Earliest Deadline First

- An important class of scheduling algorithms is the class of *dynamic priority* algorithms
  - In dynamic priority algorithms, the priority of a task can change during its execution
  - Fixed priority algorithms are a sub-class of the more general class of dynamic priority algorithms: the priority of a task does not change.

- The most important (and analyzed) dynamic priority algorithm is Earliest Deadline First (EDF)
  - The priority of a job (instance) is inversely proportional to its absolute deadline;
  - In other words, the highest priority job is the one with the earliest deadline;
  - If two tasks have the same absolute deadlines, chose one of the two at random (*ties can be broken arbitrarily*);
  - The priority is dynamic since it changes for different jobs of the same task.
Example: scheduling with RM

- We schedule the following task set with FP (RM priority assignment).
- $\tau_1 = (1, 4), \tau_2 = (2, 6), \tau_4 = (3, 8)$.
- $U = \frac{1}{4} + \frac{2}{6} + \frac{3}{8} = \frac{23}{24}$
- The utilization is greater than the bound: there is a deadline miss!

![Graph showing scheduling with RM]

- Observe that at time 6, even if the deadline of task $\tau_3$ is very close, the scheduler decides to schedule task $\tau_2$. This is the main reason why $\tau_3$ misses its deadline!

Example: scheduling with EDF

- Now we schedule the same task set with EDF.
- $\tau_1 = (1, 4), \tau_2 = (2, 6), \tau_4 = (3, 8)$.
- $U = \frac{1}{4} + \frac{2}{6} + \frac{3}{8} = \frac{23}{24}$
- Again, the utilization is very high. However, no deadline miss in the hyperperiod.

![Graph showing scheduling with EDF]

- Observe that at time 6, the problem does not appear, as the earliest deadline job (the one of $\tau_3$) is executed.
Job-level fixed priority

- In EDF, the priority of a job is *fixed*.
- Therefore some author is classifies EDF as of *job-level fixed priority* scheduling;
- LLF is a *job-level dynamic priority* scheduling algorithm as the priority of a job may vary with time;
- Another job-level dynamic priority scheduler is p-fair.

A general approach to schedulability analysis

We start from a completely aperiodic model.
- A system consists of a (infinite) set of jobs
  \[ J = \{ J_1, J_2, \ldots, J_n, \ldots \} \].
- \[ J_k = (a_k, c_k, d_k) \]
- Periodic or sporadic task sets are particular cases of this system
EDF optimality

**Theorem (Dertouzos ’73)**

*If a set of jobs \( J \) is schedulable by an algorithm \( A \), then it is schedulable by EDF.*

**Proof.**

The proof uses the exchange method.
- Transform the schedule \( \sigma_A(t) \) into \( \sigma_{EDF}(t) \), step by step;
- At each step, preserve schedulability.

**Corollary**

*EDF is an optimal algorithm for single processors.*

Schedulability bound for periodic/sporadic tasks

**Theorem**

*Given a task set of periodic or sporadic tasks, with relative deadlines equal to periods, the task set is schedulable by EDF if and only if*

\[
U = \sum_{i=1}^{N} \frac{C_i}{T_i} \leq 1
\]

**Corollary**

*EDF is an optimal algorithm, in the sense that if a task set if schedulable, then it is schedulable by EDF.*

**Proof.**

In fact, if \( U > 1 \) no algorithm can successfully schedule the task set; if \( U \leq 1 \), then the task set is schedulable by EDF (and maybe by other algorithms).
**Advantages of EDF over FP**

- EDF can schedule all task sets that can be scheduled by FP, but not vice versa.
  
  - Notice also that offsets are not relevant!
- There is not need to define priorities
  
  - Remember that in FP, in case of offsets, there is not an optimal priority assignment that is valid for all task sets
- In general, EDF has less context switches
  
  - In the previous example, you can try to count the number of context switches in the first interval of time: in particular, at time 4 there is no context switch in EDF, while there is one in FP.
- Optimality of EDF
  
  - We can fully utilize the processor, less idle times.

**Disadvantages of EDF over FP**

- EDF is not provided by any commercial RTOS, because of some disadvantage
- Less predictable
  
  - Looking back at the example, let's compare the response time of task $\tau_1$: in FP is always constant and minimum; in EDF is variable.
- Less controllable
  
  - if we want to reduce the response time of a task, in FP is only sufficient to give him an higher priority; in EDF we cannot do anything;
  
  - We have less control over the execution
Overhead

- More implementation overhead
  - FP can be implemented with a very low overhead even on very small hardware platforms (for example, by using only interrupts);
  - EDF instead requires more overhead to be implemented (we have to keep track of the absolute deadline in a long data structure);
  - There are methods to implement the queueing operations in FP in $O(1)$; in EDF, the queueing operations take $O(\log N)$, where $N$ is the number of tasks.

Domino effect

- In case of overhead ($U > 1$), we can have the domino effect with EDF: it means that all tasks miss their deadlines.
- An example of domino effect is the following;

  ![Diagram showing domino effect](image)

  All tasks missed their deadline almost at the same time.
Domino effect: considerations

- FP is more predictable: only lower priority tasks miss their deadlines! In the previous example, if we use FP:

  As you can see, while $\tau_1$ and $\tau_2$ never miss their deadlines, $\tau_3$ misses a lot of deadline, and $\tau_4$ does not execute!

- However, it may happen that some task never executes in case of high overload, while EDF is more fair (all tasks are treated in the same way).

Response time computation

- Computing the response time in EDF is very difficult, and we will not present it in this course.
  - In FP, the response time of a task depends only on its computation time and on the interference of higher priority tasks.
  - In EDF, it depends in the parameters of all tasks!
  - If all offset are 0, in FP the maximum response time is found in the first job of a task,
  - In EDF, the maximum response time is not found in the first job, but in a later job.
Generalization to deadlines different from period

- EDF is still optimal when relative deadlines are not equal to the periods
- However, the schedulability analysis formula becomes more complex
- If all relative deadlines are less than or equal to the periods, a first trivial (sufficient) test consist in substituting $T_i$ with $D_i$:

  $$U' = \sum_{i=1}^{N} \frac{C_i}{D_i} \leq 1$$

- In fact, if we consider each task as a sporadic task with interarrival time $D_i$ instead of $T_i$, we are increasing the utilization, $U < U'$. If it is still less than 1, then the task set is schedulable. If it is larger than 1, then the task set may or may not be schedulable.

Demand bound analysis

- In the following slides, we present a general methodology for schedulability analysis of EDF scheduling
- Let’s start from the concept of demand function
- **Definition:** the demand function for a task $\tau_i$ is a function of an interval $[t_1, t_2]$ that gives the amount of computation time that must be completed in $[t_1, t_2]$ for $\tau_i$ to be schedulable:

  $$df_i(t_1, t_2) = \sum_{a_{ij} \geq t_1, d_{ij} \leq t_2} c_{ij}$$

- For the entire task set:

  $$df(t_1, t_2) = \sum_{i=0}^{N} df_i(t_1, t_2)$$
Example of demand function

\[ \tau_1 = (1, 4, 6), \tau_2 = (2, 6, 8), \tau_3 = (3, 5, 10) \]

Let’s compute \( df() \) in some intervals;
- \( df(7, 22) = 2 \cdot C_1 + 2 \cdot C_2 + 1 \cdot C_3 = 9; \)
- \( df(3, 13) = 1 \cdot C_1 = 1; \)
- \( df(10, 25) = 2 \cdot C_1 + 1 \cdot C_2 + 2 \cdot C_3 = 7; \)

A necessary condition

**Theorem**

A necessary condition for any job set to be schedulable by any scheduling algorithm when executed on a single processor is that:

\[ \forall t_1, t_2 \quad df(t_1, t_2) \leq t_2 - t_1 \]

**Proof.**

By contradiction. Suppose that \( \exists t_1, t_2 \quad df(t_1, t_2) > t_2 - t_1 \). If the system is schedulable, then it exists a scheduling algorithm that can execute more than \( t_2 - t_1 \) units of computations in an interval of length \( t_2 - t_1 \). Absurd! \( \Box \)
Main theorem

Theorem

A necessary and sufficient condition for a set of jobs $\mathcal{J}$ to be schedulable by EDF is that

$$\forall t_1, t_2 \quad df(t_1, t_2) \leq t_2 - t_1$$

(1)

Proof.

The proof is based on the same technique used by Liu & Layland in their seminal paper. We only need to prove the sufficient part.

- By contradiction: assume a deadline is missed and the condition holds
- Assume the first deadline miss is at $y$
- We find an opportune $x < y$ such that $df(x, y) > y - x$.

Proof

Suppose the first deadline miss is at time $y$. Let $x$ be the last instant prior to $y$ such that:

- all jobs with arrival time before $x$ and deadline before $y$ have already completed by $x$;
- $x$ coincides with the arrival time of a job with deadline less of equal to $y$
  - Such instant always exists (it could be time 0).
- Since $x$ is the last such instant, it follows that:
  - there is no idle time in $[x, y]$
  - No job with deadline greater than $y$ executes in $[x, y]$
  - only jobs with arrival time greater or equal to $x$, and deadline less than or equal to $y$ execute in $[x, y]$
- Since there is a deadline miss in $[x, y]$, $df(x, y) > y - x$, and the theorem follows.
Feasibility analysis

- The previous theorem gives a first hint at how to perform a schedulability analysis.
  - However, the condition should be checked for all pairs \([t_1, t_2]\).
  - This is impossible in practice! (an infinite number of intervals!).
  - First observation: function \(df\) changes values only at discrete instants, corresponding to arrival times and deadline of a job set.
  - Second, for periodic tasks we could use some periodicity (hyperperiod) to limit the number of points to be checked to a finite set.

Simplifying the analysis

- A periodic task set is **synchronous** if all task offsets are equal to 0
- In other words, for a synchronous task set, all tasks start at time 0.
- A task set is **asynchronous** is some task has a non-zero offset.
Demand bound function

**Theorem**

For a set of synchronous periodic tasks (i.e. with no offset),

\[ \forall t_1, t_2 > t_1 \quad df(t_1, t_2) \leq df(0, t_2 - t_1) \]

- In plain words, the worst case demand is found for intervals starting at 0.
- **Definition:** Demand Bound function:

\[ dbf(L) = \max_t (df(t, t + L)) = df(0, L). \]

Demand bound function - II

- The maximum is when the task is activated at the beginning of the interval.
- For a periodic task \( \tau_i \):

\[ dbf_i(L) = \left( \left\lfloor \frac{L - D_i}{T_i} \right\rfloor + 1 \right) C_i \]
**Synchronous periodic task sets**

**Theorem (Baruah, Howell, Rosier ’90)**

A synchronous periodic task set $T$ is schedulable by EDF if and only if:

$$\forall L \in \text{dead}(T) \quad \text{dbf}(L) \leq L$$

where $\text{dead}(T)$ is the set of deadlines in $[0, H]$

- Proof next slide.

---

**Proof**

- **Sufficiency:** eq. holds $\rightarrow$ task set is schedulable.
  - By contradiction
  - If deadline is missed in $y$, then $\exists x, y \ y - x < \text{df}(x, y)$
  - it follows that $y - x < \text{df}(x, y) \leq \text{dbf}(y - x)$

- **Necessity:** task set is schedulable $\rightarrow$ eq. holds
  - By contradiction
  - eq. does not hold for $\bar{I}$.
  - build a schedule starting at 0, for which $\text{dbf}(\bar{I}) = \text{df}(0, \bar{I})$
  - Hence task set is not schedulable
Sporadic task

- Sporadic tasks are equivalent to synchronous periodic task sets.
- For them, the worst case is when they all arrive at their maximum frequency and starting synchronously.

Synchronous and asynchronous

- Let $T$ be an asynchronous task set.
- We call $T'$ the corresponding synchronous set, obtained by setting all offset equal to 0.

**Corollary**

If $T'$ is schedulable, then $T$ is schedulable too.

Conversely, if $T$ is schedulable, $T'$ may not be schedulable.

- The proof follows from the definition of $\text{dbf}(L)$. 
A pseudo-polynomial test

**Theorem (Baruah, Howell, Rosier, ’90)**

Given a synchronous periodic task set \( T \), with deadlines less than or equal to the period, and with load \( U < 1 \), the system is schedulable by EDF if and only if:

\[
\forall L \in \text{deadShort}(T) \quad \text{dbf}(L) \leq L
\]

where \( \text{deadShort}(T) \) is the set of all deadlines in interval \([0, L^*]\)

and

\[
L^* = \frac{U}{1 - U} \max_i (T_i - D_i)
\]

**Corollary**

The complexity of the above analysis is pseudo-polynomial.

**Example of computation of the dbf**

- \( \tau_1 = (1, 4, 6), \tau_2 = (2, 6, 8), \tau_3 = (3, 5, 10) \)
- \( U = 1/6 + 1/4 + 3/10 = 0.7167, L^* = 12.64. \)
- We must analyze all deadlines in \([0, 12]\), i.e. \((3, 5, 6, 10)\).

Let’s compute \( \text{dbf}() \)
- \( df(0, 4) = C_1 = 1 < 4; \)
- \( df(0, 5) = C_1 + C_3 = 4 < 5; \)
- \( df(0, 6) = C_1 + C_2 + C_3 = 6 \leq 6; \)
- \( df(0, 10) = 2C_1 + C_2 + C_3 = 7 \leq 10; \)
- The task set is schedulable.
Idle time and busy period

- The interval between time 0 and the first idle time is called *busy period*.
- The analysis can be stopped at the first idle time (Spuri, ’94).
- The first idle time can be found with the following recursive equations:

\[
W(0) = \sum_{i=1}^{N} C_i \\
W(k) = \sum_{i=1}^{N} \left\lceil \frac{W(k-1)}{T_i} \right\rceil C_i
\]

- The iteration stops when \( W(k - 1) = W(k) \).

Another example

- Consider the following example

<table>
<thead>
<tr>
<th></th>
<th>( C_i )</th>
<th>( D_i )</th>
<th>( T_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau_1 )</td>
<td>1</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>( \tau_2 )</td>
<td>2</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>( \tau_3 )</td>
<td>4.5</td>
<td>8</td>
<td>15</td>
</tr>
</tbody>
</table>

- \( U = 0.9; L^* = 9 \times 7 = 63; \)
- \( W = 14.5. \)
- Then we can check all deadline in interval \([0, 14.5]\).
Algorithm

- Of course, it should not be necessary to draw the schedule to see if the system is schedulable or not.
- First of all, we need a formula for the dbf:
  \[ \text{dbf}(L) = \sum_{i=1}^{N} \left( \left\lfloor \frac{L - D_i}{T_i} \right\rfloor + 1 \right) C_i \]

- The algorithm works as follows:
  - We list all deadlines of all tasks until \( L^* \).
  - Then, we compute the dbf for each deadline and verify the condition.

The previous example

- In the previous example: deadlines of the tasks:
  \[
  \begin{array}{c|c}
  \tau_1 & 4 & 10 \\
  \tau_2 & 6 \\
  \tau_3 & 5 \\
  \end{array}
  \]

- dbf in tabular form
  \[
  \begin{array}{c|c|c|c|c}
  L & 4 & 5 & 6 & 10 \\
  \text{dbf} & 1 & 4 & 6 & 7 \\
  \end{array}
  \]

- Since, for all \( L < L^* \) we have \( \text{dbf}(L) \leq L \), then the task set is schedulable.
Another example

Consider the following task set:

<table>
<thead>
<tr>
<th></th>
<th>$C_i$</th>
<th>$D_i$</th>
<th>$T_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_1$</td>
<td>1</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>$\tau_2$</td>
<td>2</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>$\tau_3$</td>
<td>4.5</td>
<td>8</td>
<td>15</td>
</tr>
</tbody>
</table>

- $U = 0.9$; $L^* = 9 \times 7 = 63$;
- hint: if $L^*$ is too large, we can stop at the first idle time.
- The first idle time can be found with the following recursive equations:

$$W(0) = \sum_{i=1}^{N} C_i$$

$$W(k) = \sum_{i=1}^{N} \left\lceil \frac{W(k-1)}{T_i} \right\rceil C_i$$

- The iteration stops when $W(k-1) = W(k)$.
- In our example $W = 14.5$. Then we can check all deadline in interval $[0, 14.5]$.

Example

- Deadlines of the tasks:

<table>
<thead>
<tr>
<th></th>
<th>2</th>
<th>6</th>
<th>10</th>
<th>14</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_1$</td>
<td>2</td>
<td>6</td>
<td>10</td>
<td>14</td>
</tr>
<tr>
<td>$\tau_2$</td>
<td>4</td>
<td>9</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>$\tau_3$</td>
<td>8</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- Demand bound function in tabular form

<table>
<thead>
<tr>
<th>t</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>14</th>
</tr>
</thead>
<tbody>
<tr>
<td>dbf</td>
<td>1</td>
<td>3</td>
<td>4</td>
<td>8.5</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- The task set is not schedulable! Deadline miss at 8.
In the schedule...

The schedule is as follows:

Let’s recall the previous Corollary and Theorem.

Let us analyze the reasons why.

When computing $\text{dbf}(L)$ we do the following steps:

- Consider any interval $[t_1, t_2]$ of length $L$.
- "push back" activations until the first jobs starts at $t_1$;
- Compute the dbf as the sum of the computation of all jobs with deadline no later than $t_2$.
- **Problem**: by “pushing back” the instance we are modifying the task set!
Example of asynchronous task set

\[ \tau_1 = (0, 4, 7, 9) \text{ and } \tau_2 = (2, 5, 8, 12) \]

- \( df(0, 8) = 4 \)
- \( df(2, 10) = 5 \)

Example of asynchronous task set

\[ \tau_1 = (0, 4, 7, 9) \text{ and } \tau_2 = (2, 5, 8, 12) \]

- \( dbf(8) = 9 \)
- The dbf is too pessimistic.
Trade off between pessimism and complexity

- The problem is that we do not know what is the worst pattern of arrivals for asynchronous task sets.
- We know for synchronous: instant 0
- For asynchronous, we should check for every possible pattern

Key observation

- The **distance** between any arrival of task $\tau_i$ and any arrival of task $\tau_j$ is:

  $$a_{j,k_1} - a_{i,k_2} = \phi_j + k_1 T_j - \phi_i - k_2 T_i = \phi_j - \phi_i + k(\gcd(T_i, T_j))$$

- Imposing that the difference must not be negative, and $k$ must be integer, we get:

  $$k \geq \frac{\phi_i - \phi_j}{\gcd(T_i, T_j)} \Rightarrow k = \left\lceil \frac{\phi_i - \phi_j}{\gcd(T_i, T_j)} \right\rceil$$

- The **minimum distance** is:

  $$\Delta_{i,j} = \phi_j - \phi_i + \left\lceil \frac{\phi_i - \phi_j}{\gcd(T_i, T_j)} \right\rceil \gcd(T_i, T_j)$$
Observations

- From the formula we can derive the following observations:
  - The value of $\Delta_{i,j}$ is an integer in interval $[0, \gcd(T_i, T_j) - 1]$
  - If $T_i$ and $T_j$ are prime between them (i.e. $\gcd = 1$), then $\Delta_{i,j} = 0$.

- Now we are ready to explain the basic idea behind the new scheduling analysis methodology.

Basic Idea

- Given an hypothetical interval $[x, y]$
- Assume task $\tau_i$ arrival time coincides with $x$
- We “push back” all other tasks until they reach the minimum distance from $\tau_i$ arrival time
  - there is no need to push it back further (it would be too pessimistic!)
- The df in all intervals starting with $x$ can only increase after the “pushing back”.
- Therefore, if no deadline is missed in $[x, y]$, then no deadline is missed in any interval of length $(y - x)$.
- We could build such interval by selecting a task $\tau_i$ to start at the beginning of the interval, and setting the arrival times of the other tasks at their minimum distances
Problem

- We do not know which task to start with in the interval
- Simple solution: just select each task in turn

Example

- $\tau_1 = (0, 4, 7, 9)$ and $\tau_2 = (2, 5, 8, 12)$
  - We select $\tau_1$ to start at 0.
  - $\tau_2$ starts at
    \[
    \phi_2 - \phi_1 + \left\lceil \frac{\phi_1 - \phi_2}{T_1 \mod T_2} \right\rceil (T_1 \mod T_2) = 2 + \left\lceil \frac{-2}{3} \right\rceil 3 = 2
    \]

![Diagram showing two tasks $\tau_1$ and $\tau_2$ with start and end times marked.](image-url)
Example

- $\tau_1 = (0, 4, 7, 9)$ and $\tau_2 = (2, 5, 8, 12)$
- Next, we select $\tau_2$ to start at 0.
- $\tau_1$ starts at

$$\phi_1 - \phi_2 + \left\lceil \frac{\phi_2 - \phi_1}{T_2 \mod T_1} \right\rceil (T_2 \mod T_1) = -2 + \left\lceil \frac{2}{3} \right\rceil 3 = 1$$

Main theorem

- Given an asynchronous task set $\mathcal{T}$
- Let $\mathcal{T}_i'$ be the task set obtained by
  - fixing the offset of $\tau_i$ at 0
  - setting the offset of all other tasks at their minimum distance from $\tau_i$

Theorem (Pellizzoni and Lipari, ECRTS '04)

*Given task set $\mathcal{T}$ with $U \leq 1$, scheduled on a single processor, if $\forall 1 \leq i \leq N$ all deadlines in task set $\mathcal{T}_i'$ are met until the first idle time, then $\mathcal{T}$ is feasible.*
Performance

![Graph showing performance](image)

**Figure:** 10 tasks with periods multiple of 10

Conclusions

- **What is this for?**
- Feasibility analysis of asynchronous task set is used for:
  - Reduction of output jitter: by setting an offset it is possible to reduce response time and jitter
  - Analysis of distributed transactions (i.e. chains of tasks related by precedence constraints).
- in both cases, the analysis must be iteratively repeated many times with different offsets;
- hence we need an efficient analysis (even though it is only sufficient)
References I

M. L. Dertouzos
Control Robotics: The Procedural Control of Physical Processes
Information Processing, 1974

@ J.Y.-T. Leung and M.L. Merril,
A Note on Preemptive Scheduling of Periodic Real-Time Tasks

S.K. Baruah, L.E. Rosier and R.R. Howell,
Algorithms and Complexity Concerning the Preemptive Scheduling of Periodic Real-Time Tasks on One Processor

References II

R. Pellizzoni and G. Lipari
Feasibility Analysis of Real-Time Periodic Tasks with Offsets
Real-Time Systems Journal, 2005