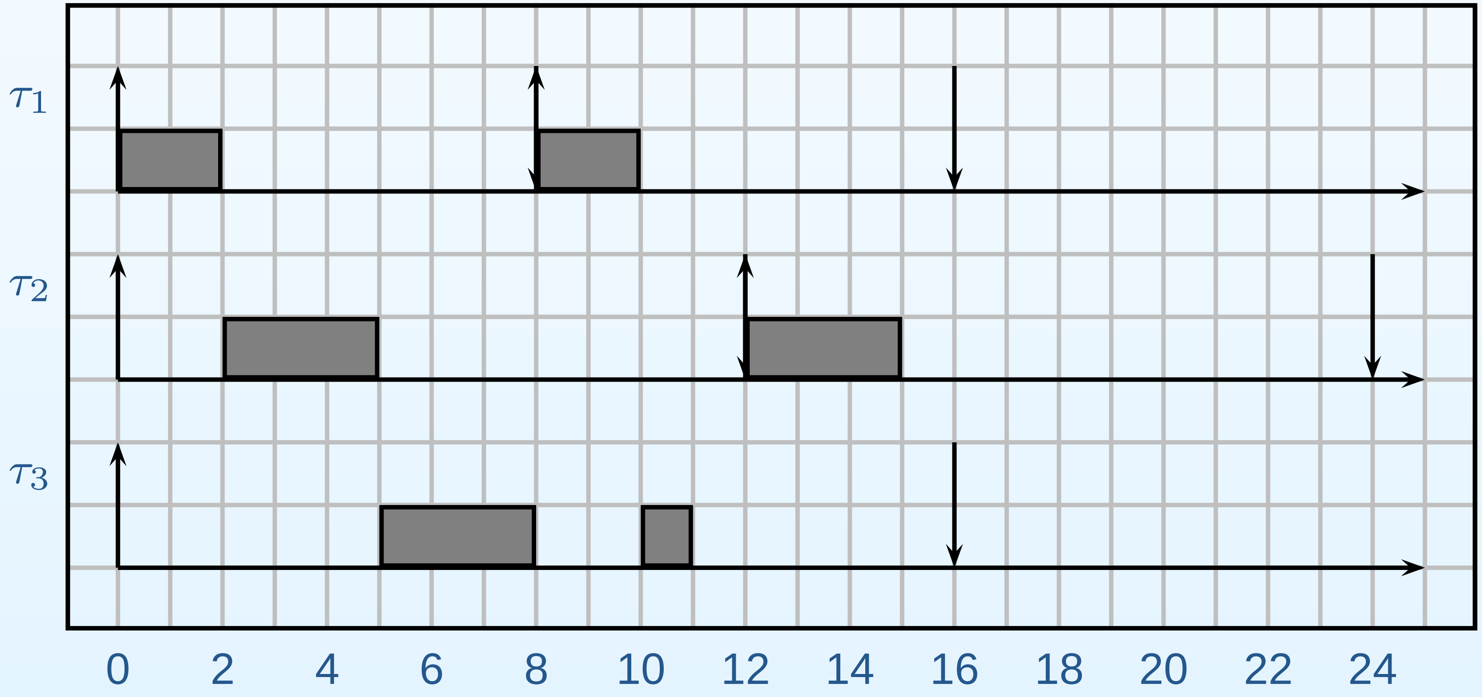

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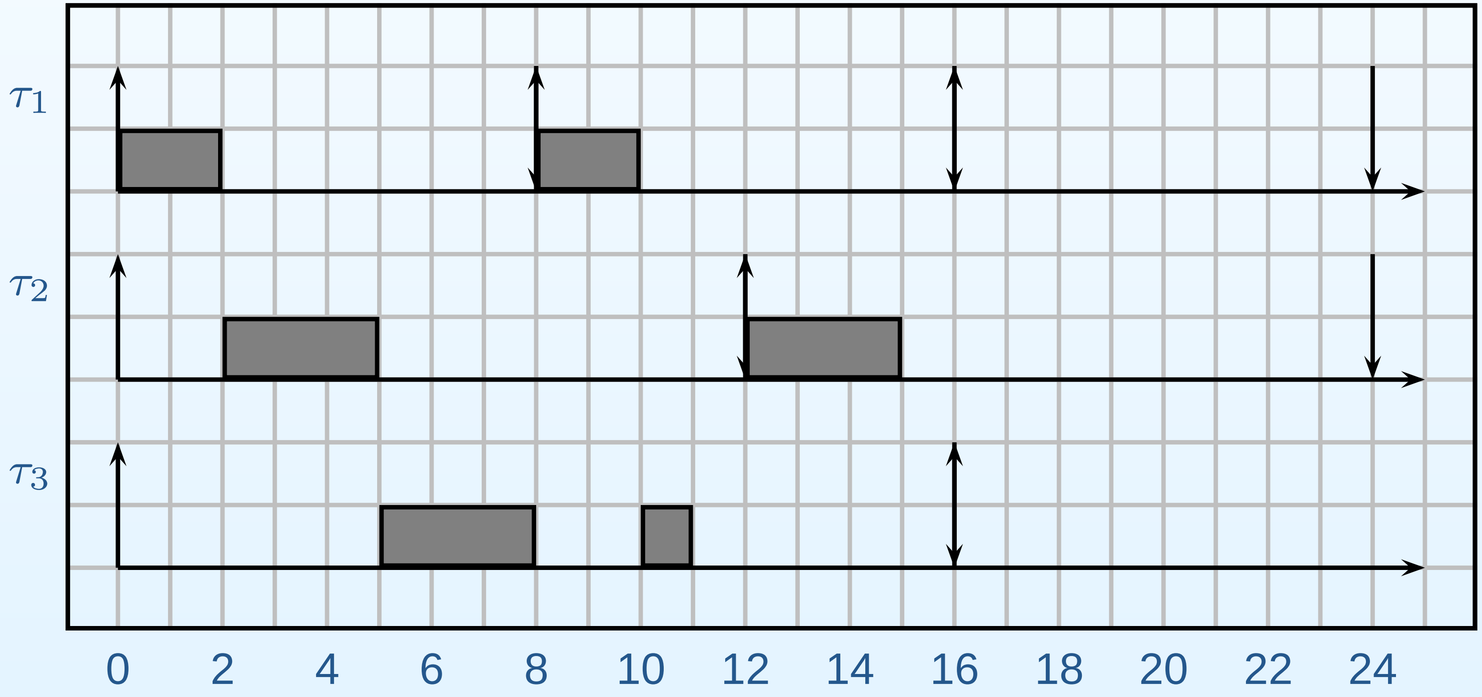

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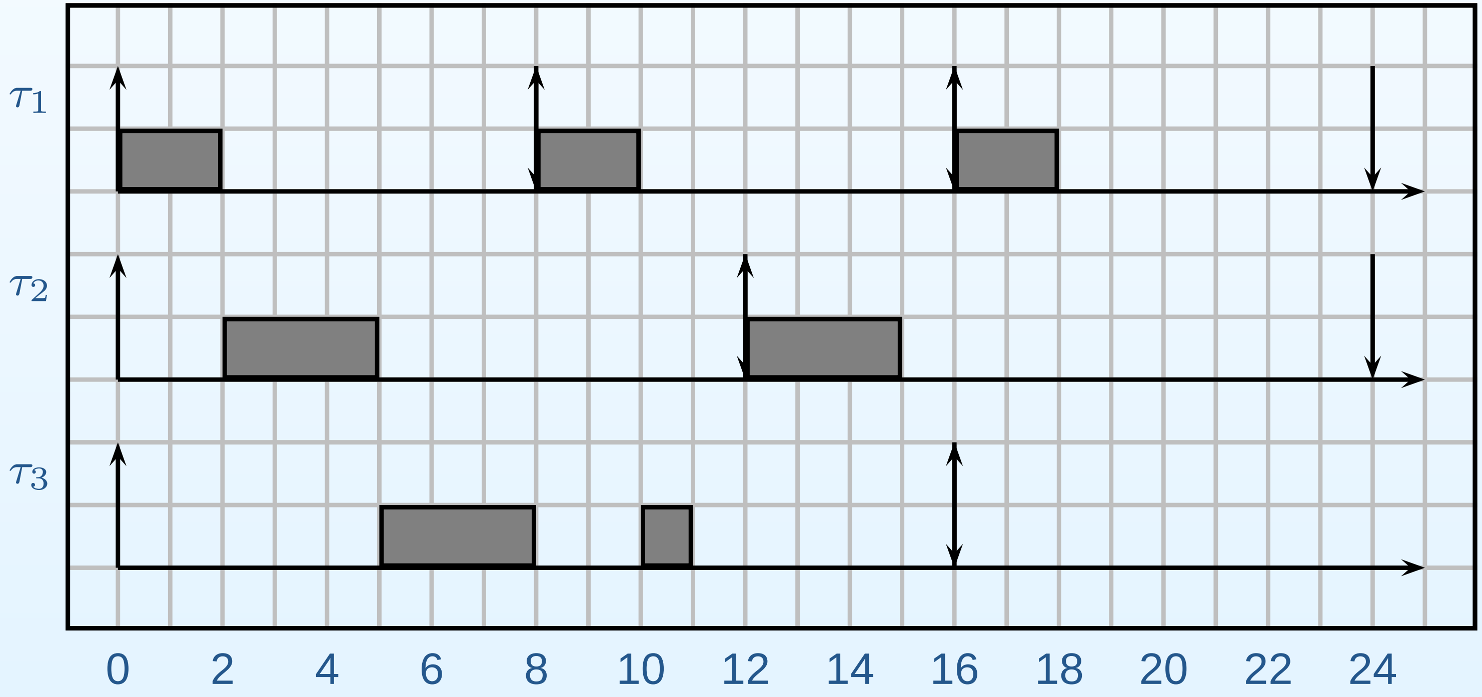

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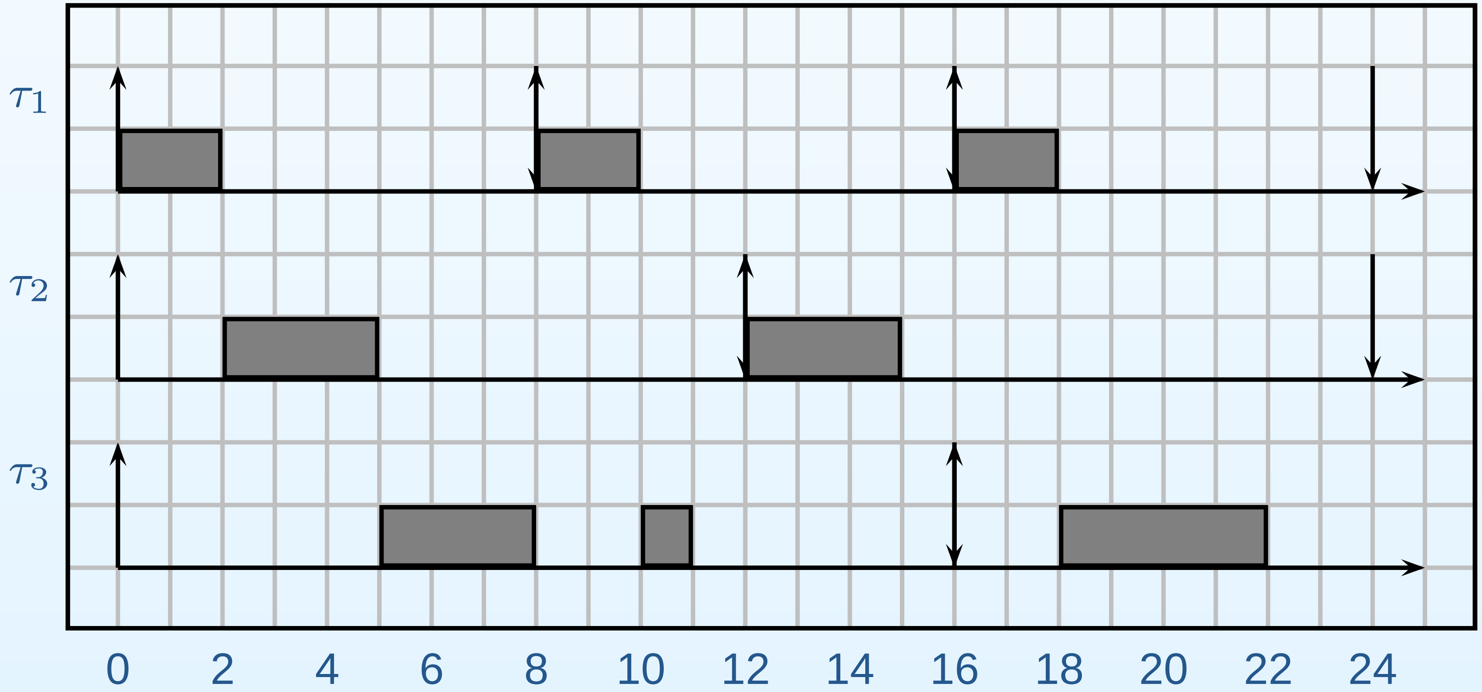
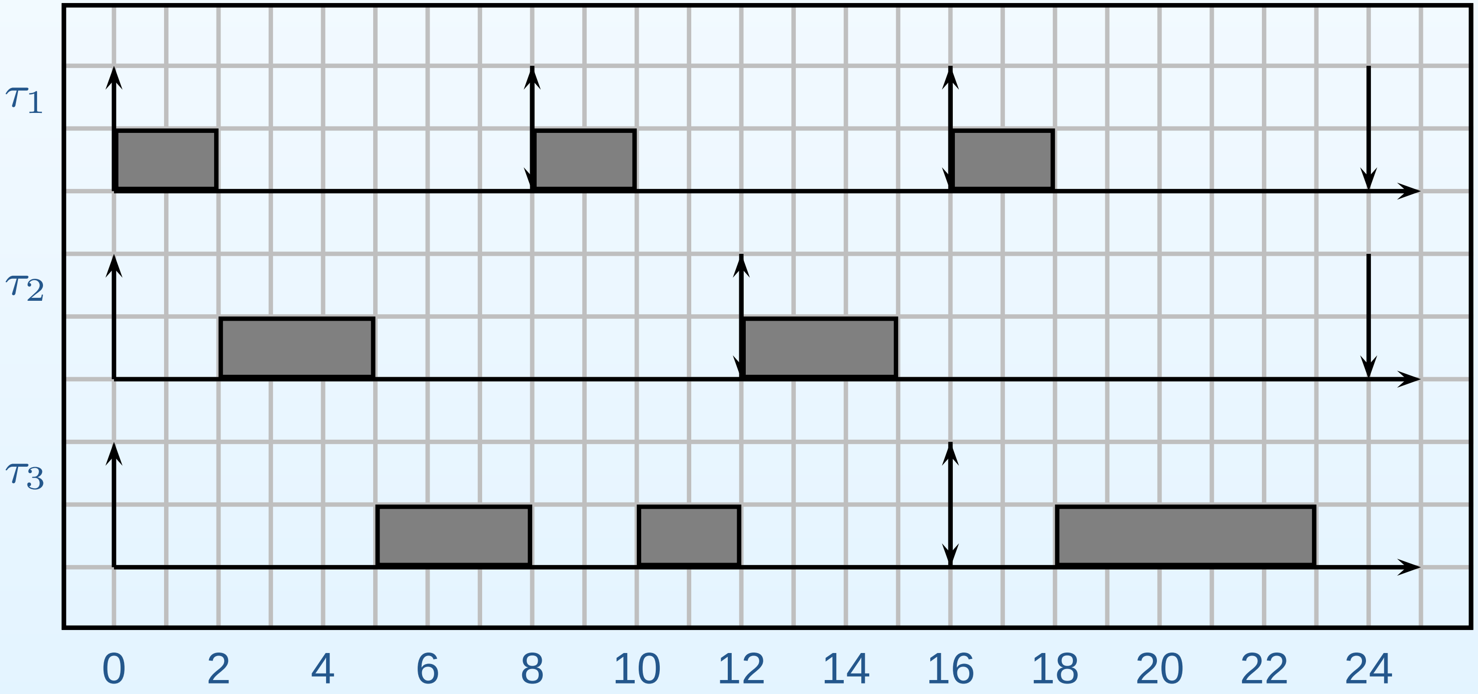


Example 2

By increasing the computation time of task τ_3, the system may still be schedulable . . .

$\tau_1 = (2, 8), \tau_2 = (3, 12), \tau_3 = (5, 16);$

$U = 0.81 > U_{lub} = 0.77$



Utilization bound for DM

- If relative deadlines are less than or equal to periods, instead of considering $U = \sum_{i=1}^{n} \frac{C_i}{T_i}$, we can consider:

$$U' = \sum_{i=1}^{n} \frac{C_i}{D_i}$$

- Then the test is the same as the one for RM (or DM), except that we must use U' instead of U.

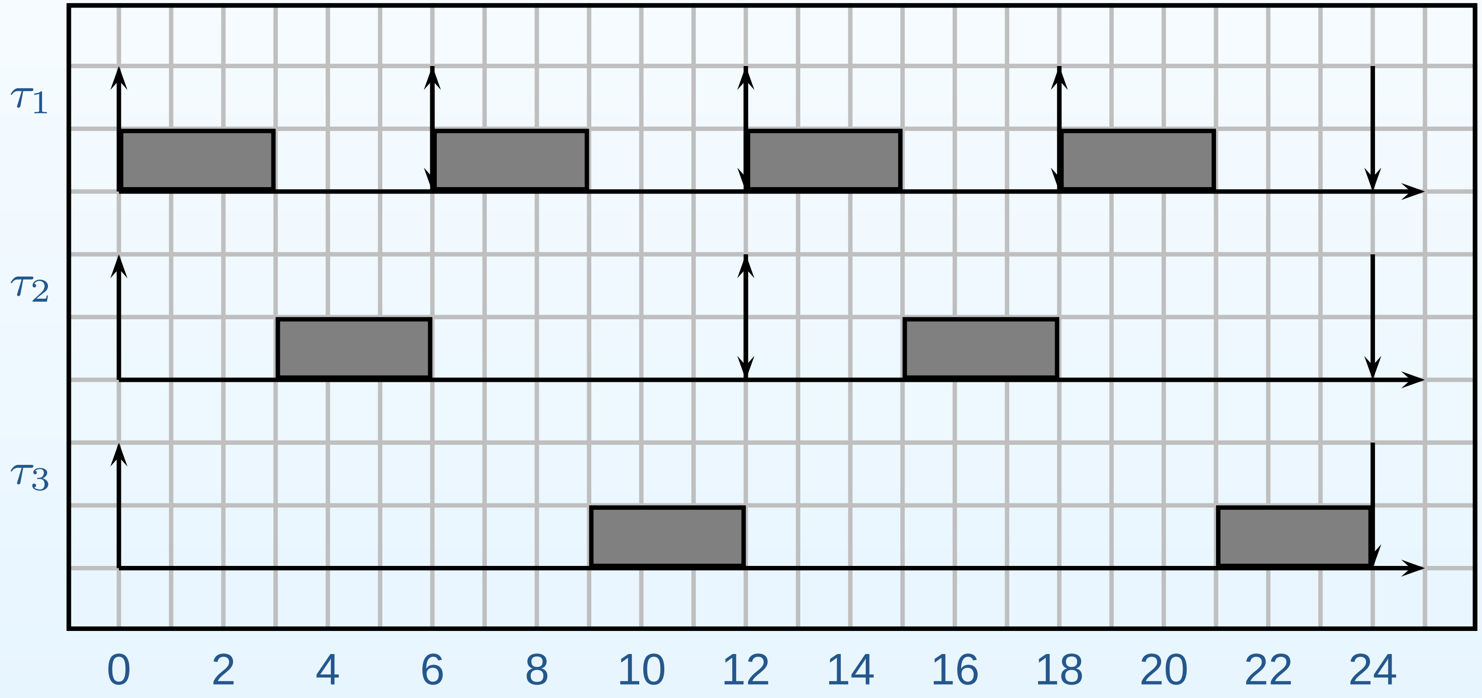
Pessimism

- The bound is very pessimistic: most of the times, a task set with $U > U_{lub}$ is schedulable by RM.
- A particular case is when tasks have periods that are *harmonic*:
  - A task set is *harmonic* if, for every two tasks τ_i, tau_j, either P_i is multiple of P_j or P_j is multiple of P_i.
- For a harmonic task set, the utilization bound is $U_{lub} = 1$.
- In other words, Rate Monotonic is an *optimal* algoritm for harmonic task sets.

Example of harmonic task set

$\tau_1 = (3, 6)$, $\tau_2 = (3, 12)$, $\tau_3 = (6, 24)$;

$U = 1$;



A necessary and sufficient test

Response time analysis

- A necessary and sufficient test is obtained by computing the *worst-case response time* (WCRT) for every task.
- For every task τ_i:
  - Compute the WCRT R_i for task τ_i;
  - If $R_i \leq D_i$, then the task is schedulable;
  - else, the task is not schedulable; we can also show the situation that make task τ_i miss its deadline!
- To compute the WCRT, we do not need to do any assumption on the priority assignment.
- The algorithm described in the next slides is valid for an arbitrary priority assignment.
- The algorithm assumes periodic tasks with no offsets, or sporadic tasks.

Response time analysis - II

- The *critical instant* for a set of periodic real-time tasks, with offset equal to 0, or for sporadic tasks, is when all jobs start at the same time.
- **Theorem:** The WCRT for a task corresponds to the response time of the job activated at the critical instant.
- To compute the WCRT of task τ_i:
  - We have to consider its computation time
  - and the computation time of the higher priority tasks (*interference*);
  - higher priority tasks can *preempt* task τ_i, and increment its response time.

Response time analysis - III

- Suppose tasks are ordered by decreasing priority. Therefore, $i < j \rightarrow prio_i > prio_j$.
- Given a task τ_i, let $R_i^{(k)}$ be the WCRT computed at step k.

$$R_i^{(0)} = C_i + \sum_{j=1}^{i-1} C_j$$

$$R_i^{(k)} = C_i + \sum_{j=1}^{i-1} \left\lceil \frac{R_i^{(k-1)}}{T_j} \right\rceil C_j$$

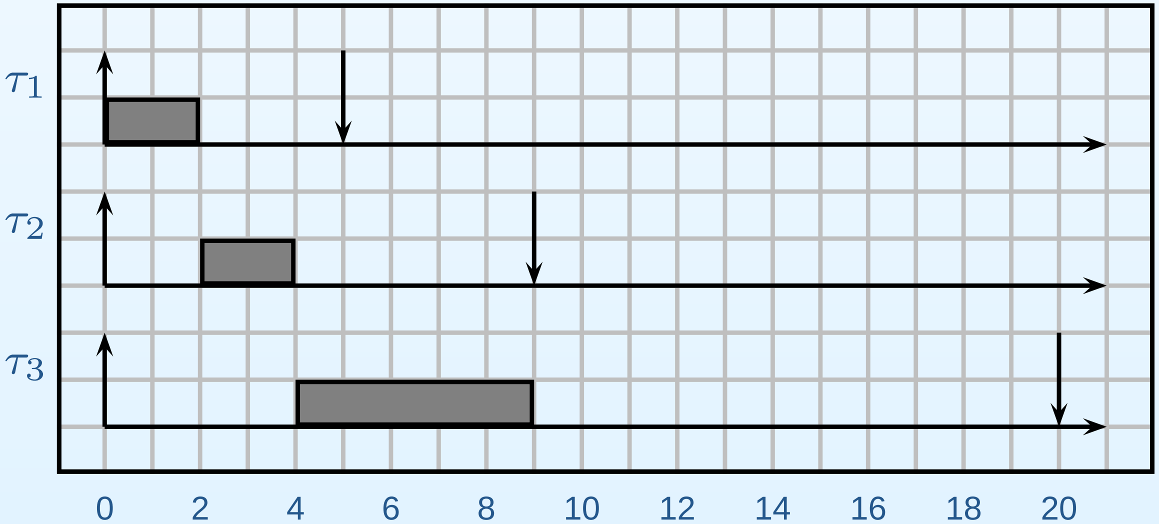
- The iteration stops when:
  - $R_i^{(k)} =_i^{(k+1)}$ *or*
  - $R_i^{(k)} > D_i$ (non schedulable);

Example

Consider the following task set: $\tau_1 = (2,5)$, $\tau_2 = (2,9)$, $\tau_3 = (5,20)$; $U = 0.872$.

$$R_i^{(k)} = C_i + \sum_{j=1}^{i-1} \left\lceil \frac{R_i^{(k-1)}}{T_j} \right\rceil C_j$$

$$R_3^{(0)} = C_3 + 1 \cdot C_1 + 1 \cdot C_2 = 9$$



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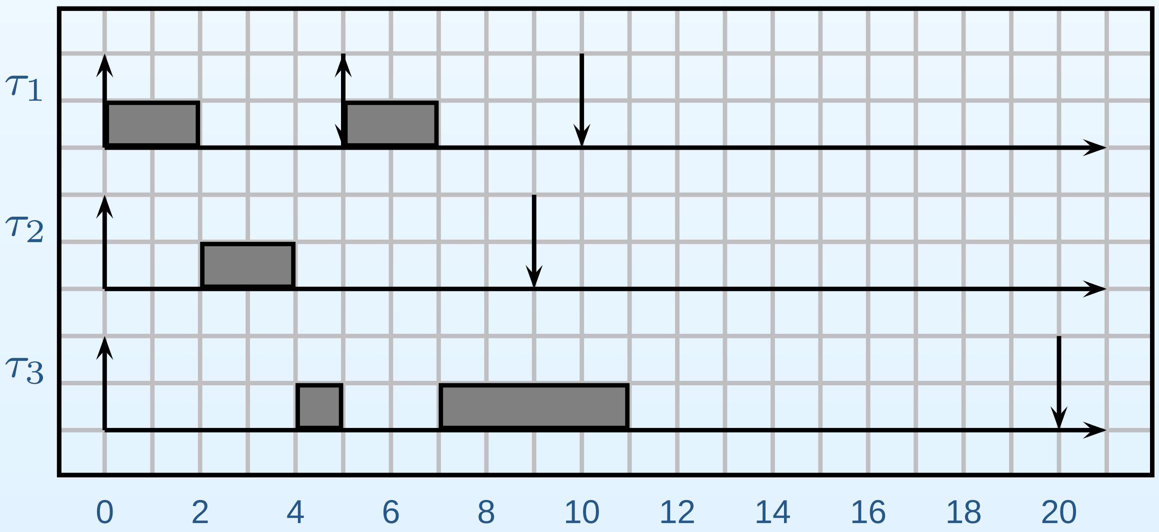
$$R_3^{(1)} = C_3 + 2 \cdot C_1 + 1 \cdot C_2 = 11$$



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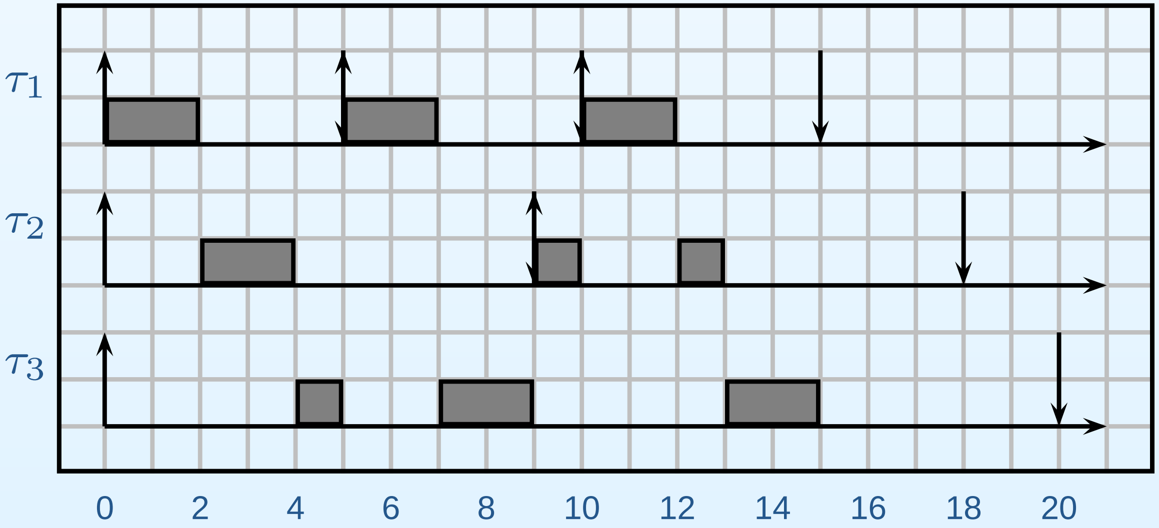
$$R_3^{(2)} = C_3 + 3 \cdot C_1 + 2 \cdot C_2 = 15$$



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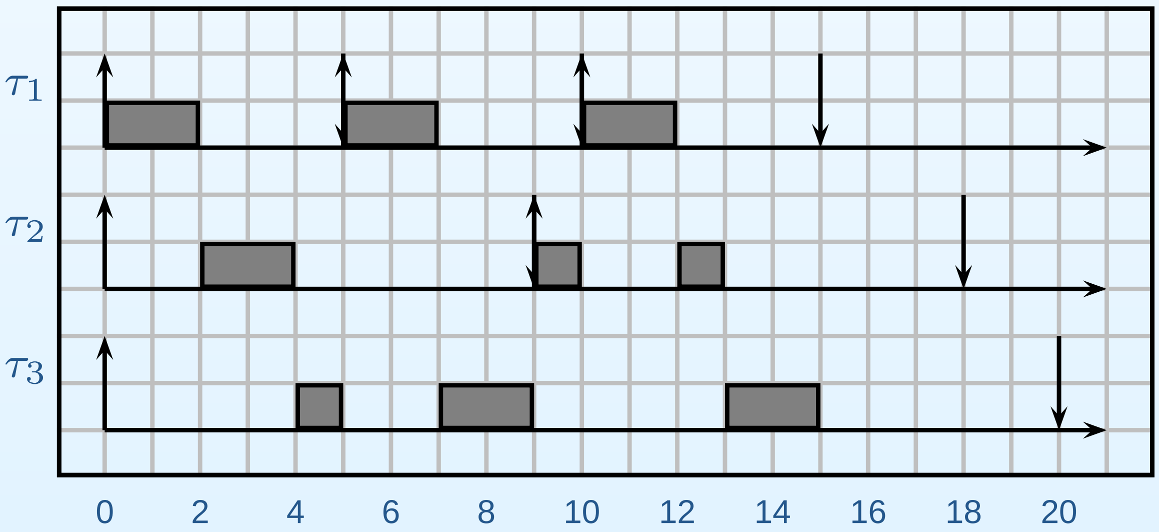


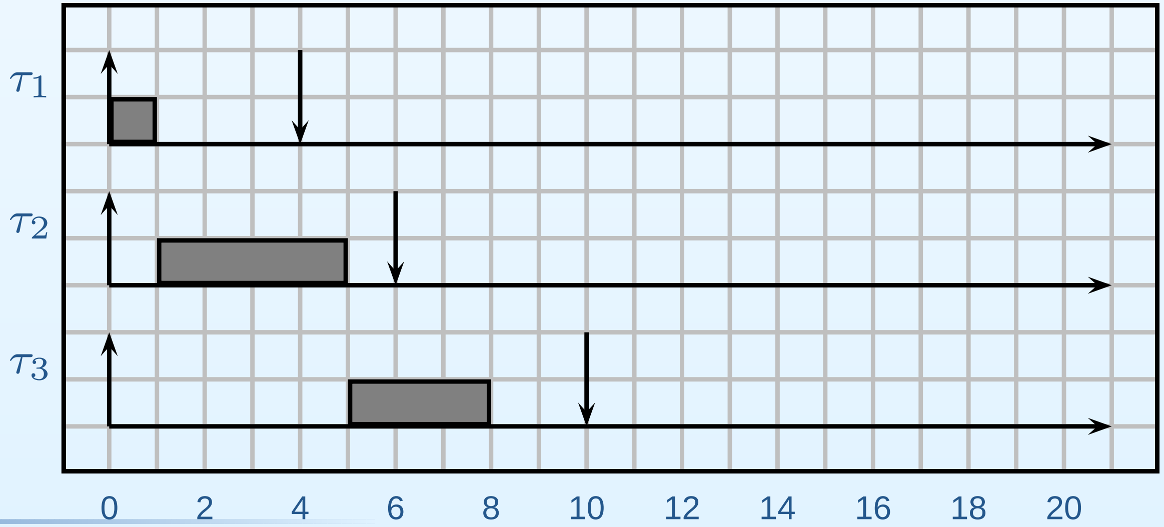
Another example with DM

The method is valid for different priority assignments and deadlines different from periods.

$\tau_1 = (1, 4, 4), p_1 = 3$, $\tau_2 = (4, 6, 15), p_2 = 2$, $\tau_3 = (3, 10, 10), p_3 = 1$; $U = 0.72$

$$R_i^{(k)} = C_i + \sum_{j=1}^{i-1} \left\lceil \frac{R_i^{(k-1)}}{T_j} \right\rceil C_j$$

$$R_3^{(0)} = C_3 + 1 \cdot C_1 + 1 \cdot C_2 = 8$$



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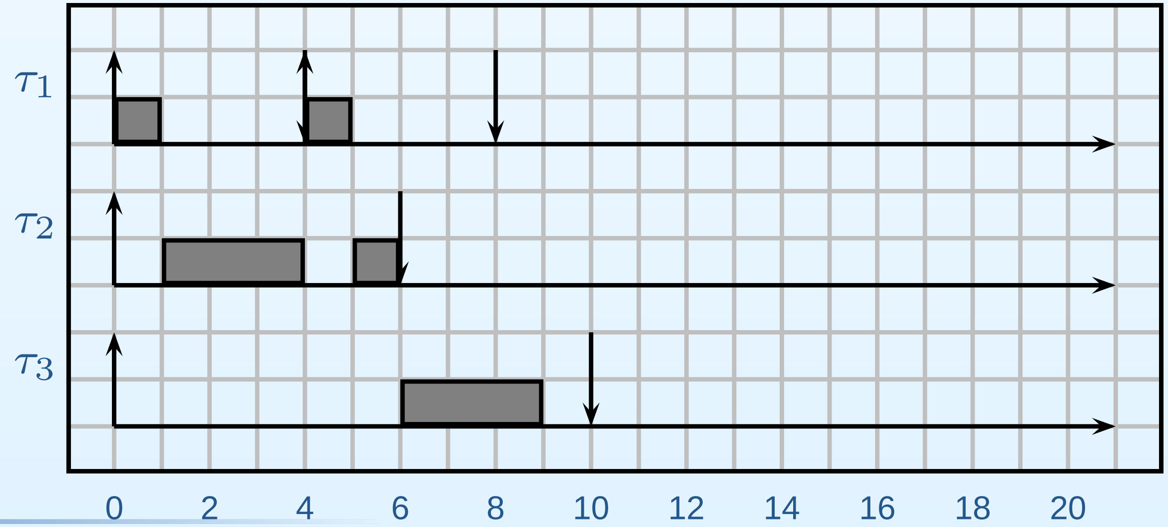

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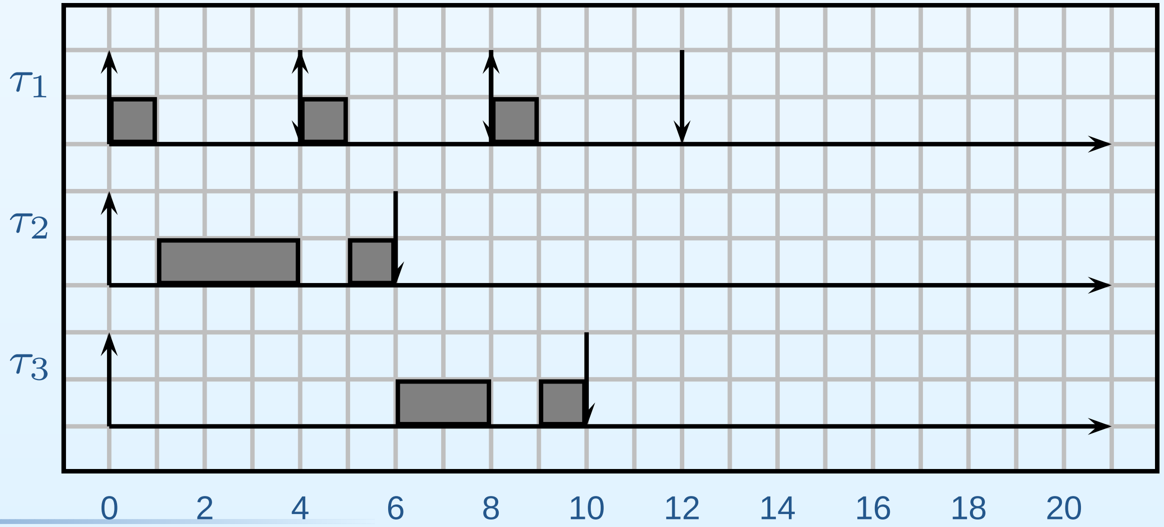
$$R_3^{(2)} = C_3 + 3 \cdot C_1 + 2 \cdot C_2 = 10$$



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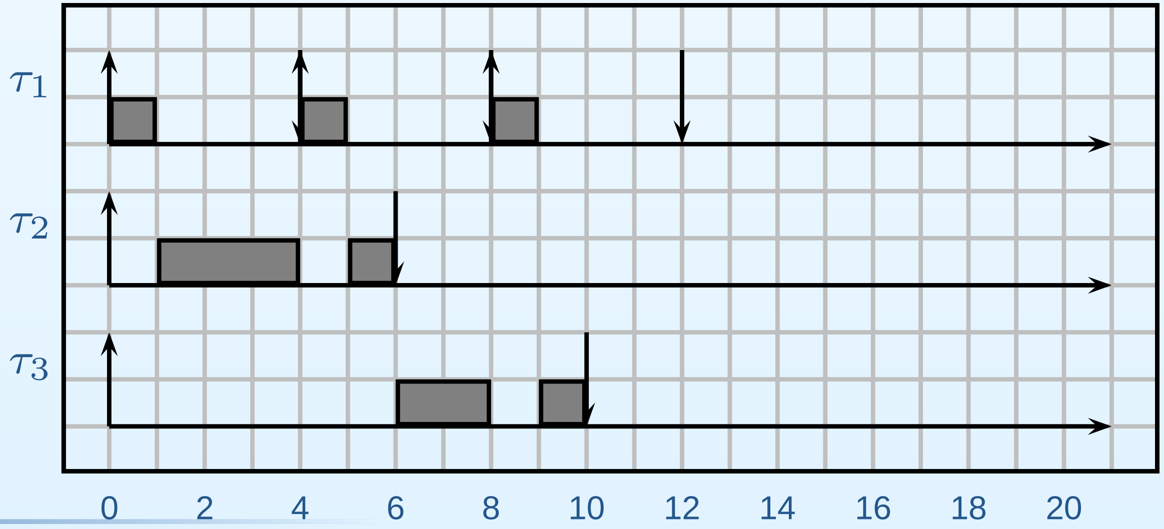


Considerations

- The response time analysis is an efficient algorithm
  - In the worst case, the number of steps N for the algorithm to converge is exponential
    - → It depends on the total number of jobs of higher priority tasks that may be contained in the interval $[0, D_i]$:

$$N \propto \sum_{j=1}^{i-1} \left\lceil \frac{D_i}{T_i} \right\rceil$$

    - → If s is the minimum granularity of the time, then in the worst case $N = \frac{D_i}{s}$;
  - However, such worst case is very rare: usually, the number of steps is low.

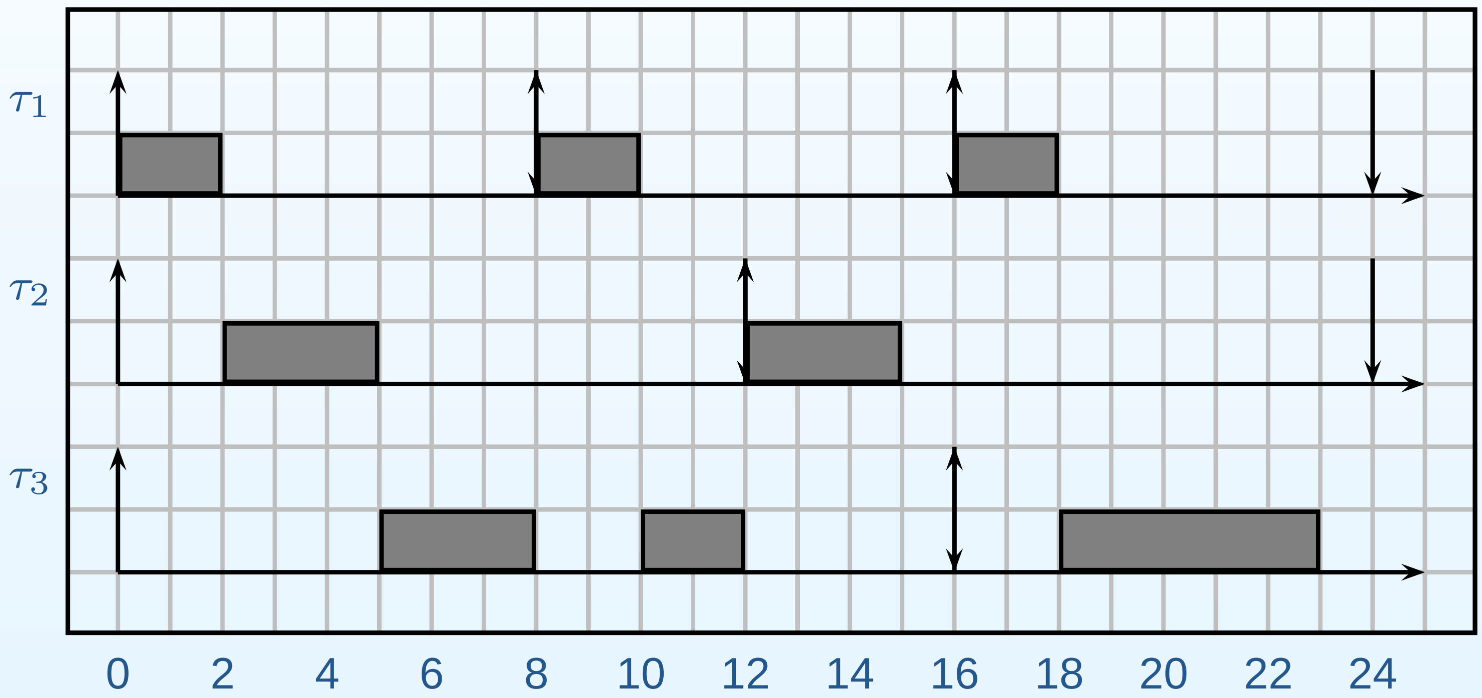
Sensitivity

Considerations on WCET

- The response time analysis is a necessary and sufficient test for fixed priority.
- However, the result is very sensitive to the value of the WCET.
  - If we are wrong in estimating the WCET (and for example we put a value that is too low), the actual system may be not schedulable.
- The value of the response time is not helpful: even if the response time is well below the deadline, a small increase in the WCET of a higher priority task makes the response time *jump*;
- We may see the problem as a sensitivity analysis problem: we have a function $R_i = f_i(C_1, T_1, C_2, T_2, \ldots, C_{i-1}, T_{i-1}, C_i)$ that is non-continuous.

Example of discontinuity

Let's consider again the example done *before*; we increment the computation time of τ_1 of 0.1.



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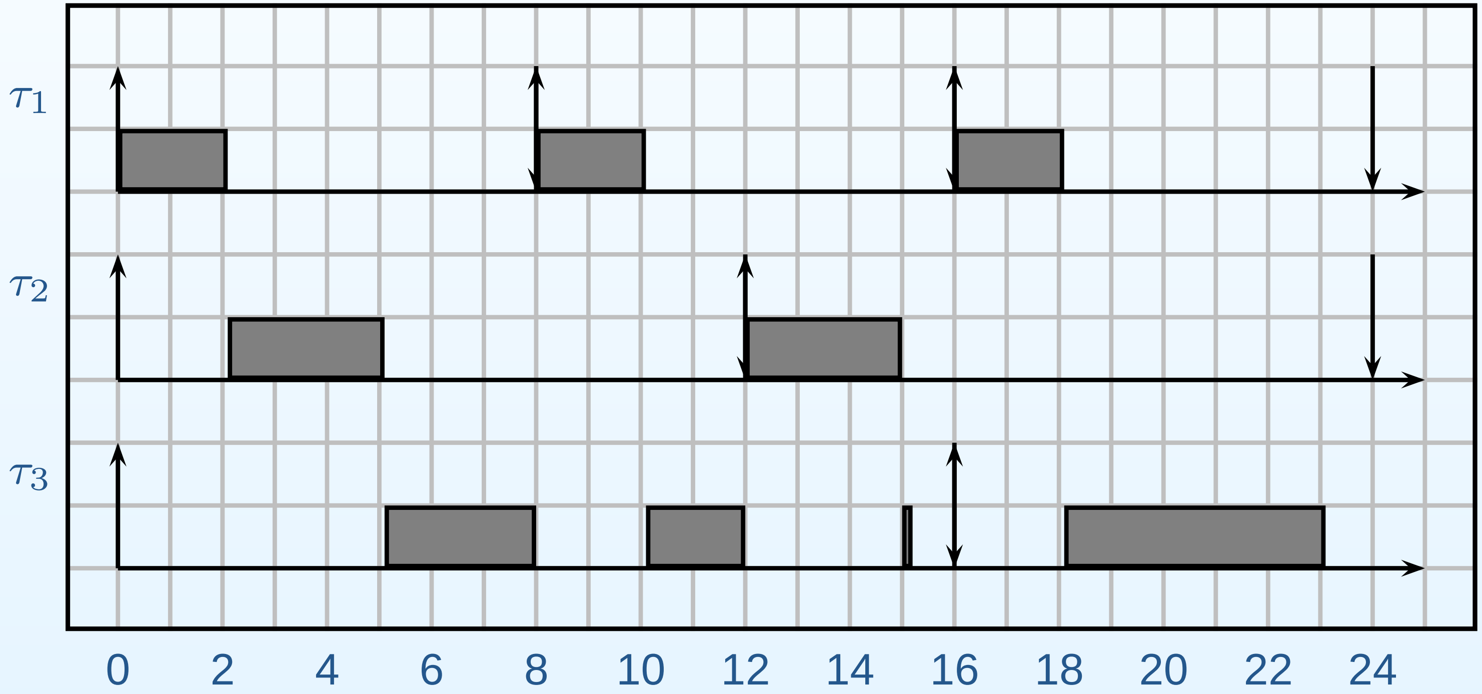


$$R_3 = 12 \rightarrow 15.2$$

Singularities

- The response time of a task τ_i is the first time at which all tasks $\tau_1, \ldots, \tau_i$ have completed;
- At this point,
  - either a lower priority task τ_j ($p_j < p_i$) is executed
  - or the system becoms idle
  - or it coincides with the arrival time of a higher priority task.
- In the last case, such an instant is also called i-level singularity point.

Sensitivity on WCETs

- Therefore, a rule of thumb is to increase the WCET by a certain percentage before doing the analysis. If the task set is still feasible, be are more confident about the schedulability of the original system.
- There are analytical methods for computing the amount of variation that it is possible to allow to a task's WCET without compromising the schedulability (see Hyperplane analysis).

Summary of schedulability tests for FP

- Utilization bound test:
  - depends on the number of tasks;
  - for large n, $U_{lub} = 0.69$;
  - only sufficient;
  - $\mathcal{O}(n)$ complexity;
- Response time analysis:
  - necessary and sufficient test for periodic tasks with arbitrary deadlines and with no offset.
  - complexity: high (*pseudo-polynomial*);

Response time analysis - extensions

- Consider offsets
- Arbitrary patterns of arrivals. Burst, quasi–periodic, etc.

Esercizio

Dato il seguente task set:

Task	C_i	D_i	T_i
τ_1	1	4	4
τ_2	2	9	9
τ_3	3	6	12
τ_4	3	20	20

Calcolare il tempo di risposta dei vari task nell'ipotesi che le priorità siano assegnate con RM o con DM.

Risposta: Nel caso di RM,

$$\tau_1 = 1 \quad \tau_2 = 3 \quad \tau_3 = 7 \quad \tau_4 = 18$$

Nel caso di DM,

$$\tau_1 = 1 \quad \tau_2 = 7 \quad \tau_3 = 4 \quad \tau_4 = 18$$

Esercizio

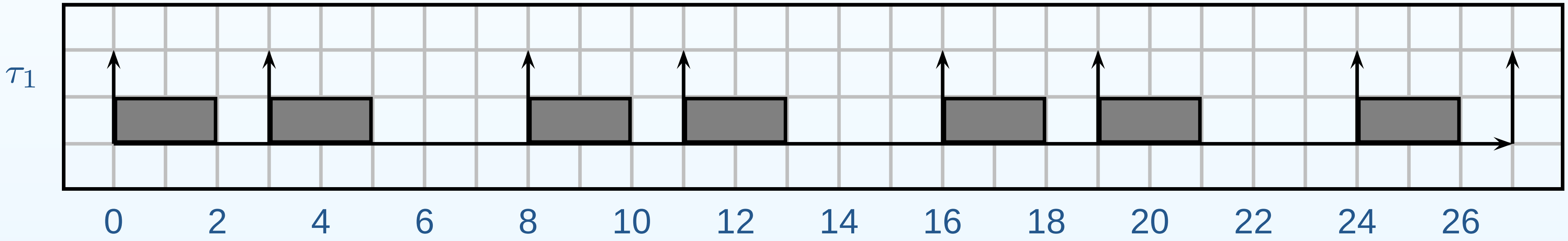
Consideriamo il seguente task τ_1 *non periodico*:

- Se j è pari, allora $a_{1,j} = 8 \cdot \frac{j}{2}$;
- Se j è dispari, allora $a_{1,j} = 3 + 8 \cdot \left\lfloor \frac{j}{2} \right\rfloor$;
- In ogni caso, $c_{1,j} = 2$;
- La priorità del task τ_1 è $p_1 = 3$.

Nel sistema, consideriamo anche i task periodici $\tau_2 = (2, 12, 12)$ e $\tau_3 = (3, 16, 16)$, con priorità $p_2 = 2$ e $p_3 = 1$. Calcolare il tempo di risposta dei task τ_2 e τ_3.

Soluzione - I

Il pattern di arrivo del task τ_1 è il seguente:



Il task τ_1 è ad alta priorità, quindi il suo tempo di risposta è pari a 2. Come questo task interferisce con gli altri due task a bassa priorità?

Soluzione - II

Bisogna estendere la formula del calcolo del tempo di risposta. La generalizzazione è la seguente:

$$R_i^{(k)} = C_i + \sum_{j=1}^{i-1} Nist_j(R_i^{(k-1)})C_j$$

dove $Nist_j(t)$ rappresenta il numero di istanze del task τ_j che “arrivano” nell’intervallo $[0, t)$.

Se il task τ_j è periodico, allora $Nist_j(t) = \left\lceil \frac{t}{T_j} \right\rceil$.

Nel caso invece del task τ_1 che non è periodico:

$$Nist_1(t) = \left\lceil \frac{t}{8} \right\rceil + \left\lceil \frac{\max(0, t-3)}{8} \right\rceil$$

Il primo termine tiene conto delle istanze con j pari, mentre il secondo termine tiene conto delle istanze con j dispari.

Soluzione - III

Applicando la formula per calcolare il tempo di risposta del task τ_2:

$$R_2^{(0)} = 2 + 2 = 4 \qquad R_2^{(1)} = 2 + 2 \cdot 2 = 6$$
$$R_2^{(2)} = 2 + 2 \cdot 2 = 6$$

Per il task τ_3:

$$R_3^{(0)} = 3 + 2 + 2 = 7 \qquad R_3^{(1)} = 3 + 2 \cdot 2 + 1 \cdot 2 = 9$$
$$R_3^{(2)} = 3 + 3 \cdot 2 + 1 \cdot 2 = 11 \qquad R_3^{(3)} = 3 + 3 \cdot 2 + 1 \cdot 2 = 11$$