EDF Scheduling

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May 11, 2008

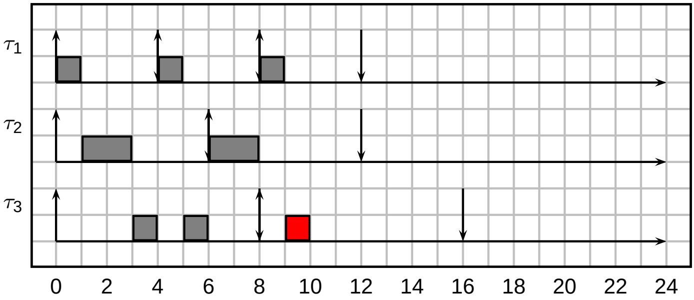
Outline

Earliest Deadline First

- An important class of scheduling algorithms is the class of *dynamic priority* algorithms
  - In dynamic priority algorithms, the priority of a task can change during its execution
  - Fixed priority algorithms are a sub-class of the more general class of dynamic priority algorithms: the priority of a task does not change.
- The most important (and analyzed) dynamic priority algorithm is Earliest Deadline First (EDF)
  - The priority of a job (istance) is inversely proportional to its absolute deadline;
  - In other words, the highest priority job is the one with the earliest deadline;
  - If two tasks have the same absolute deadlines, chose one of the two at random (*ties can be broken arbitrarly*).
  - The priority is dynamic since it changes for different jobs of the same task.

Example: scheduling with RM

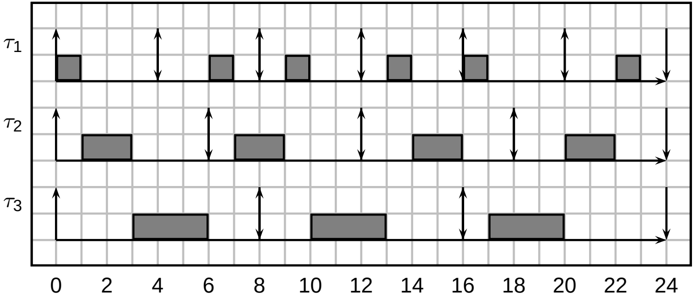
- We schedule the following task set with FP (RM priority assignment).
- $\tau_1 = (1, 4)$, $\tau_2 = (2, 6)$, $\tau_4 = (3, 8)$.
- $U = \frac{1}{4} + \frac{2}{6} + \frac{3}{8} = \frac{23}{24}$
- The utilization is greter than the bound: there is a deadline miss!



- Observe that at time 6, even if the deadline of task τ_3 is very close, the scheduler decides to schedule task τ_2. This is the main reason why τ_3 misses its deadline!

Example: scheduling with EDF

- Now we schedule the same task set with EDF.
- $\tau_1 = (1, 4)$, $\tau_2 = (2, 6)$, $\tau_4 = (3, 8)$.
- $U = \frac{1}{4} + \frac{2}{6} + \frac{3}{8} = \frac{23}{24}$
- Again, the utilization is very high. However, no deadline miss in the hyperperiod.



- Observe that at time 6, the problem does not appear, as the earliest deadline job (the one of τ_3) is executed.

Job-level fixed priority

- In EDF, the priority of a job is *fixed*.
- Therefore some author is classifies EDF as of *job-level fixed priority* scheduling;
- LLF is a *job-level dynamic priority* scheduling algorithm as the priority of a job may vary with time;
- Another job-level dynamic priority scheduler is p-fair.

Outline

A general approach to schedulability analysis

We start from a completely aperiodic model.

- A system consists of a (infinite) set of jobs $\mathcal{J} = \{J_1, J_2, \ldots, J_n, \ldots\}$.
- $J_k = (a_k, c_k, d_k)$
- Periodic or sporadic task sets are particular cases of this system

EDF optimality

Theorem (Dertouzos '73)

If a set of jobs $\mathcal{J}$ is schedulable by an algorithm A, then it is schedulable by EDF.

Proof.

The proof uses the exchange method.

- Transform the schedule $\sigma_{\mathrm{A}}(t)$ into $\sigma_{\mathrm{EDF}}(t)$, step by step;
- At each step, preserve schedulability.

□

Corollary

EDF is an optimal algorithm for single processors.

Schedulability bound for periodic/sporadic tasks

Theorem

Given a task set of periodic or sporadic tasks, with relative deadlines equal to periods, the task set is schedulable by EDF if and only if

$$U = \sum_{i=1}^{N} \frac{C_i}{T_i} \leq 1$$

Corollary

EDF is an optimal algorithm, *in the sense that if a task set if schedulable, then it is schedulable by EDF.*

Proof.

In fact, if $U > 1$ no algorithm can succesfully schedule the task set; if $U \leq 1$, then the task set is schedulable by EDF x(and maybe by other algorithms). $\square$

Outline

Advantages of EDF over FP

- EDF can schedule all task sets that can be scheduled by FP, but not vice versa.
  - Notice also that offsets are not relevant!
- There is not need to define priorities
  - Remember that in FP, in case of offsets, there is not an optimal priority assignment that is valid for all task sets
- In general, EDF has less context switches
  - In the previous example, you can try to count the number of context switches in the first interval of time: in particular, at time 4 there is no context switch in EDF, while there is one in FP.
- Optimality of EDF
  - We can fully utilize the processor, less idle times.

Disadvantages of EDF over FP

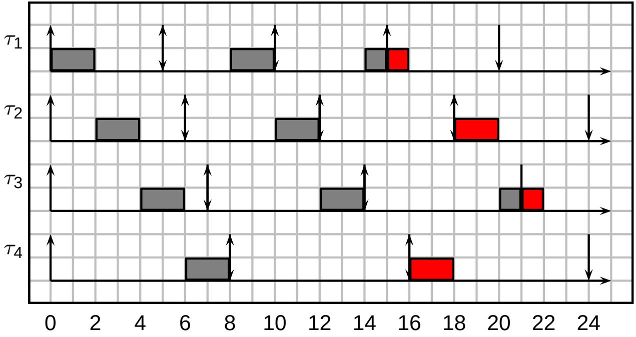
- EDF is not provided by any commercial RTOS, because of some disadvantage
- Less predictable
  - Looking back at the example, let's compare the response time of task τ_1: in FP is always constant and minimum; in EDF is variable.
- Less controllable
  - if we want to reduce the response time of a task, in FP is only sufficient to give him an higher priority; in EDF we cannot do anything;
  - We have less control over the execution

Overhead

- More implementation overhead
  - FP can be implemented with a very low overhead even on very small hardware platforms (for example, by using only interrupts);
  - EDF instead requires more overhead to be implemented (we have to keep track of the absolute deadline in a long data structure);
  - There are method to implement the queueing operations in FP in O(1); in EDF, the queueing operations take O(log N), where N is the number of tasks.

Domino effect

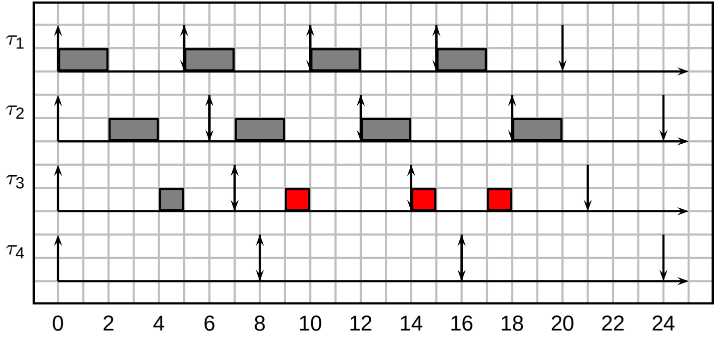
- In case of overhead ($U > 1$), we can have the *domino effect* with EDF: it means that all tasks miss their deadlines.
- An example of domino effect is the following;



- All tasks missed their deadline almost at the same time.

Domino effect: considerations

- FP is more predictable: only lower priority tasks miss their deadlines! In the previous example, if we use FP:



- As you can see, while τ_1 and τ_2 never miss their deadlines, τ_3 misses a lot of deadline, and τ_4 does not execute!
- However, it may happen that some task never executes in case of high overload, while EDF is more *fair* (all tasks are treated in the same way).

Response time computation

- Computing the response time in EDF is very difficult, and we will not present it in this course.
    - In FP, the response time of a task depends only on its computation time and on the interference of higher priority tasks
    - In EDF, it depends in the parameters of all tasks!
    - If all offset are 0, in FP the maximum response time is found in the first job of a task,
    - In EDF, the maximum response time is not found in the first job, but in a later job.

Outline

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Generalization to deadlines different from period

- EDF is still optimal when relative deadlines are not equal to the periods
- However, the schedulability analysis formula becomes more complex
- If all relative deadlines are less than or equal to the periods, a first trivial (sufficient) test consist in substituting T_i with D_i:

$$U' = \sum_{i=1}^{N} \frac{C_i}{D_i} \leq 1$$

- In fact, if we consider each task as a sporadic task with interarrival time D_i instead of T_i, we are increasing the utilization, $U < U'$. If it is still less than 1, then the task set is schedulable. If it is larger than 1, then the task set may or may not be schedulable

Demand bound analysis

- In the following slides, we present a general methodology for schedulability analysis of EDF scheduling
- Let's start from the concept of *demand function*
- **Definition:** the demand function for a task τ_i is a function of an interval $[t_1, t_2]$ that gives the amount of computation time that *must* be completed in $[t_1, t_2]$ for τ_i to be schedulable:

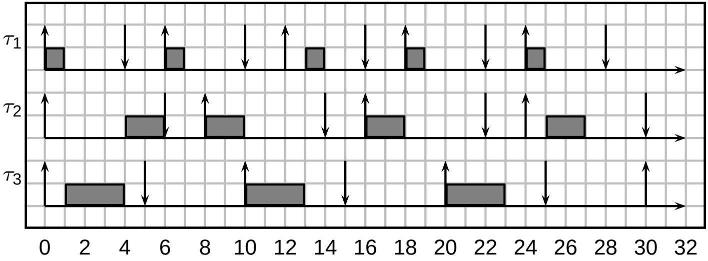
$$df_i(t_1, t_2) = \sum_{\substack{a_{ij} \geq t_1 \\ d_{ij} \leq t_2}} c_{ij}$$

- For the entire task set:

$$df(t_1, t_2) = \sum_{i=0}^{N} df_i(t_1, t_2)$$

Example of demand function

- $\tau_1 = (1, 4, 6)$, $\tau_2 = (2, 6, 8)$, $\tau_3 = (3, 5, 10)$



- Let's compute $df()$ in some intervals;

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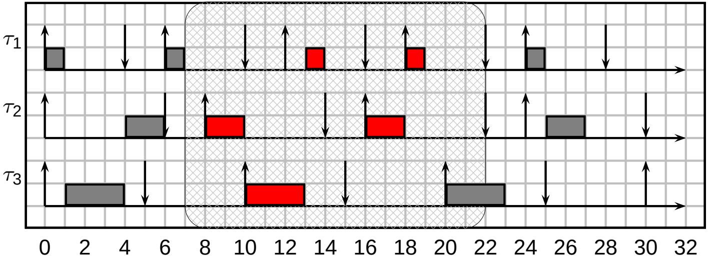

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- $df(7, 22) = 2 \cdot C_1 + 2 \cdot C_2 + 1 \cdot C_3 = 9$;

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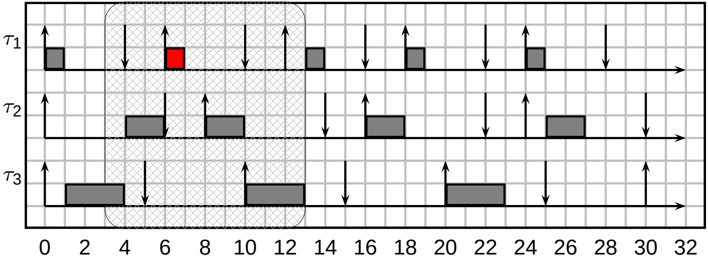

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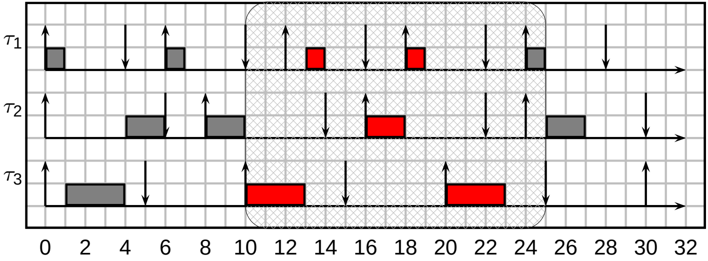

- Let's compute $df()$ in some intervals;
- $df(7, 22) = 2 \cdot C_1 + 2 \cdot C_2 + 1 \cdot C_3 = 9$;
- $df(3, 13) = 1 \cdot C_1 = 1$;
- $df(10, 25) = 2 \cdot C_1 + 1 \cdot C_2 + 2 \cdot C_3 = 7$;

A necessary condition

Theorem

A necessary condition for any job set to be schedulable by any scheduling algorithm when executed on a single processor is that:

$$\forall t_1, t_2 \quad \mathrm{df}(t_1, t_2) \leq t_2 - t_1$$

Proof.

By contradiction. Suppose that $\exists t_1, t_2 \;\; \mathrm{df}(t_1, t_2) > t_2 - t_1$. If the system is schedulable, then it exists a scheduling algorithm that can execute more than $t_2 - t_1$ units of computations in an interval of length $t_2 - t_1$. Absurd! □

Main theorem

Theorem

A necessary and sufficient condition for a set of jobs $\mathcal{J}$ to be schedulable by EDF is that

$$\forall t_1, t_2 \quad \mathrm{df}(t_1, t_2) \leq t_2 - t_1 \tag{1}$$

Proof.

The proof is based on the same technique used by Liu & Layland in their seminal paper. We only need to prove the *sufficient* part.

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- By contradiction: assume a deadline is missed and the condition holds
- Assume the first deadline miss is at y
- We find an opportune $x < y$ such that $\mathrm{df}(x, y) > y - x$.

□

Proof

- Suppose the first deadline miss is at time y. Let x be the **last instant prior to** y such that:
  - all jobs with arrival time before x and deadline before y have already completed by x;
  - x coincides with the arrival time of a job with deadline less of equal to y
  - Such instant always exists (it could be time 0).

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- Since x is the last such instant, it follows that:
  - there is no idle time in $[x, y]$
  - No job with deadline greater than y executes in $[x, y]$
  - only jobs with arrival time greater or equal to x, and deadline less than or equal to y execute in $[x, y]$

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  - No job with deadline greater than y executes in $[x, y]$
  - only jobs with arrival time greater or equal to x, and deadline less than or equal to y execute in $[x, y]$
- Since there is a deadline miss in $[x, y]$, $\mathrm{df}(x, y) > y - x$, and the theorem follows.

Feasibility analysis

- The previous theorem gives a first hint at how to perform a schedulability analysis.
  - However, the condition should be checked for all pairs $[t_1, t_2]$.
  - This is impossible in practice! (an infinite number of intervals!).
  - First observation: function df changes values only at discrete instants, corresponding to arrival times and deadline of a job set.

Feasibility analysis

- The previous theorem gives a first hint at how to perform a schedulability analysis.
  - However, the condition should be checked for all pairs $[t_1, t_2]$.
  - This is impossible in practice! (an infinite number of intervals!).
  - First observation: function df changes values only at discrete instants, corresponding to arrival times and deadline of a job set.
  - Second, for periodic tasks we could use some periodicity (hyperperiod) to limit the number of points to be checked to a finite set.

Outline

Simplifying the analysis

- A periodic task set is *synchronous* if all task offsets are equal to 0
- In other words, for a synchronous task set, all tasks start at time 0.
- A task set is *asynchronous* is some task has a non-zero offset.

Demand bound function

Theorem

For a set of synchronous periodic tasks (i.e. with no offset),

$$\forall t_1, t_2 > t_1 \quad df(t_1, t_2) \leq df(0, t_2 - t_1)$$

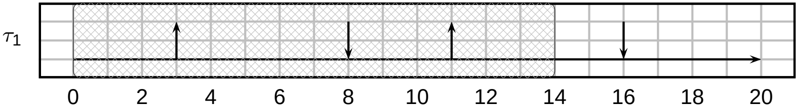
- In plain words, the worst case demand is found for intervals starting at 0.
- **Definition:** Demand Bound function:

$$dbf(L) = \max_t (df(t, t + L)) = df(0, L).$$

Demand bound function - II

- The maximum is when the task is activated at the beginning of the interval.
- For a periodic task τ_i:

$$\mathrm{dbf}_i(L) = \left(\left\lfloor \frac{L - D_i}{T_i} \right\rfloor + 1 \right)_0 C_i$$



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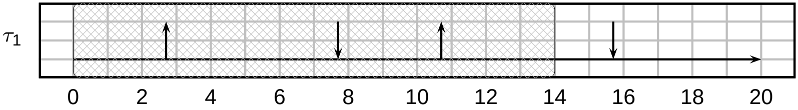

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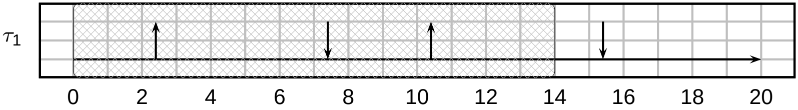

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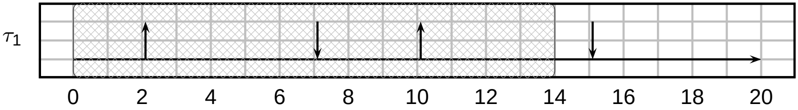

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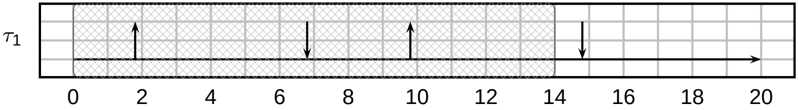

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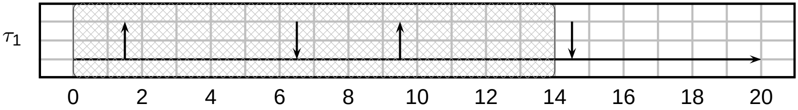

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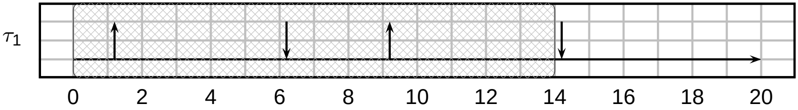

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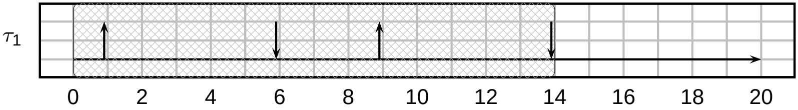

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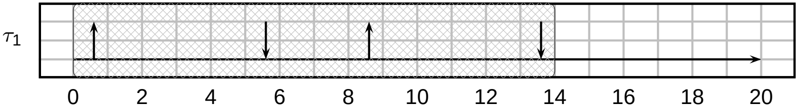

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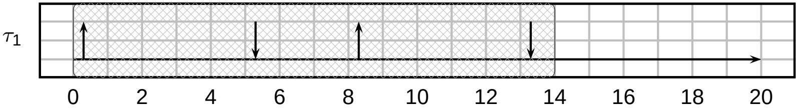

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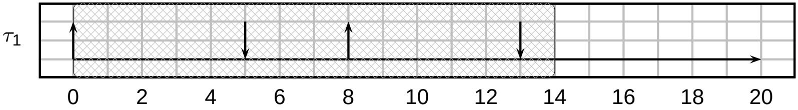

Synchronous periodic task sets

Theorem (Baruah, Howell, Rosier '90)

A synchronous periodic task set $\mathcal{T}$ is schedulable by EDF *if and only if:*

$$\forall L \in \mathrm{dead}(\mathcal{T}) \quad \mathrm{dbf}(L) \leq L$$

where $\mathrm{dead}(\mathcal{T})$ *is the set of deadlines in* $[0, H]$

- Proof next slide.

Proof

- Sufficiency: eq. holds $\longrightarrow$ task set is schedulable.
  - By contradiction
- Necessity: task set is schedulable $\longrightarrow$ eq. holds

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  - eq. does not hold for $\overline{L}$.
  - build a schedule starting at 0, for which $\mathrm{dbf}(\overline{L}) = \mathrm{df}(0, \overline{L})$
  - Hence task set is not schedulable $\square$

Sporadic task

- Sporadic tasks are equivalent to synchronous periodic task sets.
- For them, the worst case is when they all arrive at their maximum frequency and starting synchronously.

Synchronous and asynchronous

- Let $\mathcal{T}$ be a asynchronous task set.
- We call $\mathcal{T}'$ the corresponding synchronous set, obtained by setting all offset equal to 0.

Corollary

If $\mathcal{T}'$ is schedulable, then $\mathcal{T}$ is schedulable too.

Conversely, if $\mathcal{T}$ is schedulable, $\mathcal{T}'$ may not be schedulable.

- The proof follows from the definition of $\mathrm{dbf}(L)$.

A pseudo-polynomial test

Theorem (Baruah, Howell, Rosier, '90)

Given a synchronous periodic task set $\mathcal{T}$, with deadlines less than or equal to the period, and with load $U < 1$, the system is schedulable by EDF if and only if:

$$\forall L \in \mathrm{deadShort}(\mathcal{T}) \quad \mathrm{dbf}(L) \leq L$$

where $\mathrm{deadShort}(\mathcal{T})$ *is the set of all deadlines in interval* $[0, L^*]$ *and*

$$L^* = \frac{U}{1-U} \max_i (T_i - D_i)$$

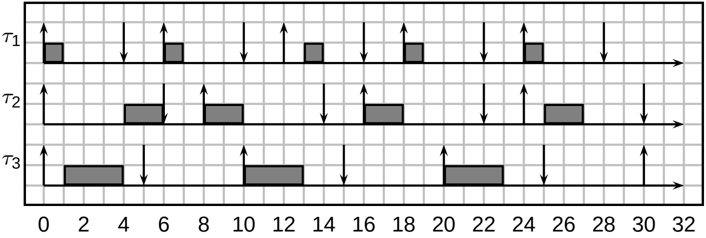
Corollary

The complexity of the above analysis is pseudo-polynomial.

Outline

Example of computation of the *dbf*

- $\tau_1 = (1, 4, 6)$, $\tau_2 = (2, 6, 8)$, $\tau_3 = (3, 5, 10)$
- $U = 1/6 + 1/4 + 3/10 = 0.7167$, $L^* = 12.64$.
- We must analyze all deadlines in $[0, 12]$, i.e. $(3, 5, 6, 10)$.



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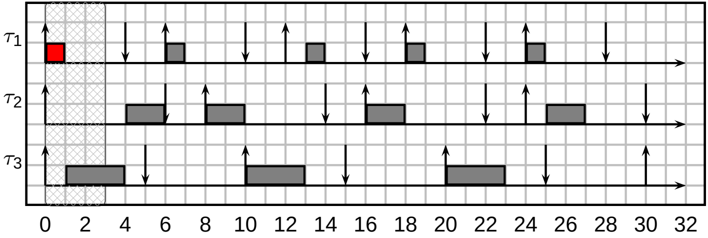

- Let's compute $dbf()$
- $df(0, 4) = C_1 = 1 < 4$;

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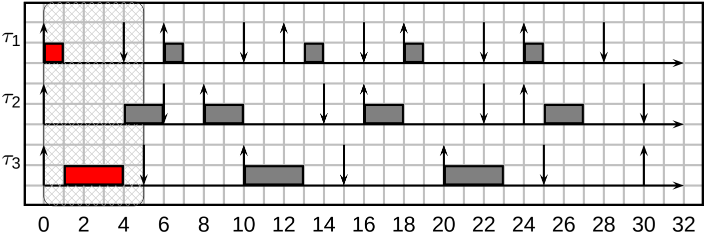

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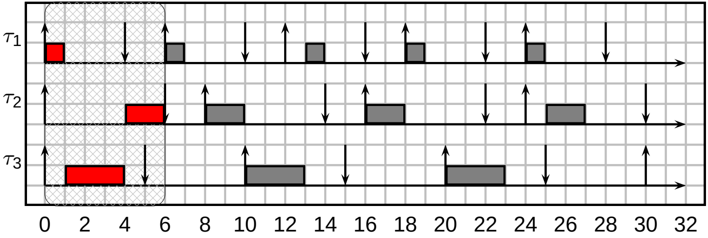

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- $df(0, 6) = C_1 + C_2 + C_3 = 6 \leq 6$;

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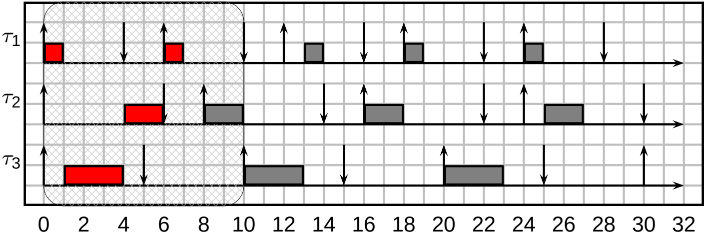

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- $df(0, 10) = 2C_1 + C_2 + C_3 = 7 \leq 10$;

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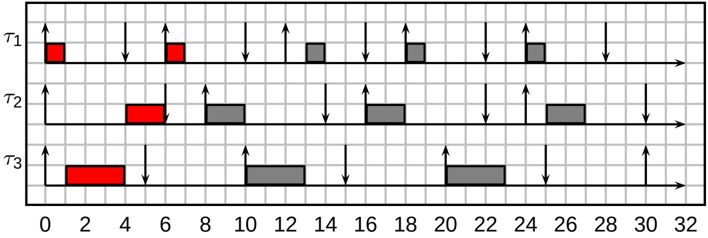

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- $df(0, 10) = 2C_1 + C_2 + C_3 = 7 \leq 10$;
- The task set is schedulable.

Idle time and busy period

- The interval between time 0 and the first idle time is called *busy period*.
- The analysis can be stopped at the first idle time (Spuri, '94).

- The first idle time can be found with the following recursive equations:

$$W(0) = \sum_{i=1}^{N} C_i$$

$$W(k) = \sum_{i=1}^{N} \left\lceil \frac{W(k-1)}{T_i} \right\rceil C_i$$

- The iteration stops when $W(k-1) = W(k)$.

Another example

- Consider the following example

	C_i	D_i	T_i
τ_1	1	2	4
τ_2	2	4	5
τ_3	4.5	8	15

- $U = 0.9$; $L^* = 9 * 7 = 63$;

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- $U = 0.9$; $L^* = 9 * 7 = 63$;
- $W = 14.5$.
- Then we can check all deadline in interval $[0, 14.5]$.

Outline

Algorithm

- Of course, it should not be necessary to draw the schedule to see if the system is schedulable or not.
- First of all, we need a formula for the *dbf*:

$$dbf(L) = \sum_{i=1}^{N} \left(\left\lfloor \frac{L - D_i}{T_i} \right\rfloor + 1 \right) C_i$$

- The algorithm works as follows:
  - We list all deadlines of all tasks until L^*.
  - Then, we compute the *dbf* for each deadline and verify the condition.

The previous example

- In the previous example: deadlines of the tasks:

τ_1	4	10
τ_2	6	
τ_3	5	

- dbf in tabular form

L	4	5	6	10
dbf	1	4	6	7

- Since, for all $L < L^*$ we have $dbf(L) \leq L$, then the task set is schedulable.

Another example

- Consider the followin task set

	C_i	D_i	T_i
τ_1	1	2	4
τ_2	2	4	5
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- $U = 0.9$; $L^* = 9 * 7 = 63$;
- hint: if L^* is too large, we can stop at the first idle time.
- The first idle time can be found with the following recursive equations:

$$
\begin{aligned}
W(0) &= \sum_{i=1}^{N} C_i \\
W(k) &= \sum_{i=1}^{N} \left\lceil \frac{W(k-1)}{T_i} \right\rceil C_i
\end{aligned}
$$

- The iteration stops when $W(k-1) = W(k)$.
- In our example $W = 14.5$. Then we can check all deadline in interval $[0, 14.5]$.

Example

- Deadlines of the tasks:

τ_1	2	6	10	14
τ_2	4	9	14	
τ_3	8			

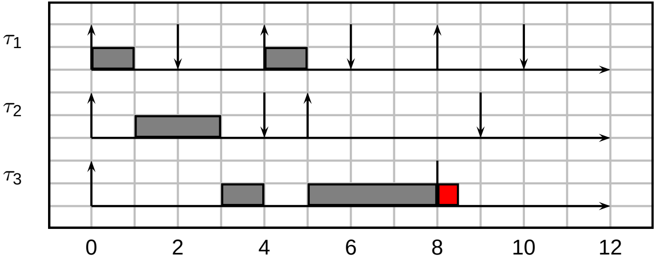
- Demand bound function in tabular form

t	2	4	6	8	9	10	14
dbf	1	3	4	8.5			

- The task set is not schedulable! Deadline miss at 8.

In the schedule...

- The schedule is as follows:



Outline

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1. Dynamic priority
2. Basic analysis
3. FP vs EDF
4. Processor demand bound analysis
   - Generalization to deadlines different from period
   - Synchronous and asynchronous tasks
   - Examples
   - Testing algorithm
5. A sufficient pseudo-polynomial test for synchronous sets
   - Basic idea

Differences between synchronous and asynchronous sets

- Let's recall the previous Corollary and Theorem

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  - "push back" activations until the first jobs starts at t_1;
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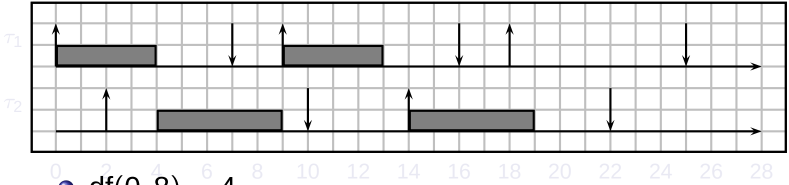
  - "push back" activations until the first jobs starts at t_1;
  - Compute the dbf as the sum of the computation of all jobs with deadline no later than t_2.
  - Problem: by "pushing back" the instance we are modyfing the task set!

Example of asynchronous task set

- $\tau_1 = (0, 4, 7, 9)$ and $\tau_2 = (2, 5, 8, 12)$



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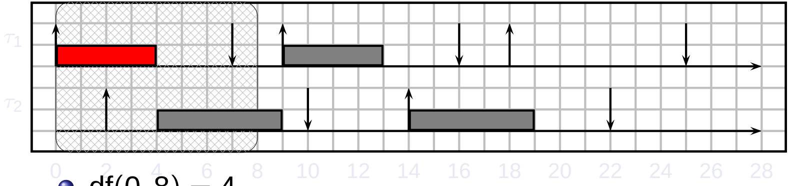

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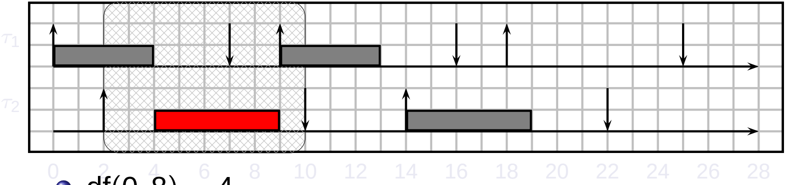

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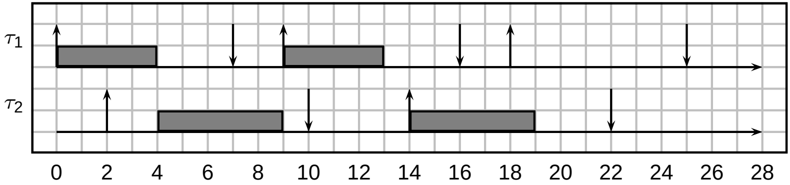

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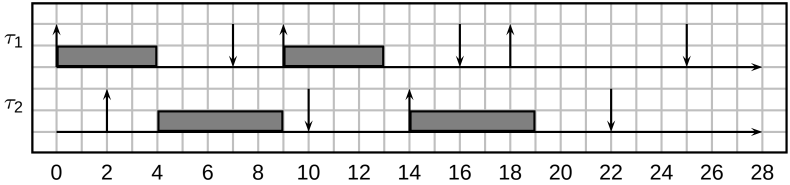

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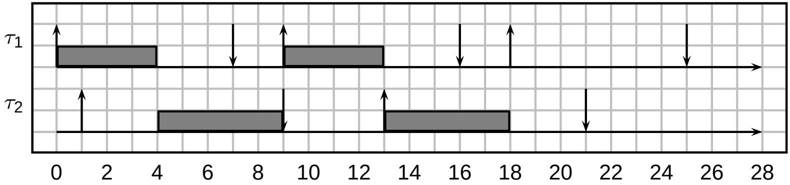

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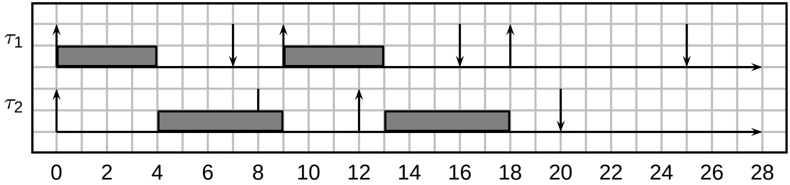

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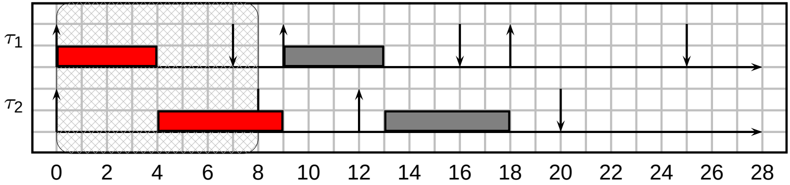

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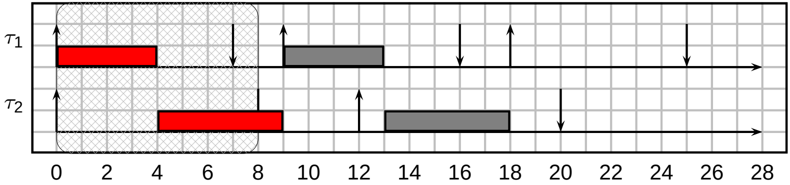

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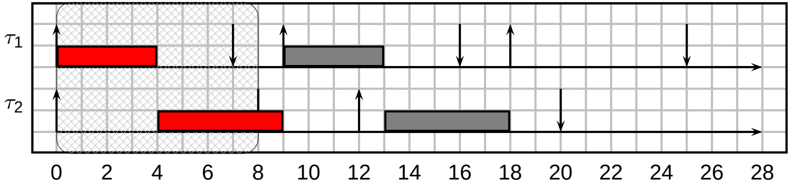

- dbf(8) $= 9$
- The dbf is too pessimistic.

Trade off between pessimism and complexity

- The problem is that we do not know what is the worst pattern of arrivals for asynchronous task sets.
- We know for synchronous: instant 0
- For asynchronous, we should check for every possible pattern

Key observation

- The distance between any arrival of task τ_i and any arrival of task τ_j is:

$$a_{j,k_1} - a_{i,k_2} = \phi_j + k_1 T_j - \phi_i - k_2 T_i = \phi_j - \phi_i + k(\gcd(T_i, T_j))$$

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- Imposing that the difference must not be negative, and k must be integer, we get:

$$k \geq \frac{\phi_i - \phi_j}{\gcd(T_i, T_j)} \Rightarrow k = \left\lceil \frac{\phi_i - \phi_j}{\gcd(T_i, T_j)} \right\rceil$$

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- The minimum distance is:

$$\Delta_{i,j} = \phi_j - \phi_i + \left\lceil \frac{\phi_i - \phi_j}{\gcd(T_i, T_j)} \right\rceil \gcd(T_i, T_j)$$

Observations

- From the formula we can derive the following observations:
  - The value of $\Delta_{i,j}$ is an integer in interval $[0, \mathrm{gcd}(T_i, T_j) - 1]$
  - If T_i and T_j are prime between them (i.e. gcd = 1), then $\Delta_{i,j} = 0$.
- Now we are ready to explain the basic idea behind the new scheduling analysis methodology.

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- Given an hypothetical interval $[x, y]$
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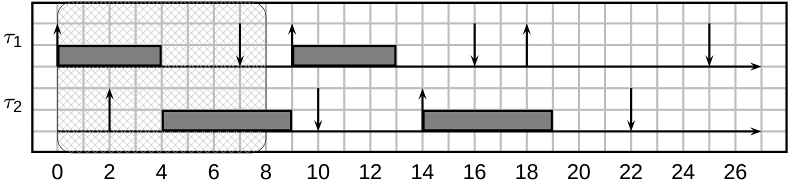
- The df in all intervals starting with x can only increase after the "pushing back".
- Therefore, if no deadline is missed in $[x, y]$, then no deadline is missed in any interval of length $(y - x)$.
- We could build such interval by selecting a task τ_i to start at the beginning of the interval, and setting the arrival times of the other tasks at their minimum distances

Problem

- We do not know which task to start with in the interval
- Simple solution: just select each task in turn

Example

- $\tau_1 = (0, 4, 7, 9)$ and $\tau_2 = (2, 5, 8, 12)$
  - We select τ_1 to start at 0.



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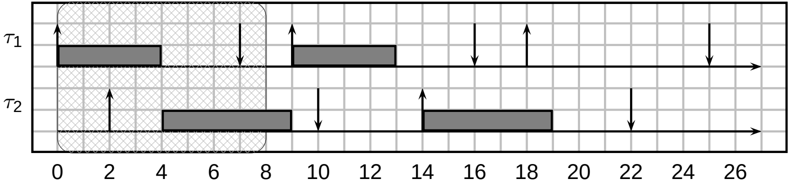
$$\phi_2 - \phi_1 + \left\lceil \frac{\phi_1 - \phi_2}{T_1 mod T_2} \right\rceil (T_1 \mod T_2) = 2 + \left\lceil \frac{-2}{3} \right\rceil 3 = 2$$



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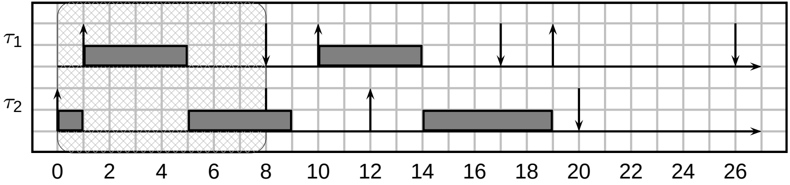

Main theorem

- Given an asynchronous task set $\mathcal{T}$
- Let $\mathcal{T}_i'$ be the task set obtained by
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Theorem (Pellizzoni and Lipari, ECRTS '04)

Given task set $\mathcal{T}$ with $U \leq 1$, scheduled on a single processor, if $\forall\ 1 \leq i \leq N$ all deadlines in task set $\mathcal{T}_i'$ are met until the first idle time, then $\mathcal{T}$ is feasible.

Performance

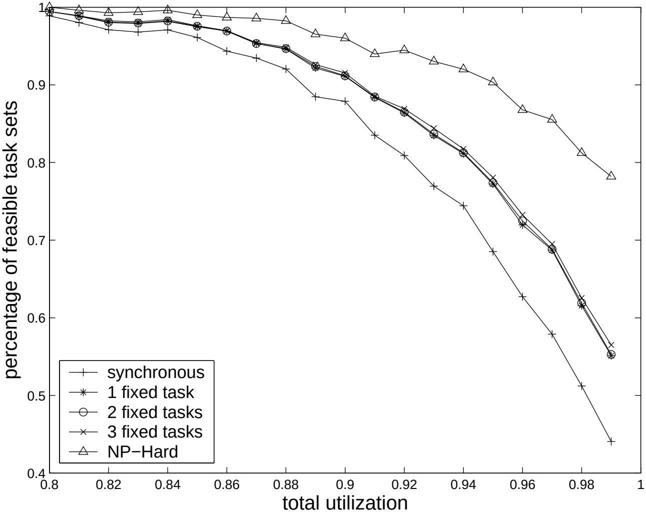

Figure: 10 tasks with periods multiple of 10

Conclusions

- What is this for?
- Feasibility analysis of asynchronous task set is used for:
  - Reduction of output jitter: by setting an offset it is possible to reduce response time and jitter
  - Analysis of distributed transactions (i.e. chains of tasks related by precedence constraints).

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  - Analysis of distributed transactions (i.e. chains of tasks related by precedence constraints).
- in both cases, the analysis must be iteratively repeated many times with different offsets;
- hence we need an efficient analysis (even though it is only sufficient)

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