Laurea Specialistica in Ingegneria dell'Automazione

Sistemi in Tempo Reale

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Introduzione

Lecture schedule

- Introduction
- Selected topics on discrete-time and sampled data systems
- Discrete Equivalents
- Design
- Quantisation effects
- Sampling rate selection and multirate systems
- FSM
- Hybrid systems

Outline

- Generalities
- The development flow
- An introductory example

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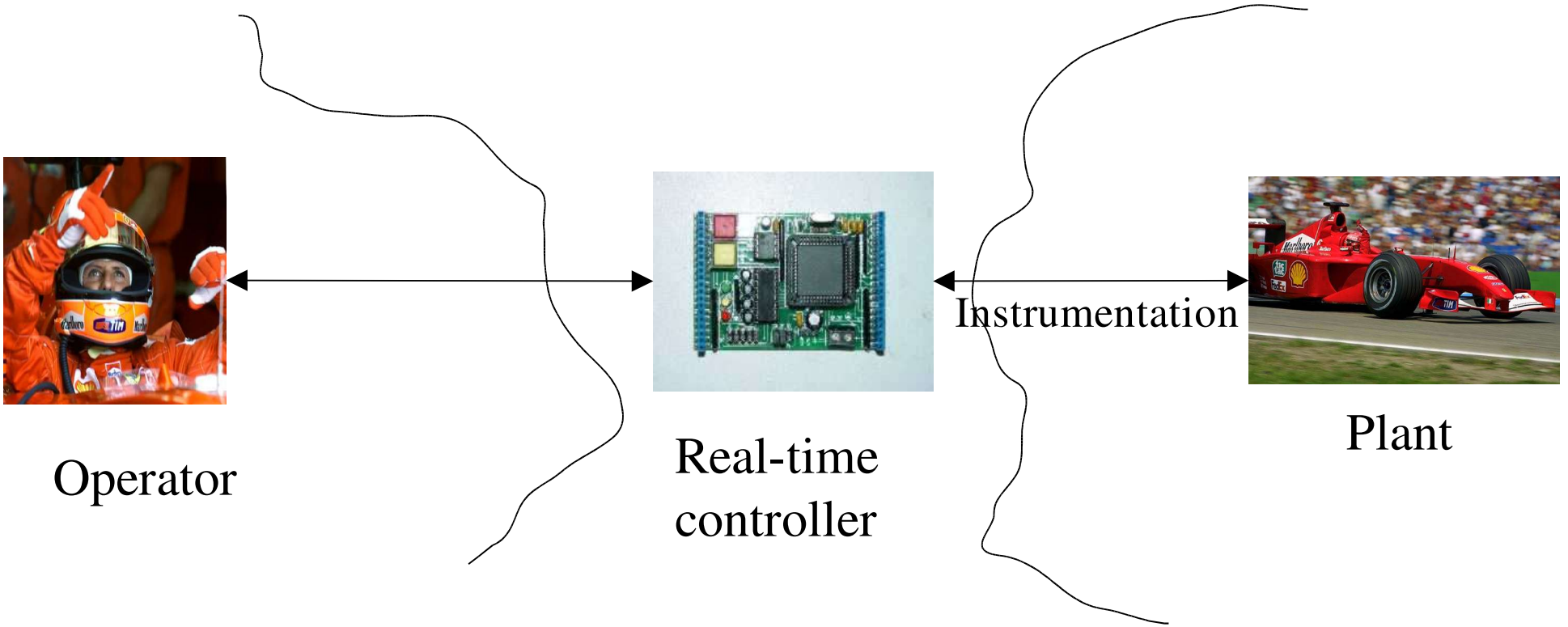
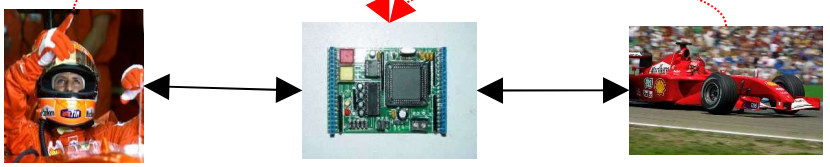
A real-time system

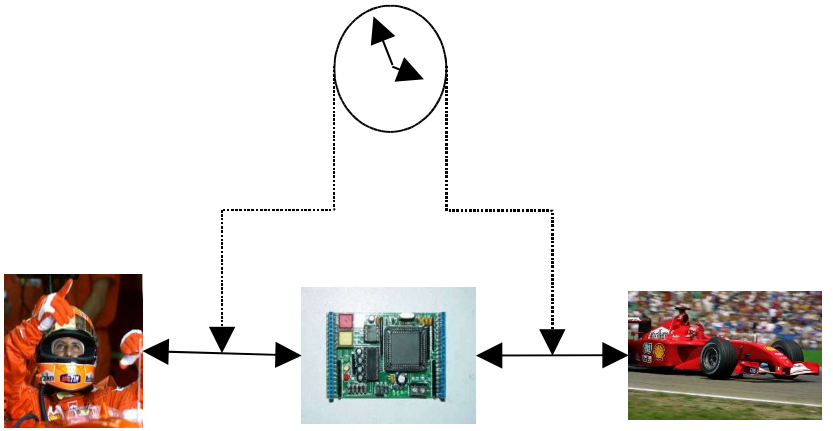


A real-time controller is a computer based system, which produces results to inputs complying with some temporal constraints

Some useful definitions

- *Event triggered vs Time-triggered*



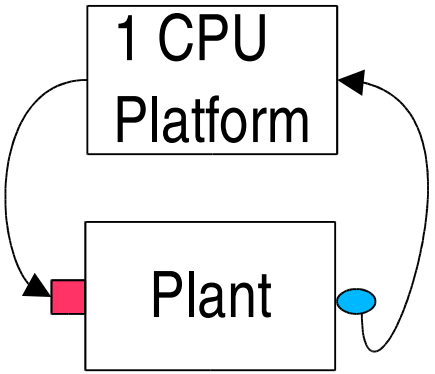


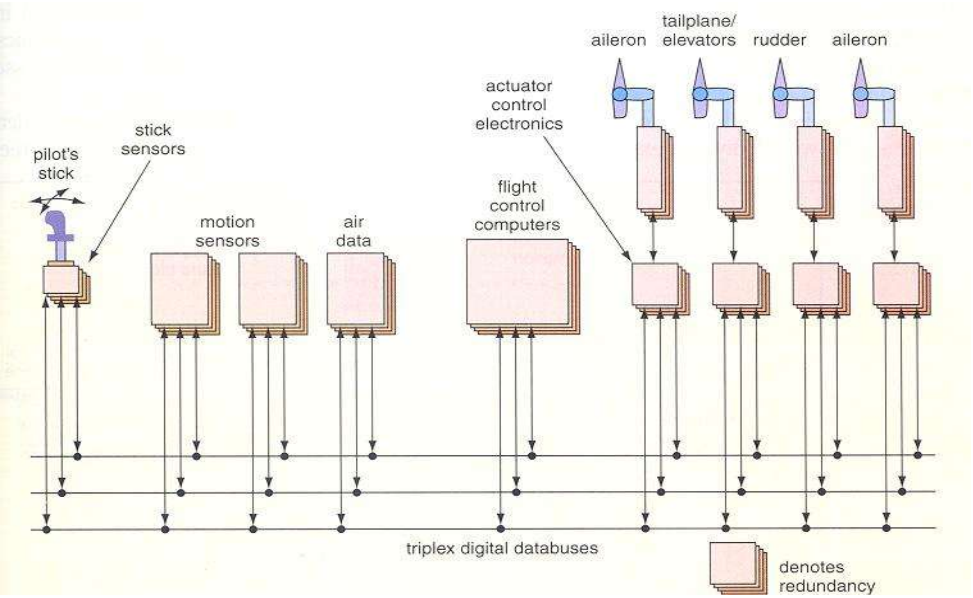
Event-triggered: system's reactions are elicited by the occurrence of certain events in the environment

Time-triggered: interactions with the environment take place upon well defined instants

Some useful definitions I

- *Single node vs distributed*



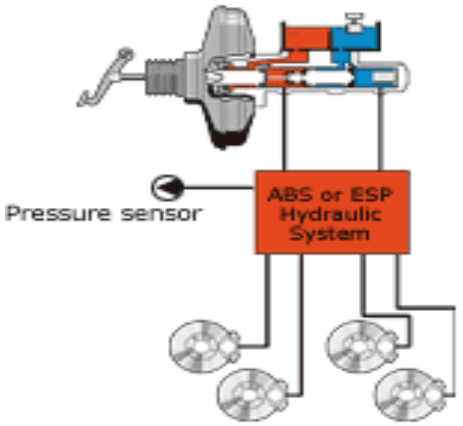


Single node: computation is concentrated in one node, which has direct access to sensors and actuators

Distributed: computation is distributed across different nodes which communicate by means of a bus

Some useful definitions ... II

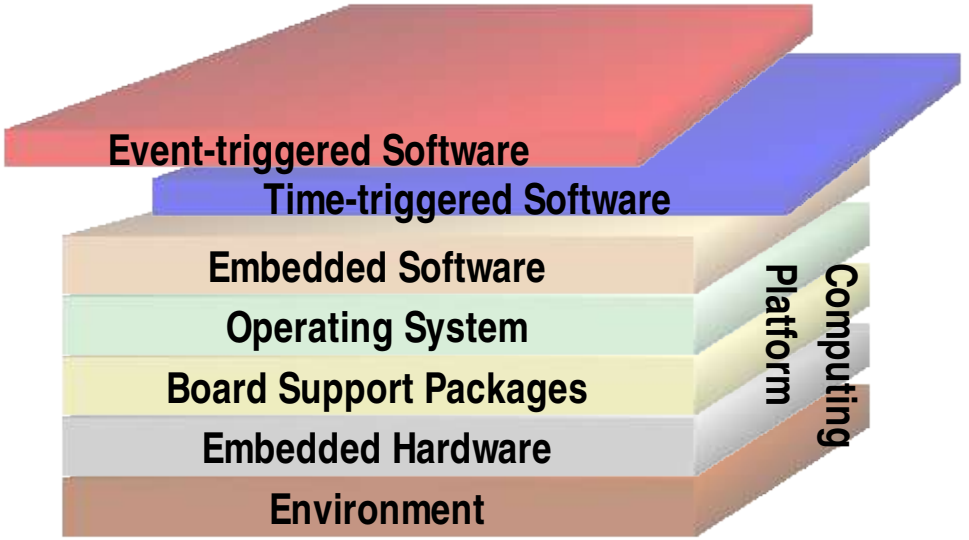
- *Hard real-time vs Soft real-time*



Hard real-time: computation must terminate within certain deadlines

Soft real-time: deadlines can occasionally be missed but the anomaly has to be kept in check, lest the Quality of Service be severely degraded

Real-time systems: what's in a name?



- A an embedded controller is a complex ensemble of software and hardware components:
  - Hardware devices
  - Software support components
  - Software applications
- Event-triggered and Time-triggered semantics are often intertwined

Why should a control engineer care about real-time software?

- The pure “springs” of control engineer:
  - instantaneous computations and communication
  - infinite bandwidth links and nodes
  - ideal sampling
  - infinite precision

- The polluted “delta” of system engineer
  - communication and computation take a (random) time
  - links and nodes have finite bandwidth
  - there is sampling and actuation jitter
  - information is quantised
- Performance can be severely degraded

Design

- Design: act of defining a system or a subsystem
- Usually design amounts to:
  - defining one or more models of the system
  - refining the models until a desired functionality is obtained
- Informal specification and design always lead to problems

Example

- Where is the problem in the following specification?

The system has two inputs, *reset* and *next*, and three outputs: *a*, *b* and *c*. Whenever *reset* appears, *a* is emitted. After this, the first *next* signal produces *b* and the second *next* signal produces *c*.

- What if *reset* and *next* are present together?

A formal model for design

- **Functional Specification**
  - relations between inputs outputs, inputs and internal state
  - describe the system's behaviour
- **Properties**
  - a set of relations between inputs, outputs and states that should always hold true
  - assertions on the system's behaviour
- **Performance indexes**
  - Cost, reliability, speed, size...
- **Constraints**
  - inequalities on performance indexes

Our goal

- To devise design techniques and tools that preserve properties
  - If a property has been proved correct at a certain step, it has to be preserved in the next phases

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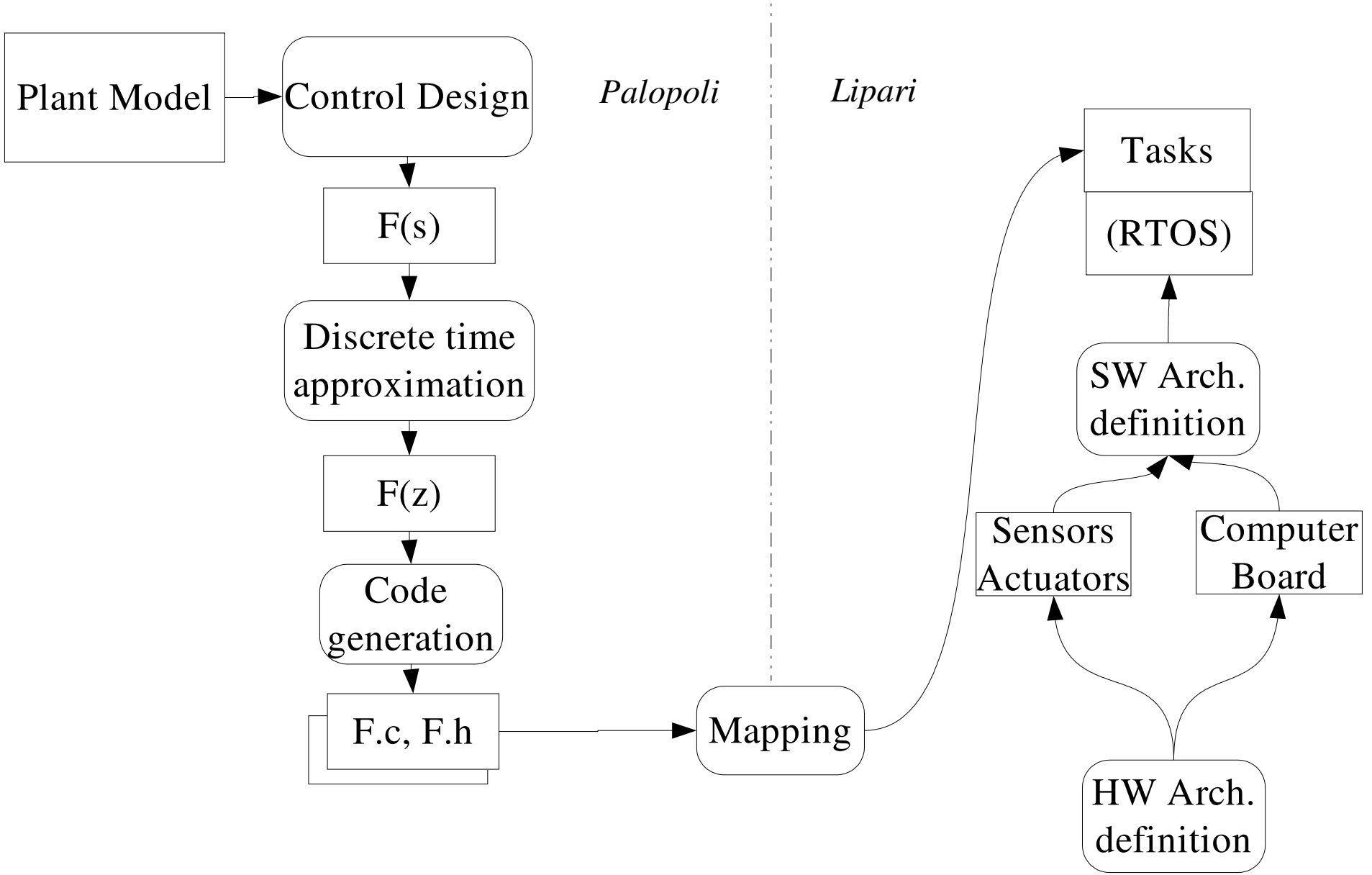
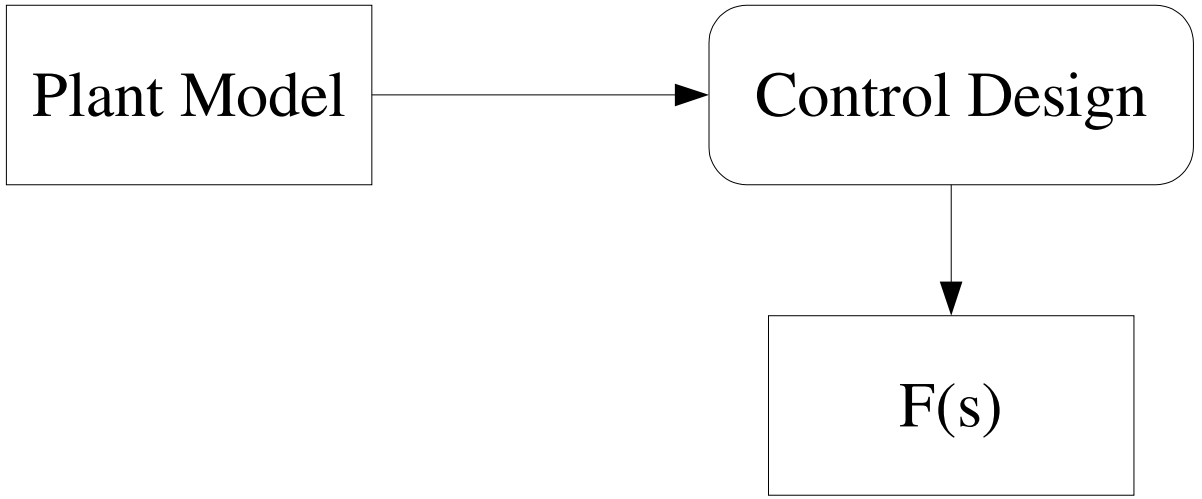
A meet-in-the-middle approach

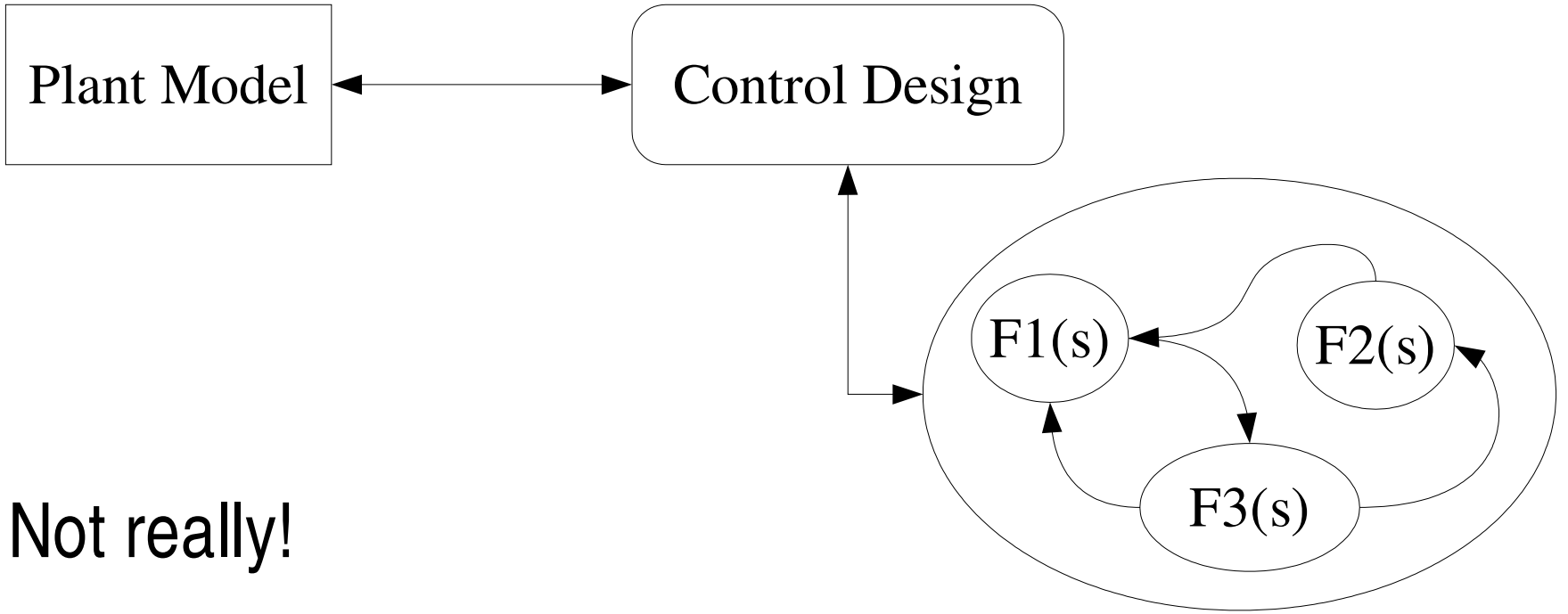


First step: control law design



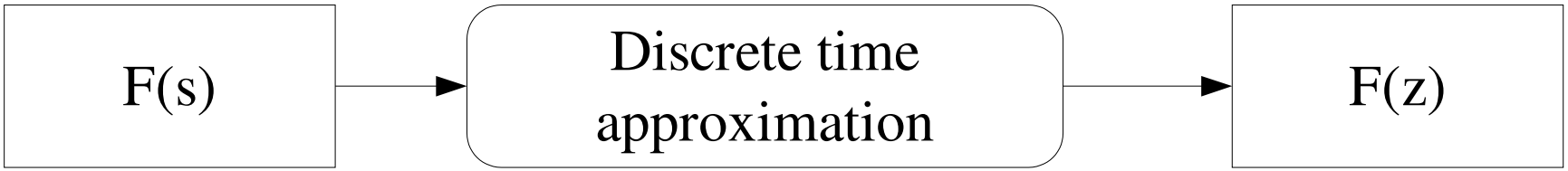
- Control law design: analytical and semi-analytical procedures
- Validation by Matlab/Simulink tools

Is that all?



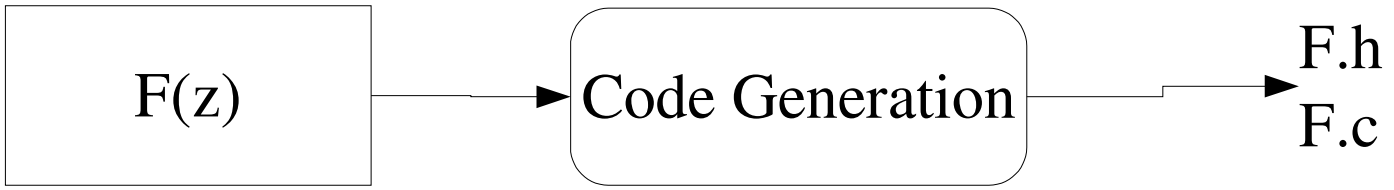
- Not really!
- We can have multiple controllers to choose between according to the occurrence of certain events
- Combination of FSM + continuous time controller: HYBRID systems

Second Step: a discrete time model for the controller



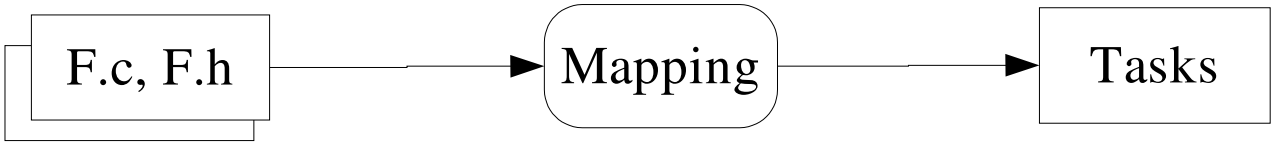
- Problems
  - Effects of sampling
  - Choice of the sampling period
  - Numerical problems
- Matlab control toolbox is a nice friend...

Third step: generating C code



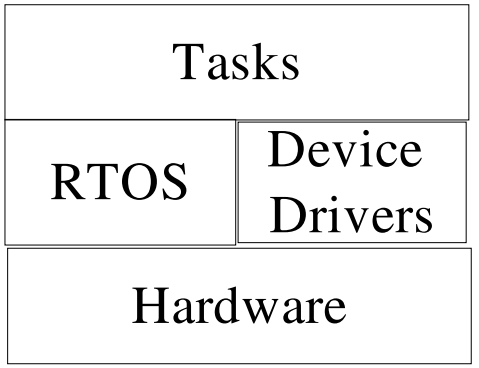
- This is easy, ain't it?
- No!
  - Quantisation (finite precision)
  - Computation delays!
- Done by hand or by CAD tools
  - RTW, Embedded Coder, Targetlink

Fourth step: mapping



- Critical passage
  - Scheduling delays and jitter
  - Enforcement of real-time constraints

Architecture selection



- Further delays
- Quantisation effects

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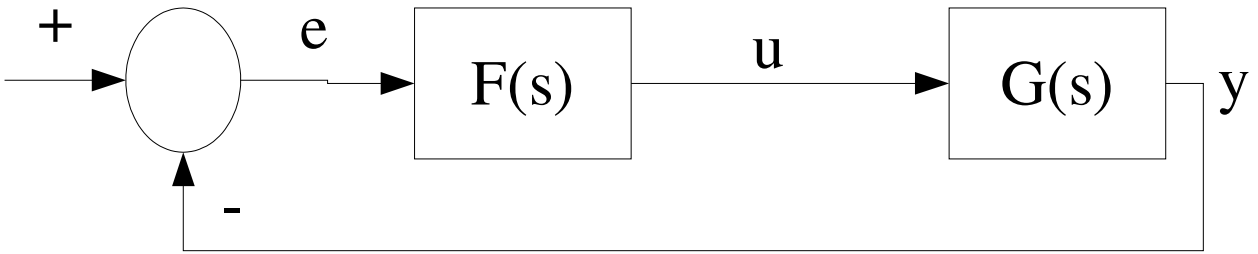
Example



$$G(s) = \frac{1}{s(s+1)}$$

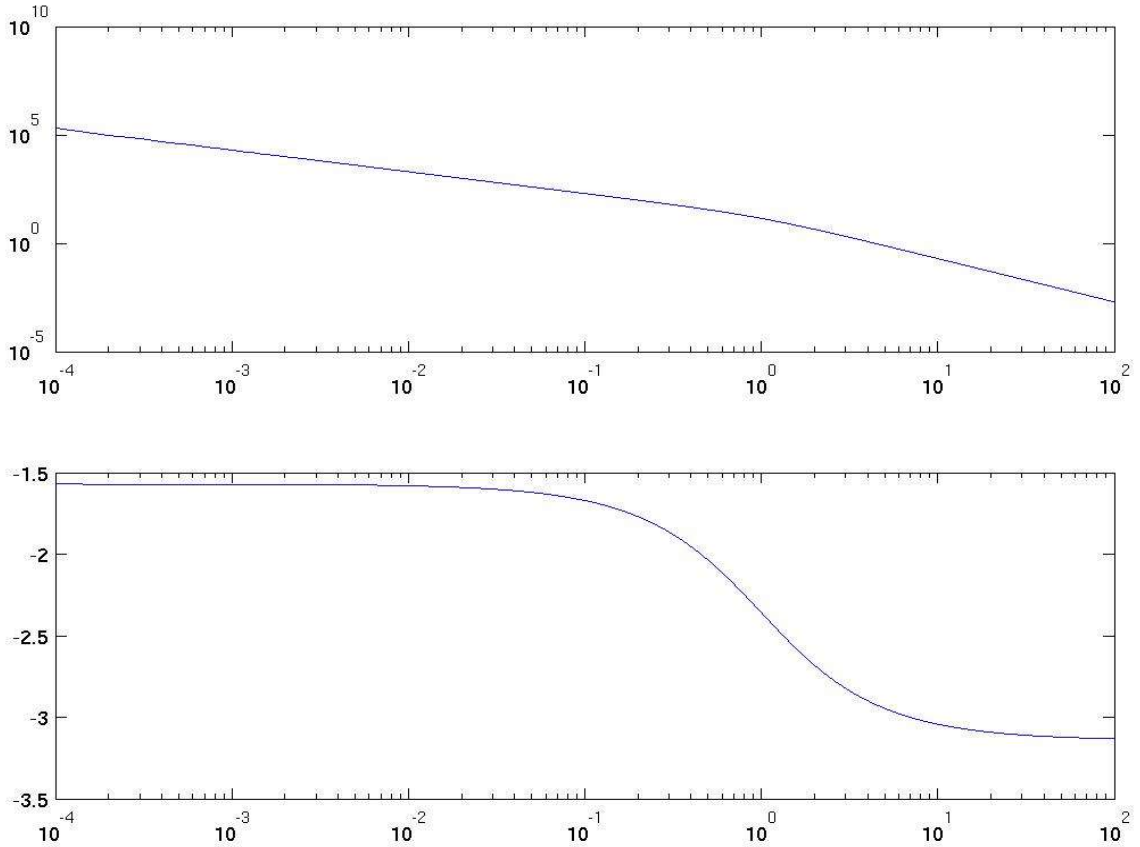
Recall definitions for 2nd order systems:

$$\frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

ω_n: Natural frequency

ζ: Damping

Bode plot



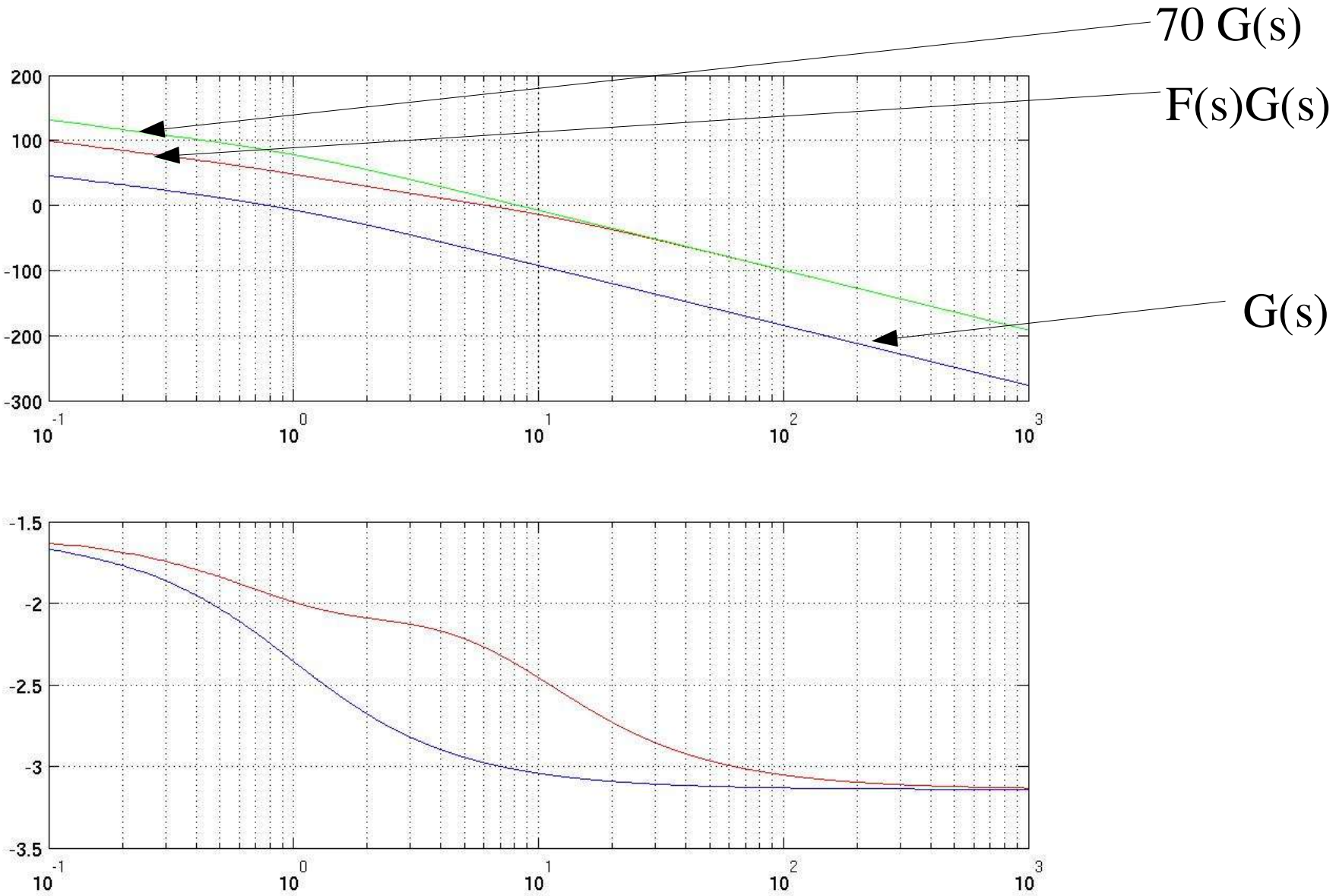
Phase Margin =51°

Control design

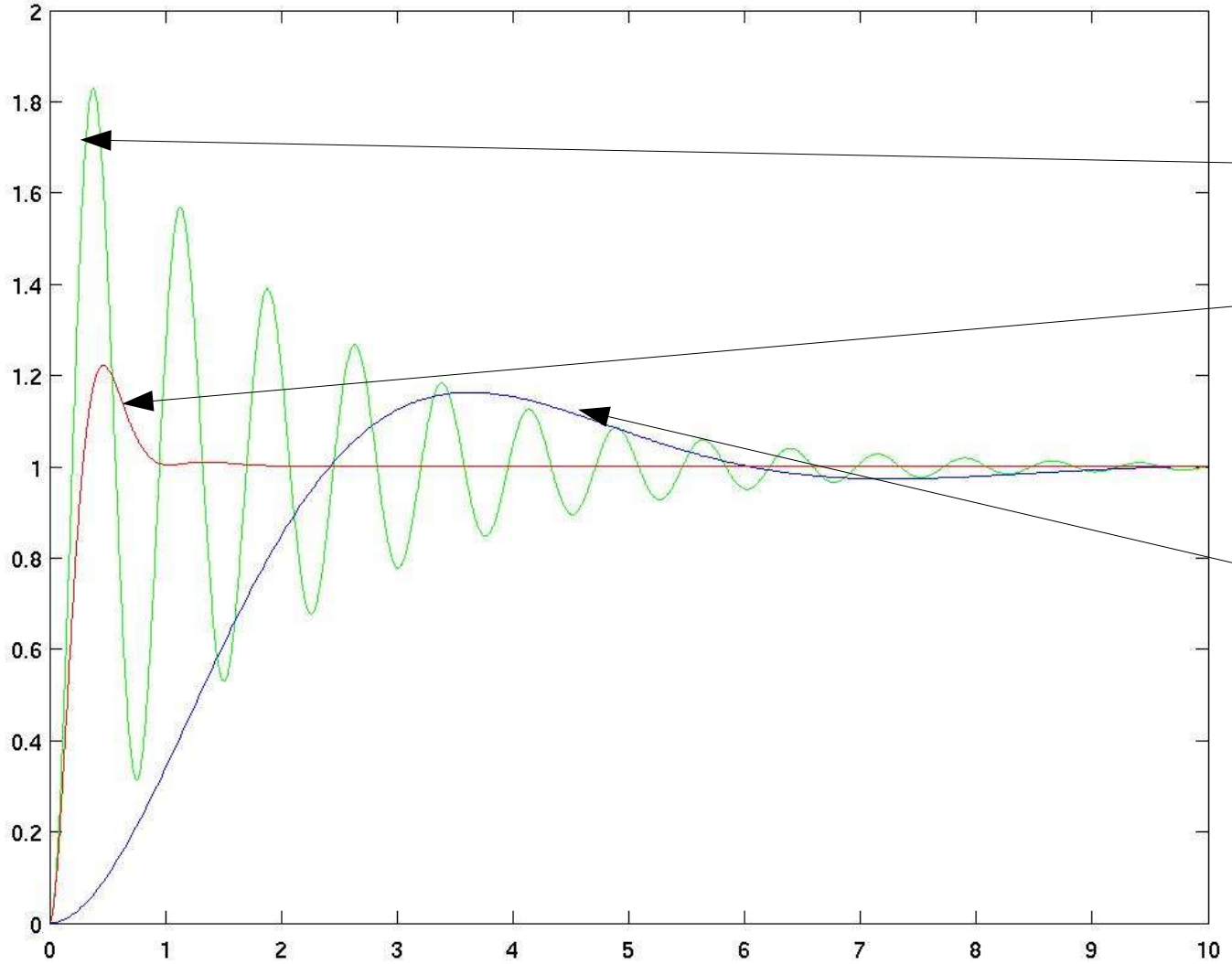
- We use a gain to increase the badwidth
- A lead compensantor around the cross over frequency helps us improve the phase margin

$$F(s) = 70 \frac{s+2}{s+10}$$

Bode plots



Step responses

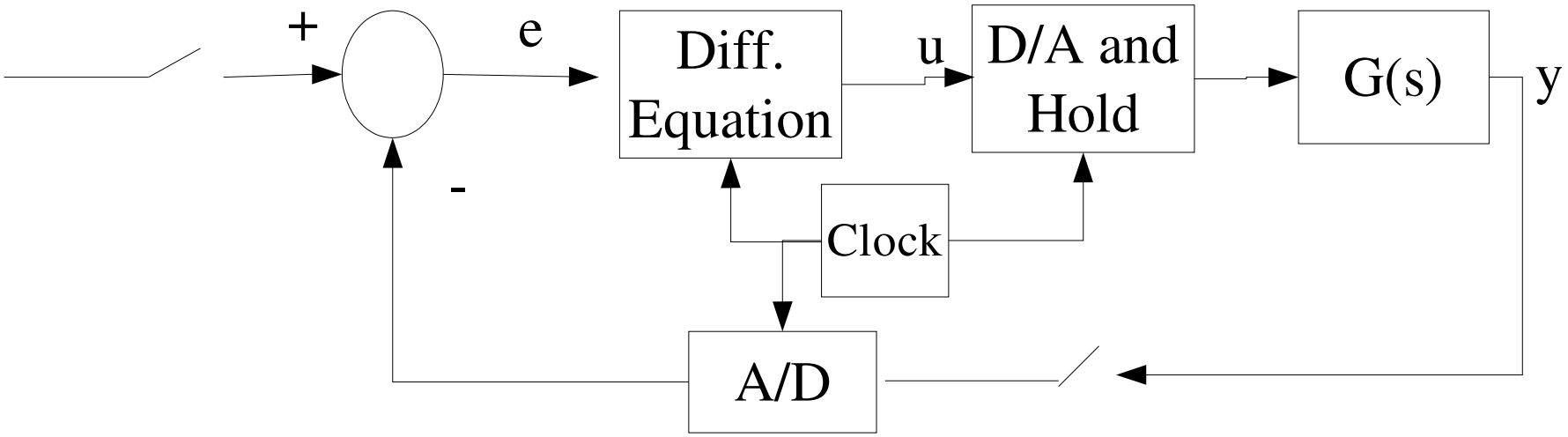


(70 G)/(70 G+1)

(F G)/(F G+1)

G/(G+1)

Let's go to implementation



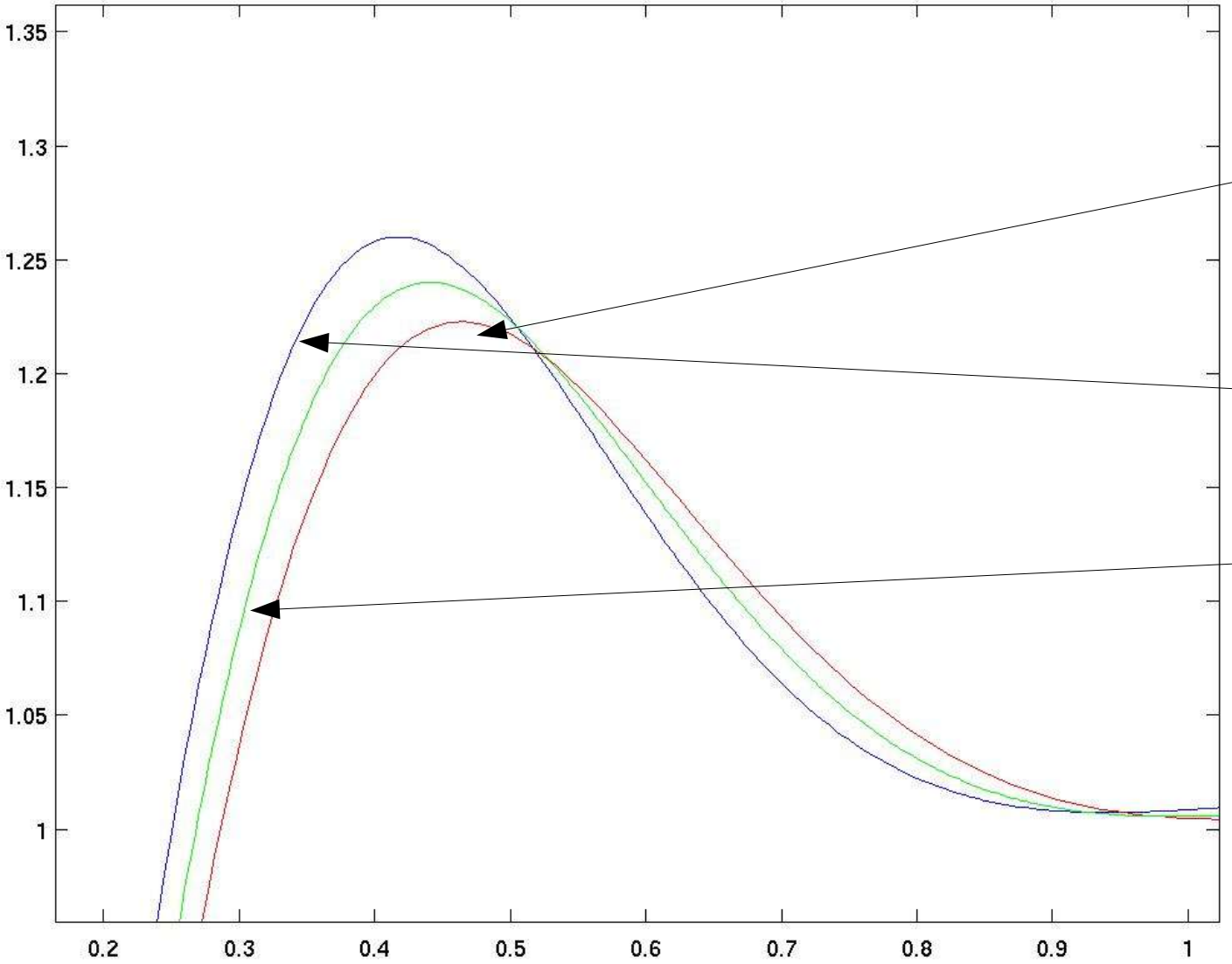
$$F(s)=\frac{U(s)}{E(s)}=70\frac{s+2}{s+10}$$

$$\dot{u}+10u=70\dot{e}+140e$$

$$FW\ Euler\ Approximation: \dot{u}\approx\frac{u_{k+1}-u_k}{T},\ \dot{e}\approx\frac{e_{k+1}-e_k}{T}$$

$$u_{k+1}=(1-10T)u_k+70e_{k+1}+(140T-70)e_k$$

Step Responses



Analog controller

f=1/T=20hz

f=1/T=40hz

Sampling decreases damping

Why?

- The transfer function of a ZoH

$$Z(s) = \frac{1 - e^{-sT}}{s}$$

$$e^{-sT} \approx \frac{1 - sT/2}{1 + sT/2} \rightarrow Z(s) \approx \frac{T}{1 + sT/2}$$

- The ZoH results into a phase decrease of - ωT/2
- The Phase Margin (PM) is reduced and so is the damping ζ = PM/100

Conclusions

- Sampling introduces an inherent delays
- There are problems for stability and performance (damping is reduced)
- We will present techniques to analyse and reduce these problems that will be presented through the course