Sistemi in tempo reale
Anno accademico 2006 - 2007

Introduction to FSMs

Giuseppe Lipari
http://feanor.sssup.it/~lipari

Scuola Superiore Sant'Anna

Outline

1. Generalities on Finite State Machines (FSMs)
   - State Diagrams
   - Mealy machines
   - Non deterministic FSMs

Introduction

State machines are basic building blocks for computing theory.

- very important in theoretical computer science
- many applications in practical systems
- There are many slightly different definitions, depending on the application area
- A state machine is a Discrete Event Discrete State system
  - transitions from one state to another only happen on specific events
  - events do not need to occur at specific times
  - we only need a temporal order between events (events occur one after the other), not the exact time at which they occur

Definition

A deterministic finite state machine (DFSM) is a 5-tuple:

- S (finite) set of states
- I set of possible input symbols (also called input alphabet)
- s_0 initial state
- ϕ transitions: a function from (state,input) to a new state

$$\phi : S \times I \rightarrow S$$

- ω output function (see later)

An event is a new input symbol presented to the machine.

- In response, the machine will react by updating its state and possibly producing an output. This reaction is istantaneous (synchronous assumption).

Output function

Two types of machines:

Moore output only depends on state:

$$\omega_{\text{moore}} : S \to \Omega$$

Where Ω is the set of output symbols. In this case, the output only depends on the state, and it is produced upon entrance on a new state.

Mealy output depends on state and input:

$$\omega_{\text{mealy}} : S \times I \to \Omega$$

In this case, the output is produced upon occurrence of a certain transaction.

Moore machines

- Moore machines are the simplest ones
- If $\Omega = \{\text{yes}, \text{no}\}$, the machine is a recognizer
- A recognizer is able to accept or reject sequences of input symbols
- The set of sequences accepted by a recognizer is a regular language

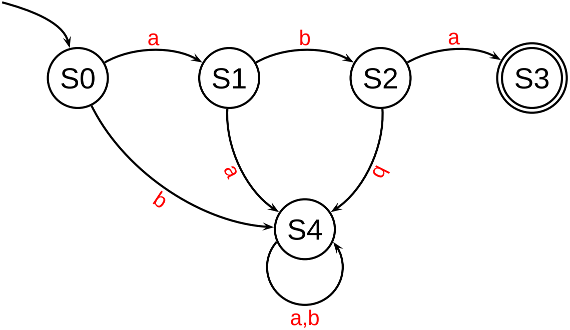
State diagrams

- FSM can be represented by State Diagrams

- final states are identified by a double circle

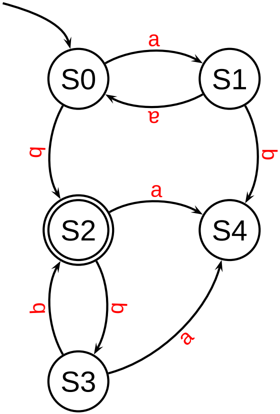
Example: language recognizer

- In this example $I = \{a, b\}$. The following state machine recognizes string *aba*



Example: language recognizer II

- recognize string $a^n b^m$ with n even and m odd (i.e. *aabbb*, *b*, *aab* are all legal sequences, while *a*, *aabb*, are non legal)



- S4 is an error state. It is not possible to go out from an error state (for every input, no transaction out of the state)
- S2 is an accepting state, however we do not know the length of the input string, so it is possible to exit from the accepting state if the input continues
- If we want to present a new string we have to reset the machine to its initial state

Non regular language

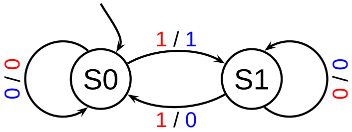
- FSM are not so powerful. They can only recognize simple languages
- Example:
  - strings of the form $a^n b^n$ for all $n \geq 0$ cannot be recognized by a FSM (because they only have a finite number of states)
  - they could if we put a limit on n. For example, $0 \leq n \leq 10$.

Mealy machines

- in Mealy machines, output is related to both state and input.
- in practice, output can be associated to a transition
- given the synchronous assumption, the Moore's model is equivalent to the Mealy's model
- for every Moore machine, it is possible to derive an equivalent Mealy machine, and viceversa

Example: parity check

- In this example, we have a Mealy machine that outputs 1 if the number of symbols 1 in input so far is odd; it outputs 0 otherwise.



- Usually, Mealy machines have a more compact representation than Moore machines (i.e. they perform the same task with a number of states that is no less than the equivalent Moore machine).

Table representation

- A FSM can be represented through a table
- The table shown below corresponds to the parity-check Mealy FSM shown just before.

	0	1
S_0	S_0 / 0	S_1 / 1
S_1	S_1 / 1	S_0 / 0

Stuttering symbol

- Input and output alphabets include the absent symbol ϵ
- It correspond to a null input or output
- When the input is absent, the state remains the same, and the output is absent
- Any sequence of inputs can be interleaved or extended with an arbitrary number of absent symbols without changing the behavior of the machine
- the absent symbol is also called the stuttering symbol

Abbreviations

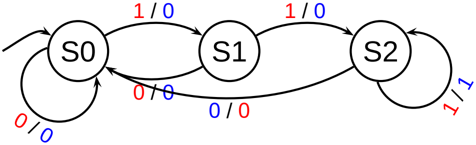
- If no guard is specified for a transition, the transition is taken for every possible input (except the absent symbol ϵ)
- If no output is specified for a transition, the output is ϵ
- given a state S_0, if a symbol α is not used a guard of any transition going out of S_0, then an implicit transition from S_0 to itself is defined with α as guard and ϵ as output

Exercise

- Draw the state diagram of a FSM with $I = \{0, 1\}$, $\Omega = \{0, 1\}$
- let $x(k)$ be the sequence of inputs
- the output $\omega(k) = 1$ iff $x(k-2) = x(k-1) = x(k) = 1$

Solution

- three states: S0 is the initial state, S1 if last input was 1, S2 if last two inputs were 1



Deterministic machines

- Transitions are associated with
  - a source state
  - a guard (i.e. a input value)
  - a destination state
  - a output
- in deterministic FSM, a transition is uniquely identified by the first two.
- in other words, given a source state and a input, the destination and the output are uniquely defined

Non deterministic FSMs

- A non deterministic finite state machine is identified by a 5-tuple:

I	set of input symbols
Ω	set of output symbols
S	set of states
S_0	set of initial states
ϕ	transition function:

  $$\phi : S \times I \to (S \times \Omega)^*$$

  where S^* denotes the power set of S, i.e. the set of all possible subsets of S.

- In other words, given a state and an input, the transition returns a set of possible pairs (new state, output).

Non determinism

- Non determinism is used in many cases:
  - to model randomness
  - to build more compact automata

Non determinism

- Non determinism is used in many cases:
  - to model randomness
  - to build more compact automata
- Randomness is when there is more than one possible behaviour and the system follows one specific behavior at random

Non determinism

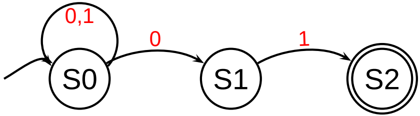
- Non determinism is used in many cases:
  - to model randomness
  - to build more compact automata
- Randomness is when there is more than one possible behaviour and the system follows one specific behavior at random
- Randomness has nothing to do with probability! we do not know the probability of occurrence of every behavior, we only know that they are possible

Non determinism

- Non determinism is used in many cases:
  - to model randomness
  - to build more compact automata
- Randomness is when there is more than one possible behaviour and the system follows one specific behavior at random
- Randomness has nothing to do with probability! we do not know the probability of occurrence of every behavior, we only know that they are possible
- A more abstract model of a system hides *unnecessary* details, and it is more compact (less states)

Example of non deterministic state machine

- We now build an automata to recognize all input strings (of any lenght) that end with a `01`



Equivalence between D-FSM and N-FSM

- It is possible to show that Deterministic FSMs (D-FSMs) are equivalent to non deterministic ones(N-FSMs)
- Proof sketch
  - Given a N-FSM $\mathcal{A}$, we build an equivalent D-FSM $\mathcal{B}$ (i.e. that recognizes the same strings recognized by the N-FSM. For every subset of states of the $\mathcal{A}$, we make a state of $\mathcal{B}$. Therefore, the maximum number of states of $\mathcal{B}$ is $2^{|S|}$. The start state of $\mathcal{B}$ is the one corresponding to the $\mathcal{A}$. For every subset of states that are reachable from the start state of state of $\mathcal{A}$ with a certain symbol, we make one transition in $\mathcal{B}$ to the state corresponding to the sub-set. The procedure is iterated until all transitions have been covered.

Example

- As an exercise, build the D-FSM equivalent to the previous example of N-FSM

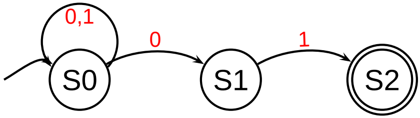


Figure: The N-FSM

Solution

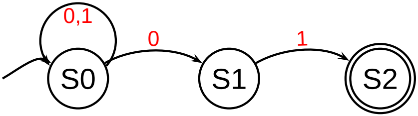


Figure: The N-FSM

- Initial state: $\{S0\}$

state name	subset	0	1
q0	{S0}	{S0, S1}	{S0}
q1	{S0,S1}	{S0, S1}	{S0, S2}
q2	{S0,S2}	{S0, S1}	{S0}

Solution

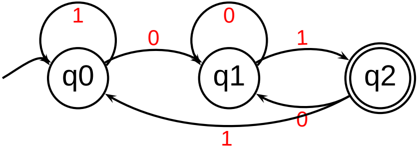


Figure: The N-FSM