Shared Resources and Blocking in Real-Time Systems

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Outline

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Interacting tasks

- Until now, we have considered only independent tasks
  - a task never blocks or suspends
  - it can only be suspended when it finishes its istance (job)
- However, in reality, many tasks exchange data through shared memory
- Consider as an example three periodic tasks:
  - One reads the data from the sensors and applies a filter. The results of the filter are stored in memory.
  - The second task reads the filtered data and computes some control law (updating the state and the outputs); both the state and the outputs are stored in memory;
  - finally, a third periodic task reads the outputs from memory and writes on the actuator device.
- All three tasks access data in the shared memory
- Conflicts on accessing this data in concurrency could make the data structures inconsistent.

Resources and critical sections

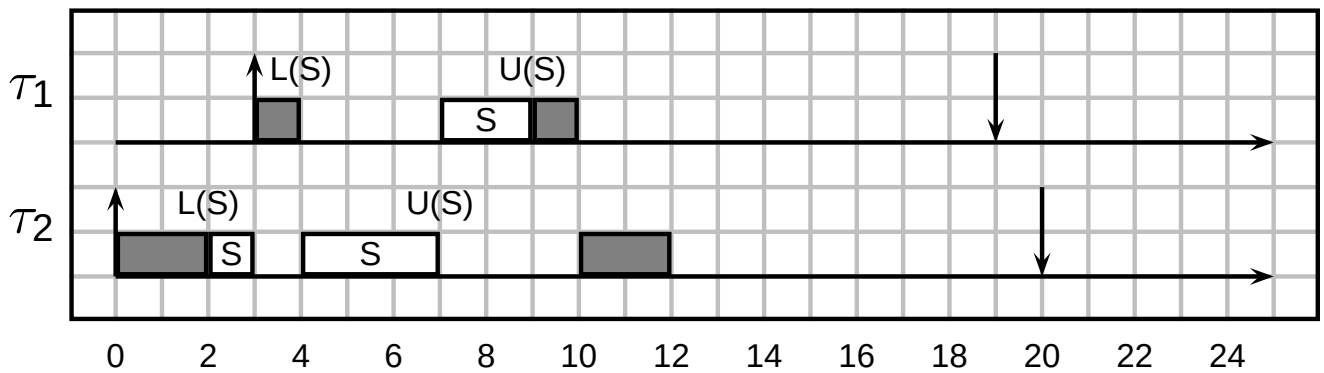
- The shared data structure is called *resource*;
- A piece of code accessing the data structure is called *critical section*;
- Two or more critical sections on the same resource must be executed in *mutual exclusion*;
- Therefore, each data structure should be *protected* by a mutual exclusion mechanism;
- In this lecture, we will study what happens when resources are protected by mutual exclusion semaphores.

Notation

- The resource and the corresponding mutex semaphore will be denoted by symbol S_j.
- A system consists of:
  - A set of N periodic (or sporadic) tasks $\mathcal{T} = \{\tau_1, \ldots, \tau_N\}$;
  - A set of shared resources $\mathcal{S} = \{S_1, \ldots, S_M\}$;
  - We say that a task τ_i *uses* resource S_j if it accesses the resource with a critical section.
  - The k-th critical of τ_i on S_j is denoted with $cs_{i,j}(k)$.
  - The length of the longest critical section of τ_i on S_j is denoted by $\xi_{i,j}$.

Blocking and priority inversion

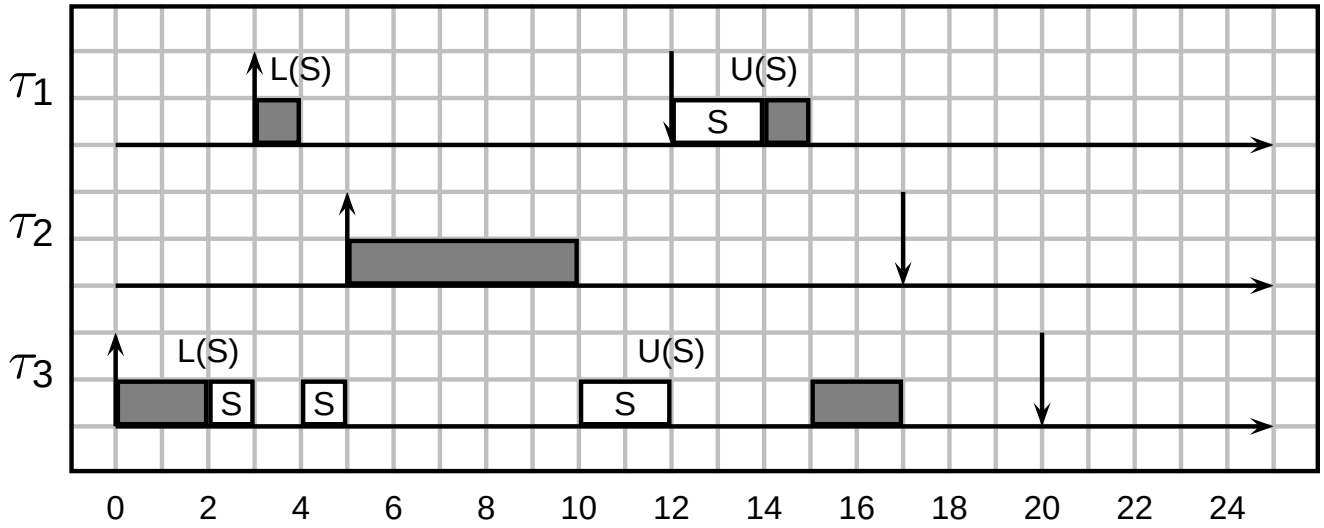
- A blocking condition happens when a high priority tasks wants to access a resource that is held by a lower priority task.
- Consider the following example, where $p_1 > p_2$.



- From time 4 to 7, task τ_1 is blocked by a lower priority taskτ_2; this is a *priority inversion*.
- Priority inversion is not avoidable; in fact, τ_1 must wait for τ_2 to leave the critical section.
- However, in some cases, the priority inversion could be too large.

Example of priority inversion

- Consider the following example, with $p_1 > p_2 > p_3$.



- This time the priority inversion is very large: from 4 to 12.
- The problem is that, while τ_1 is blocked, τ_2 arrives and preempt τ_3 before it can leave the critical section.
- If there are other medium priority tasks, they could preempt τ_3 as well.
- Potentially, the priority inversion could be unbounded!

The Mars Pathfinder

- This is not only a theoretical problem. It may happen in real cases.
- The most famous example of such problem was found during the Mars Pathfinder mission.

Outline

The Priority Inheritance protocol

- The solution of the problem is rather simple;
  - While the low priority task blocks an higher priority task, it *inherits* the priority of the higher priority task;
  - In this way, every medium priority task cannot make preemption.

Example

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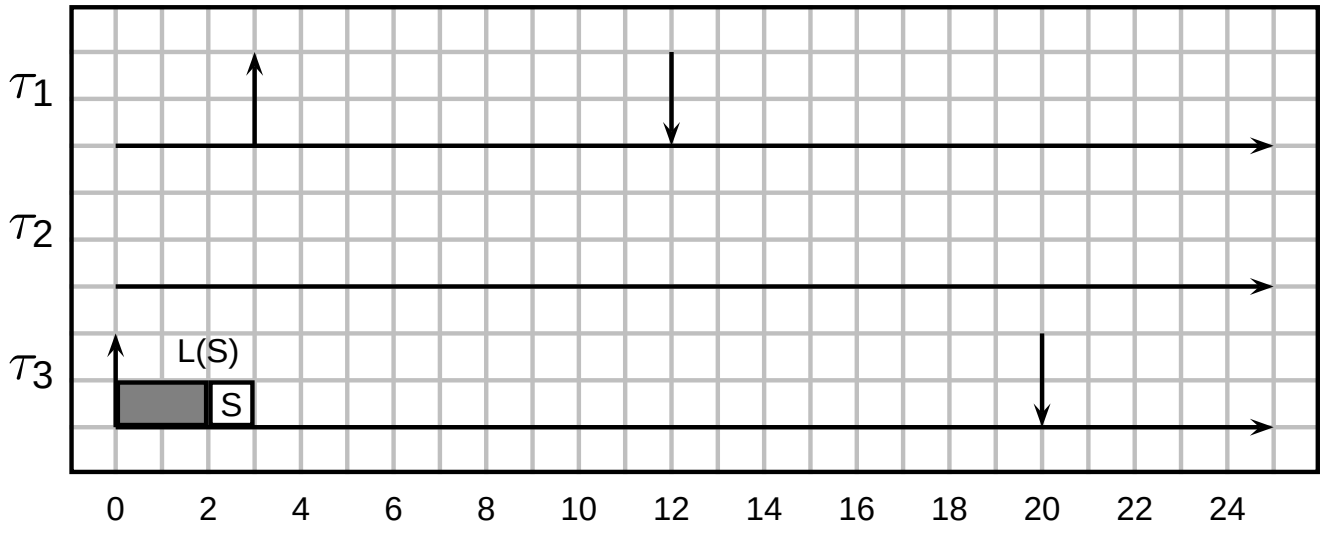

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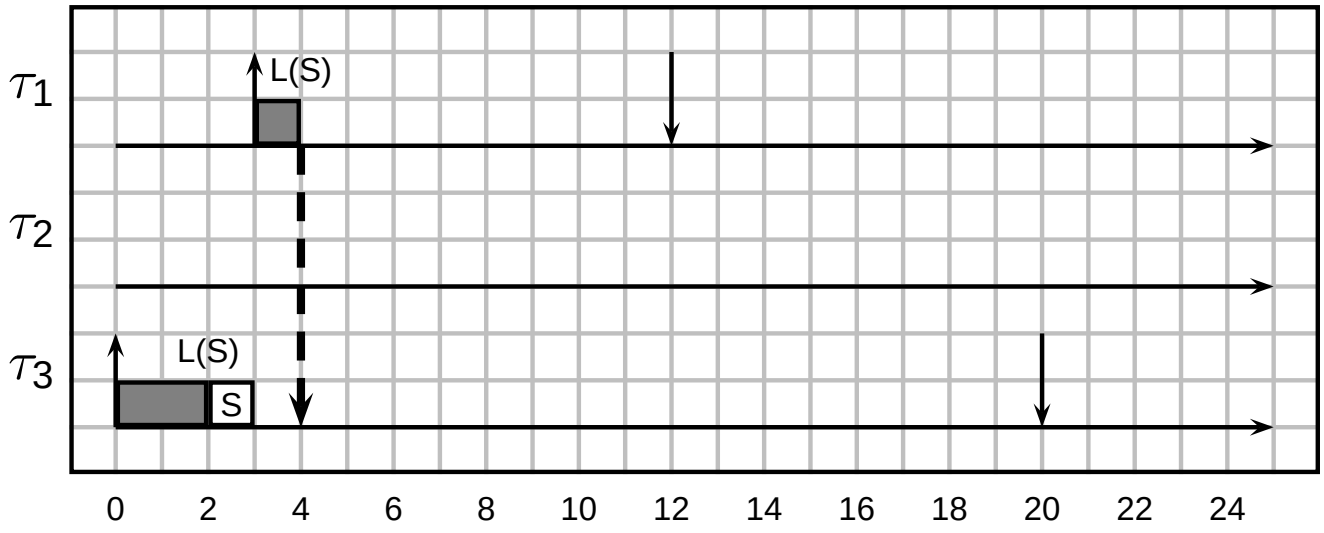

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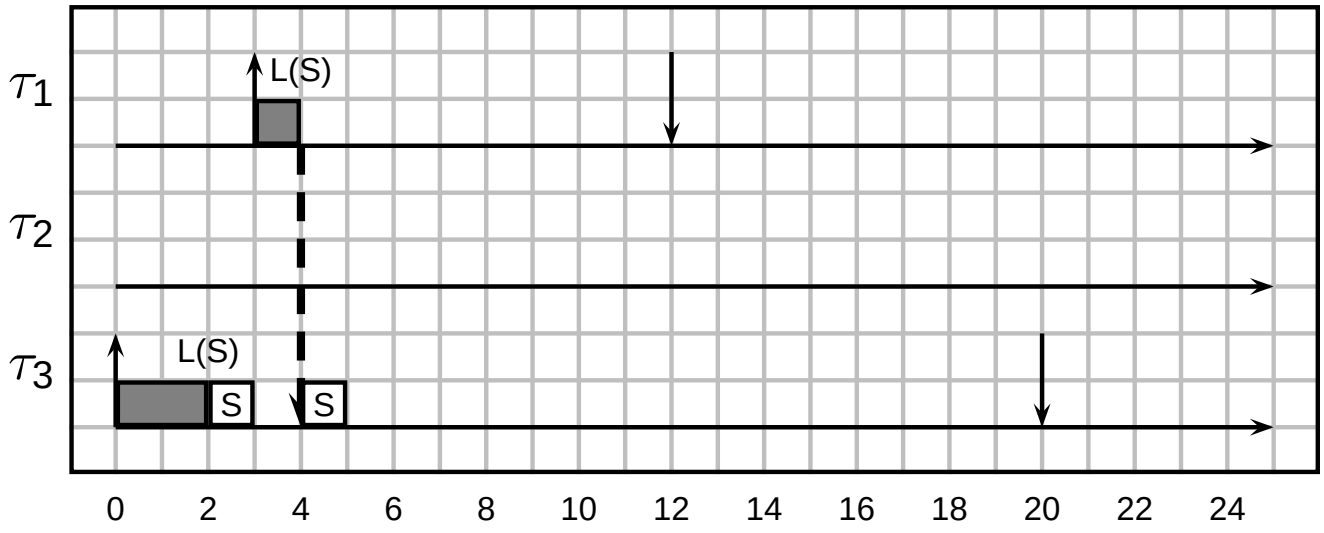

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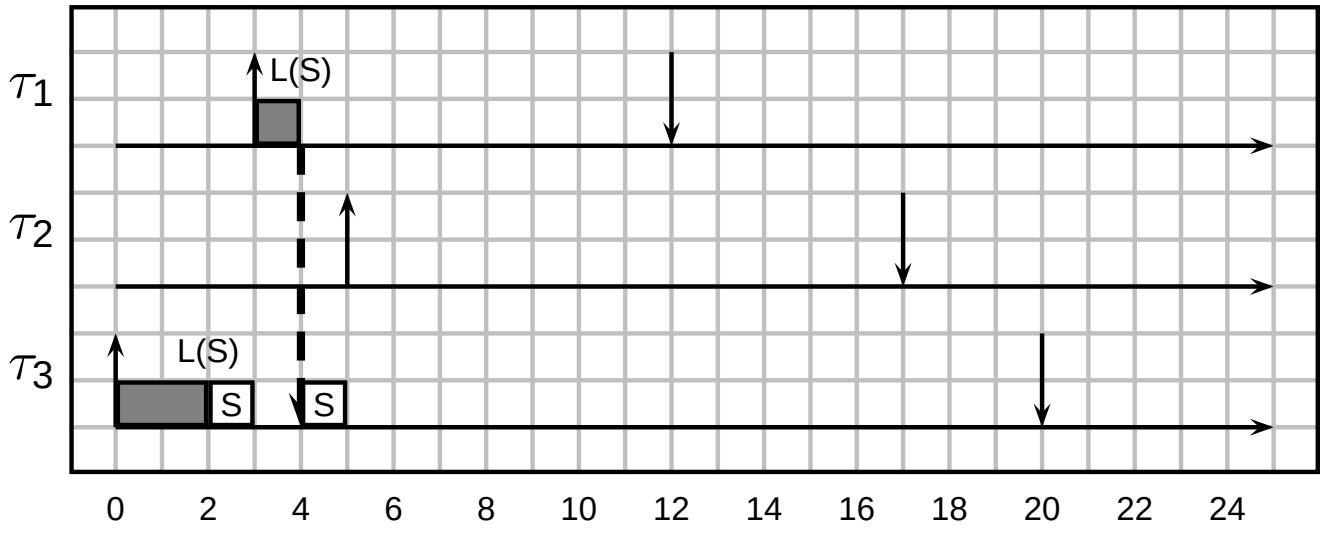

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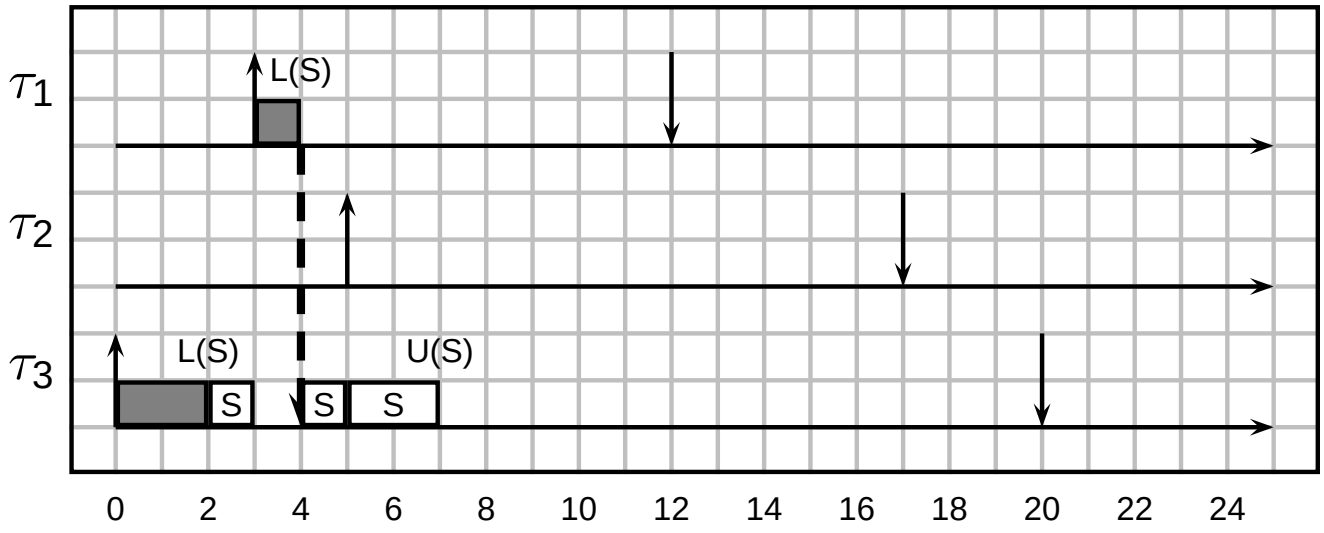

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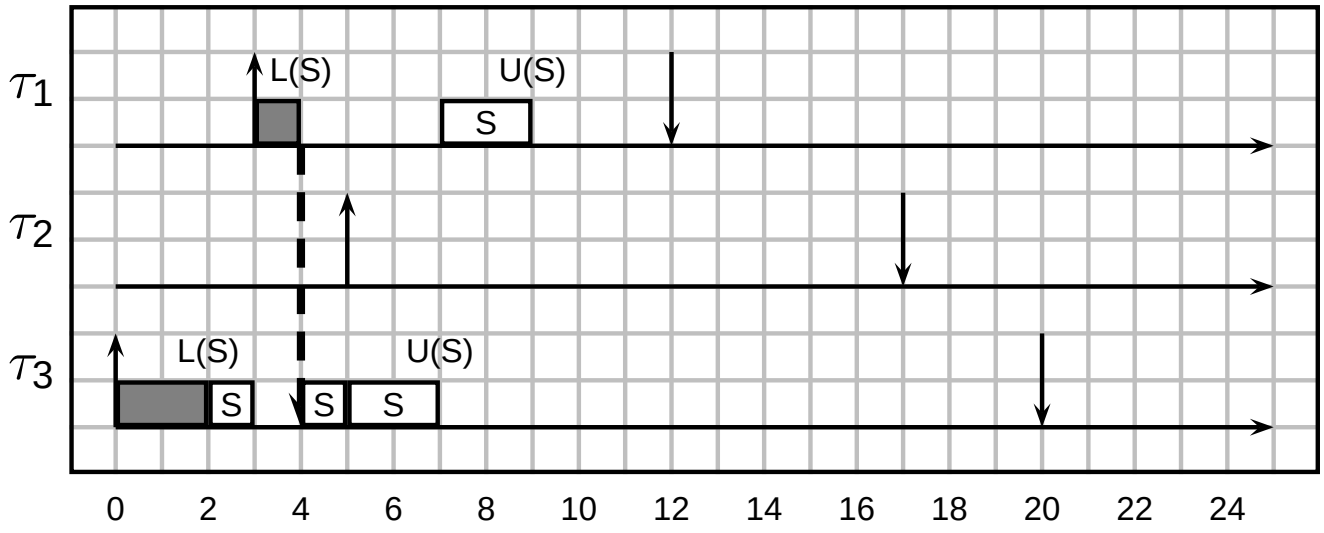

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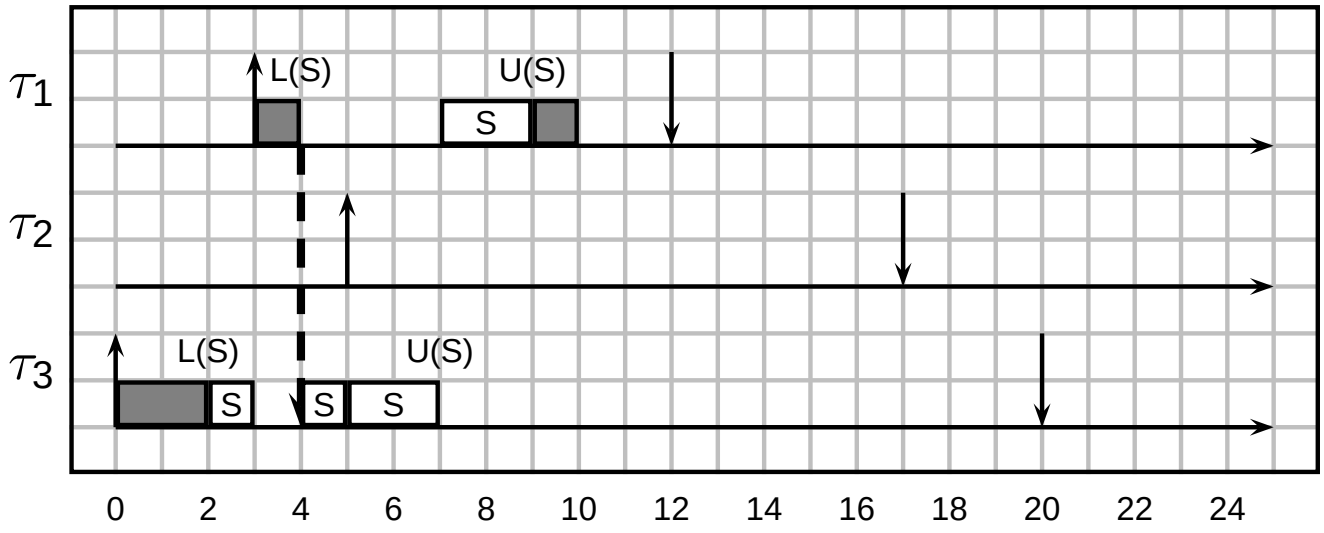

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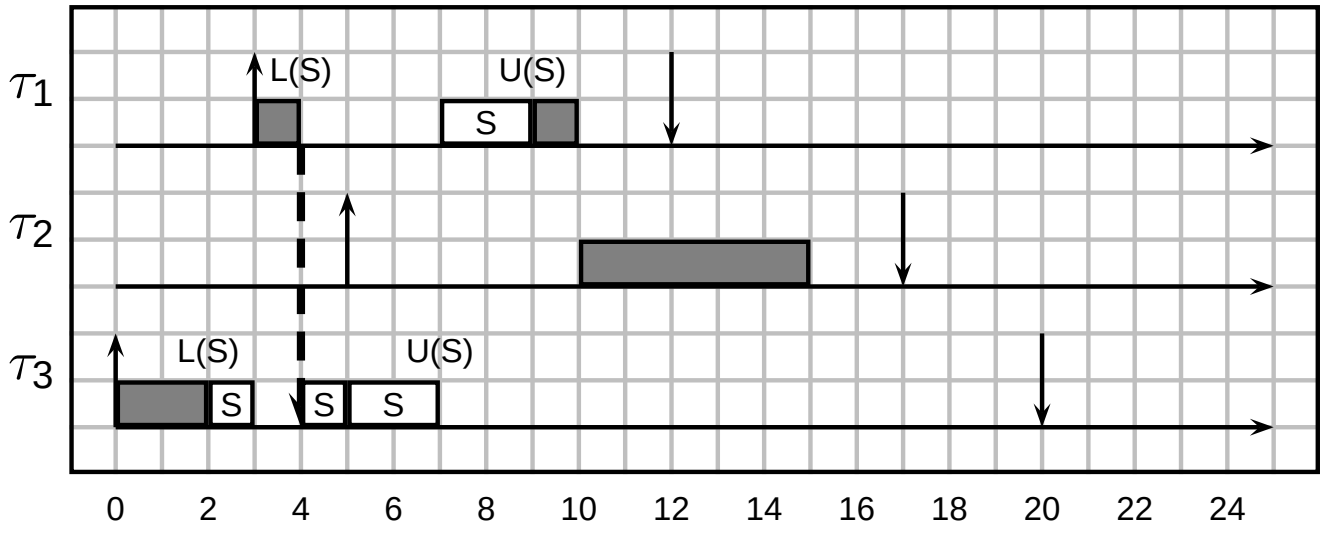

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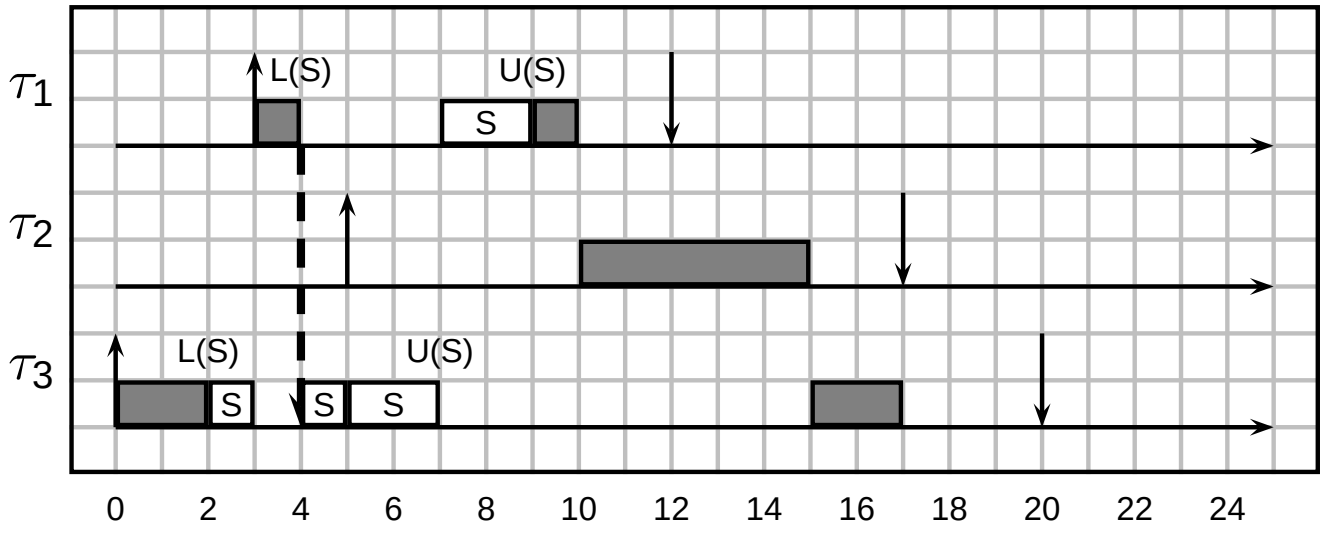

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Comments

- The blocking (priority inversion) is now bounded to the length of the critical section of task τ_3
- Tasks with intermediate priority τ_2 cannot interfere with τ_1
- However, τ_2 has a blocking time, even if it does not use any resource
  - This is called *indirect blocking* (or push-through)
  - due to the fact that τ_2 is *in the middle between* τ_1 and τ_3 which use the same resource.
  - This blocking time must be computed and taken into account in the formula

To be solved

- It remains to understand:
  - What is the maximum blocking time for a task
  - How we can account for blocking times in the schedulability analysis
- From now on, the maximum blocking time for a task τ_i is denoted by B_i.

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Nested critical sections

- Critical sections can be nested:
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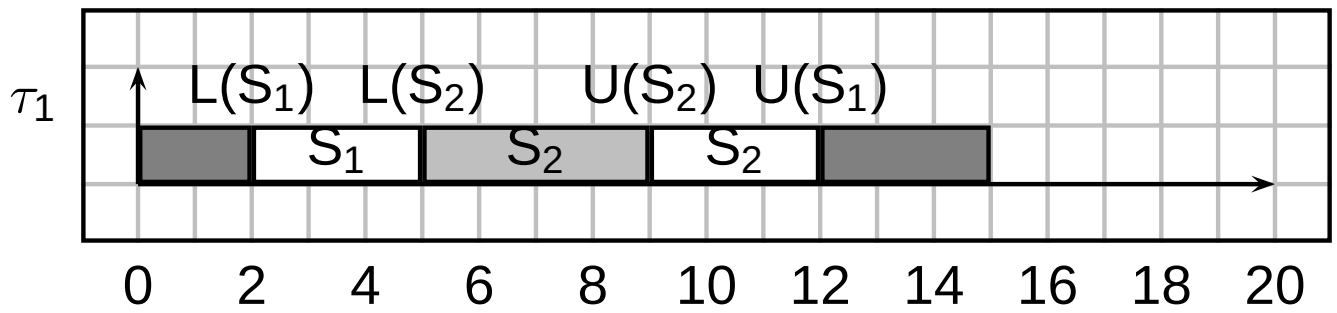

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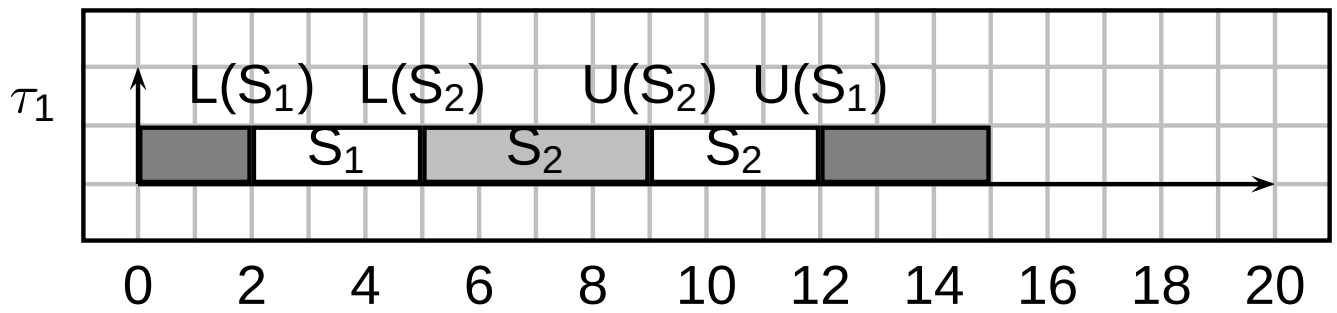
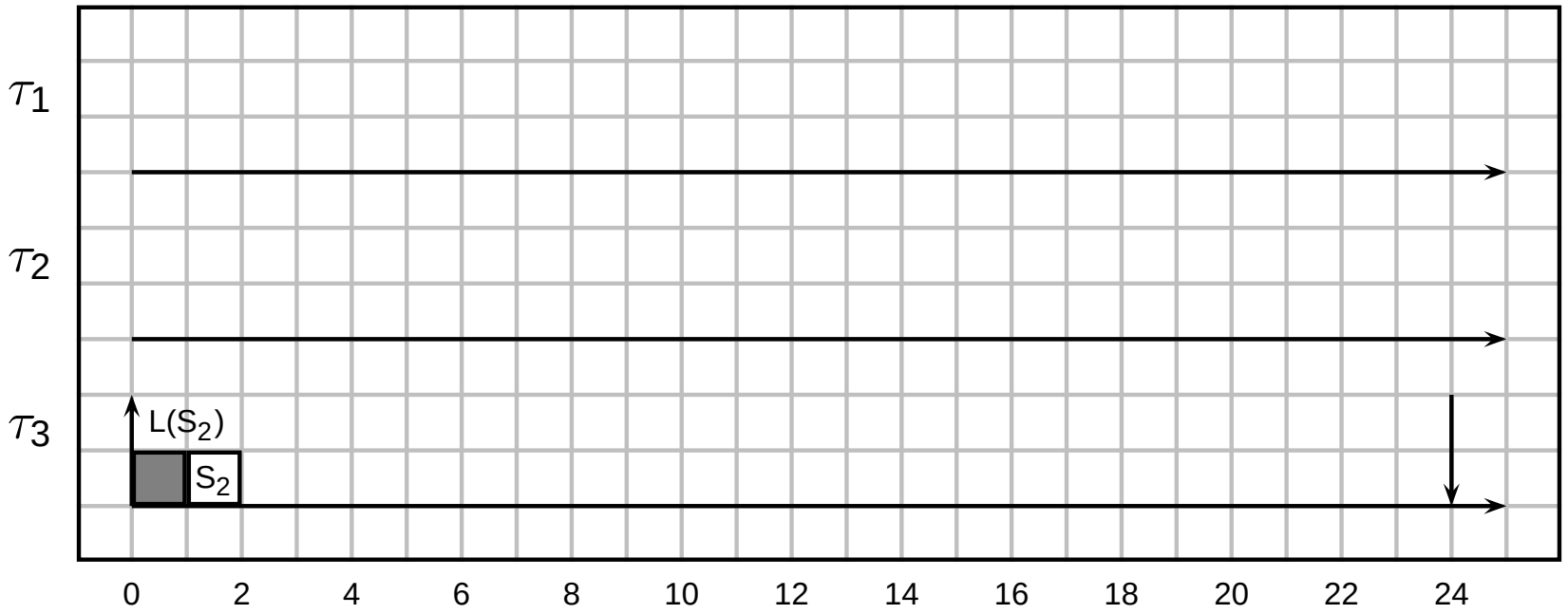


- When critical sections are nested, we can have *multiple inheritance*

Multiple inheritance

- Task τ_1 uses resource S_1; Task τ_2 uses S_1 and S_2 nested inside S_1; Task τ_3 uses only S_2.
- $p_1 > p_2 > p_3$;



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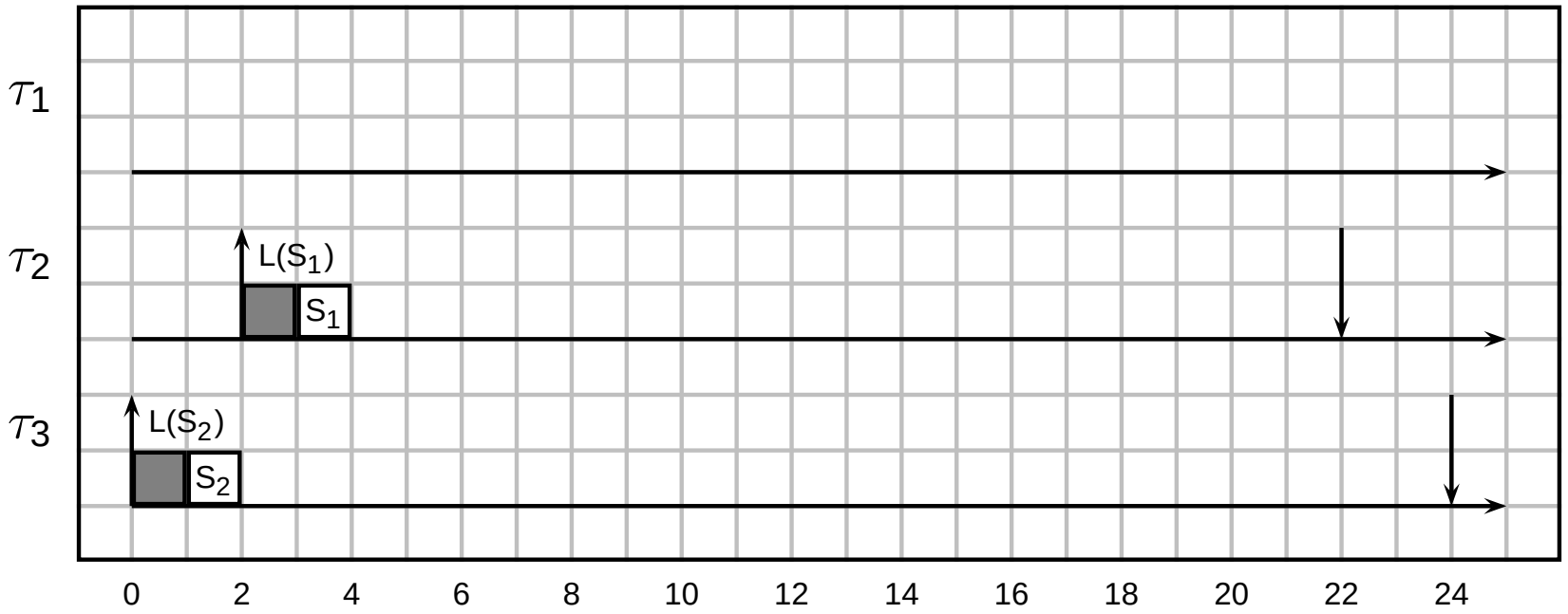

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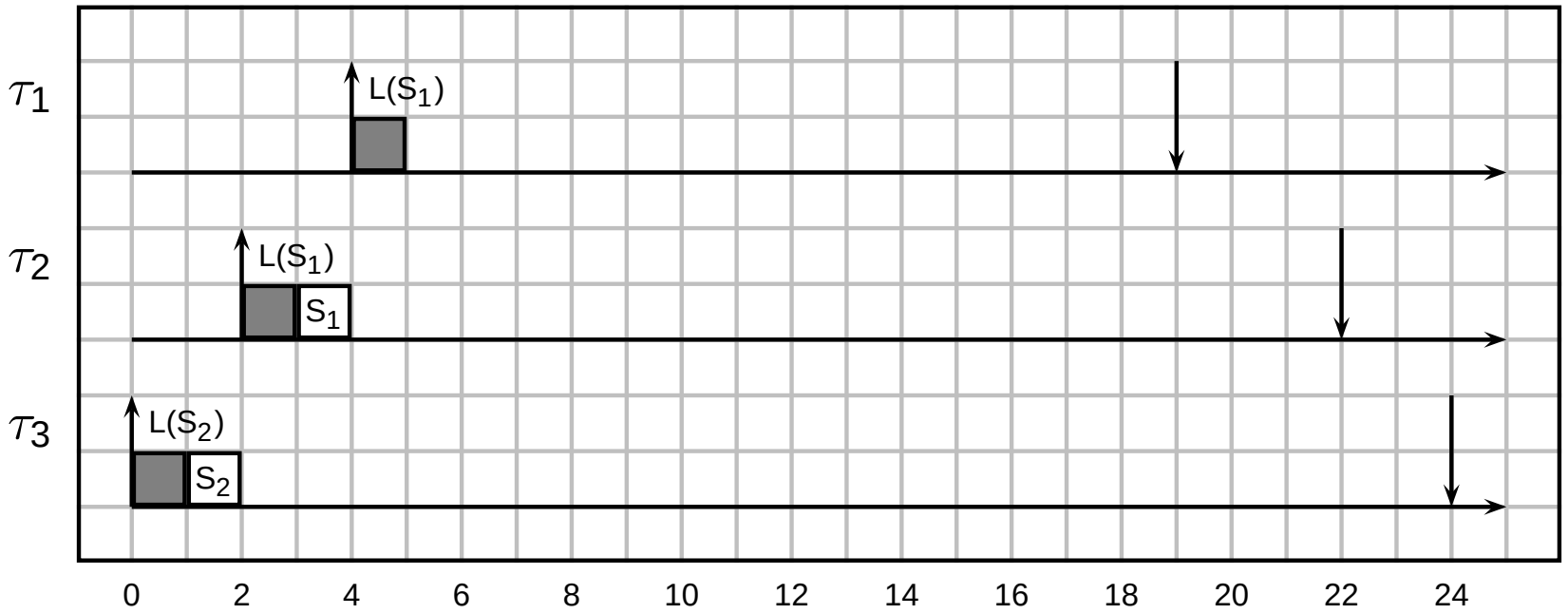

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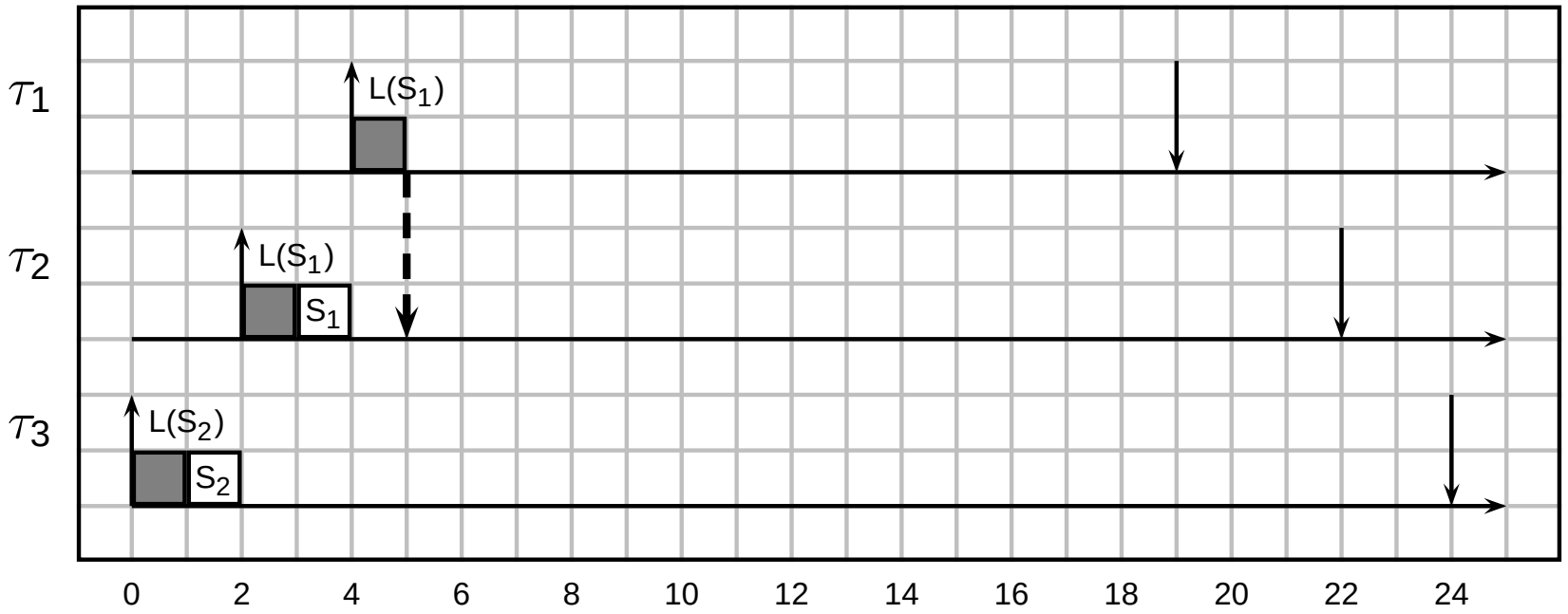

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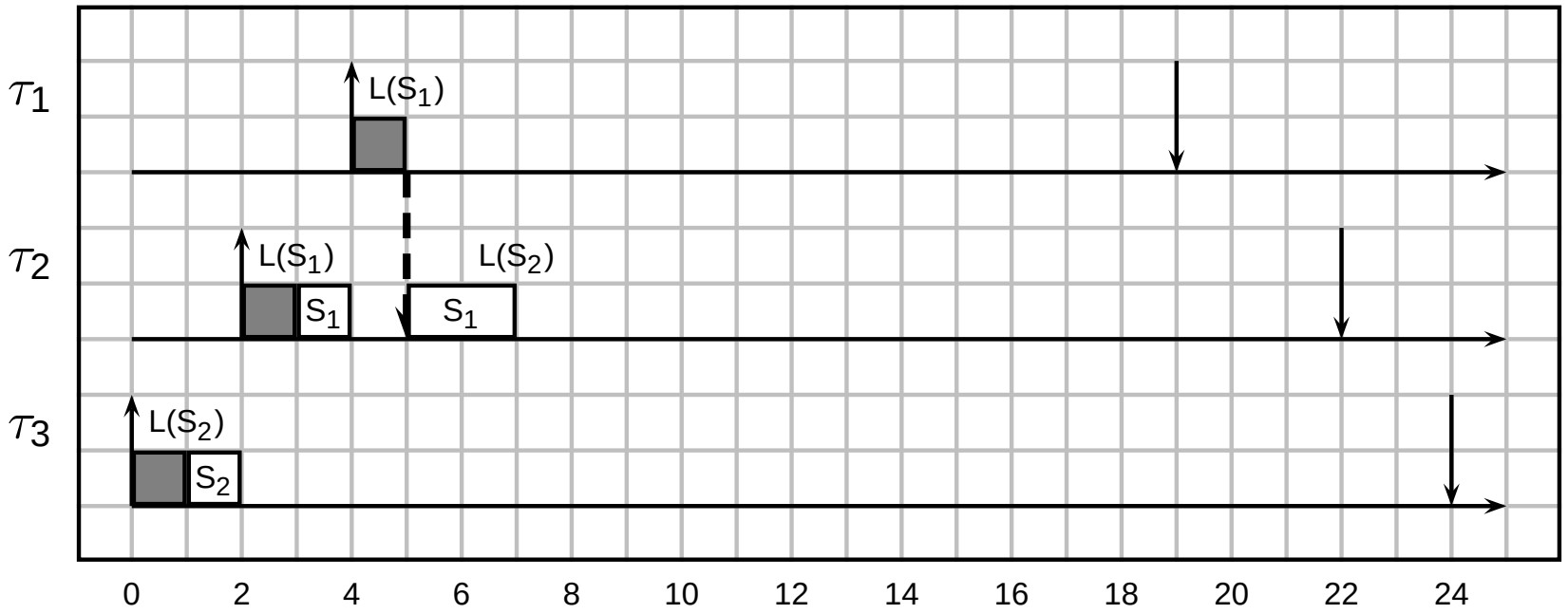

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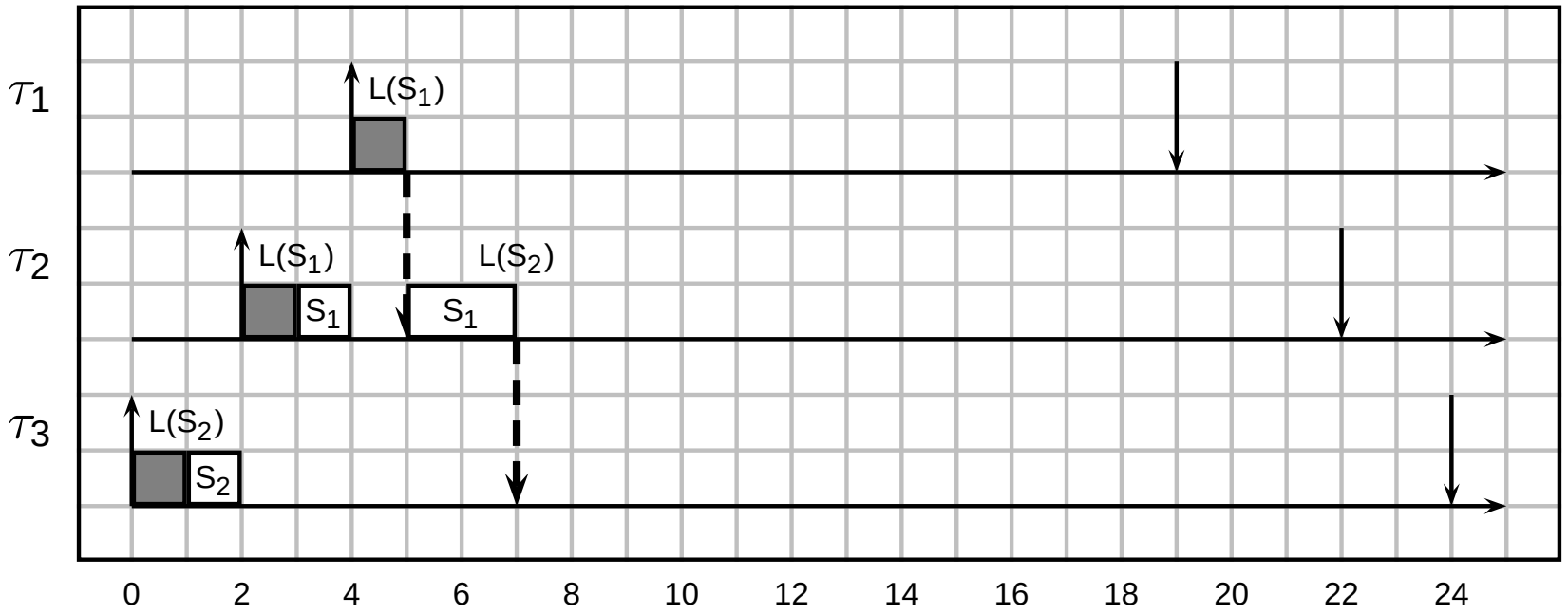

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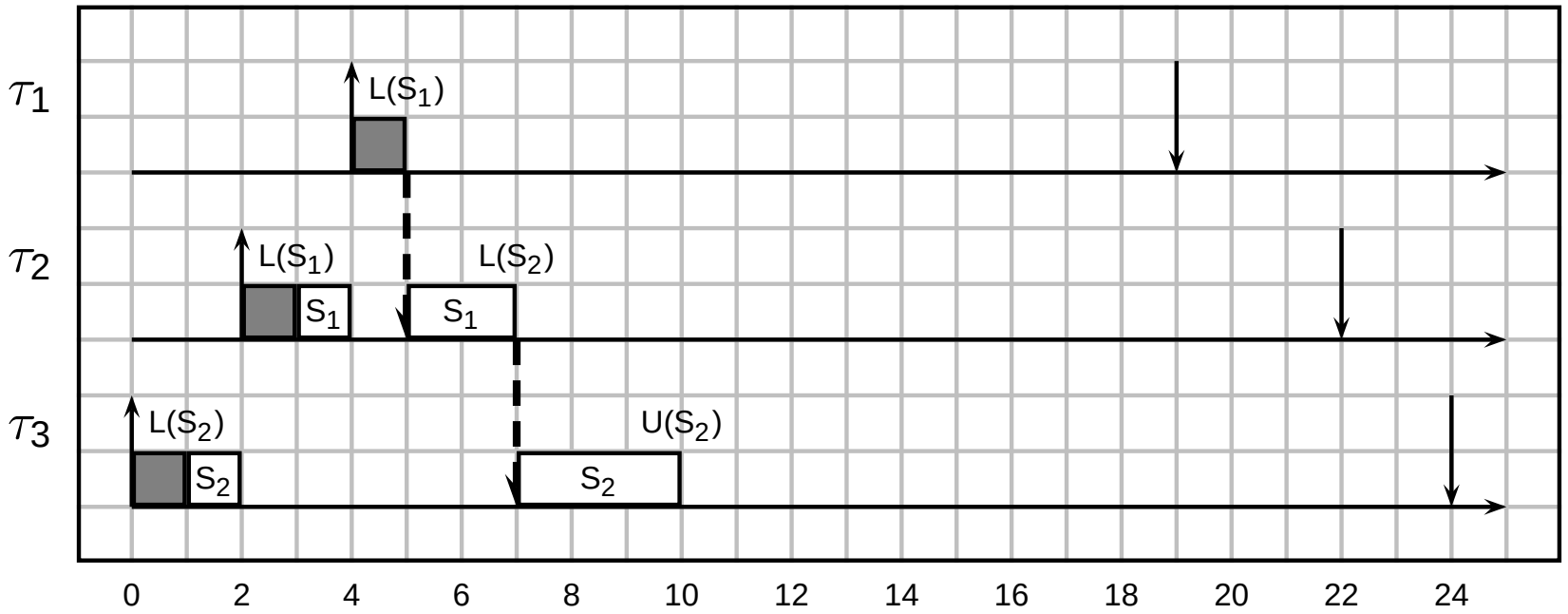

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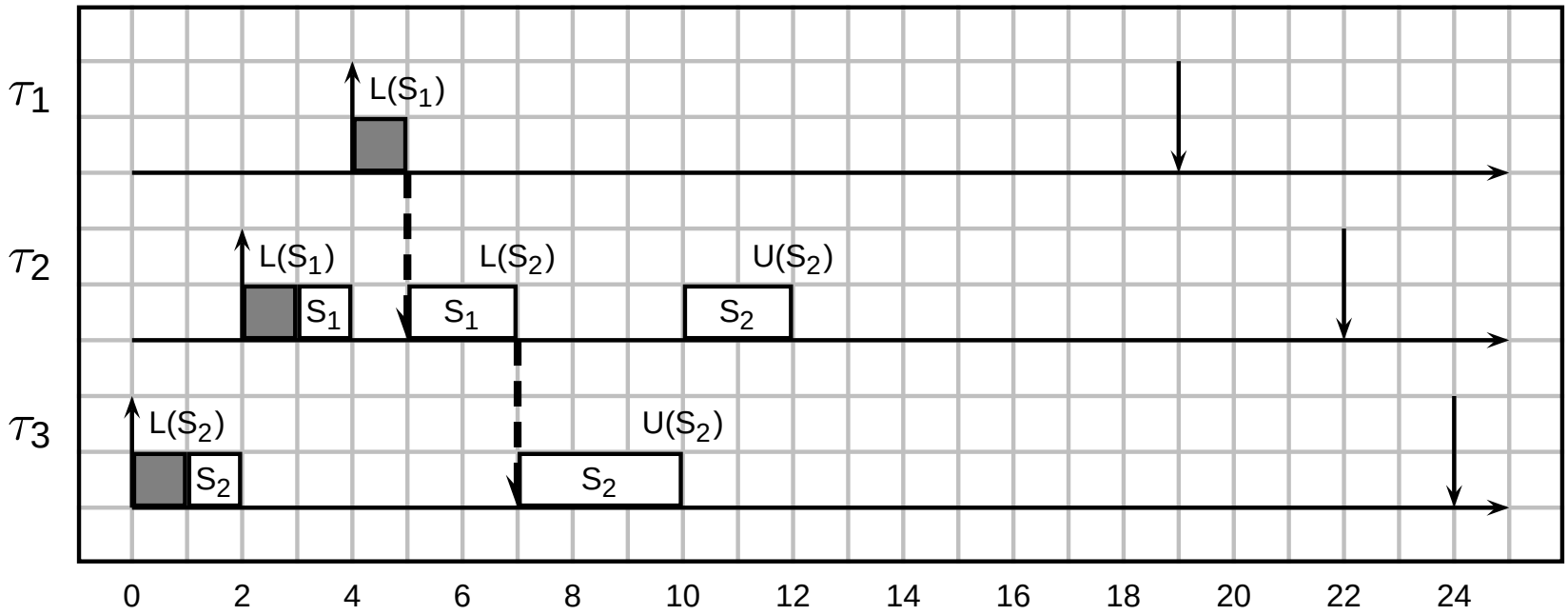

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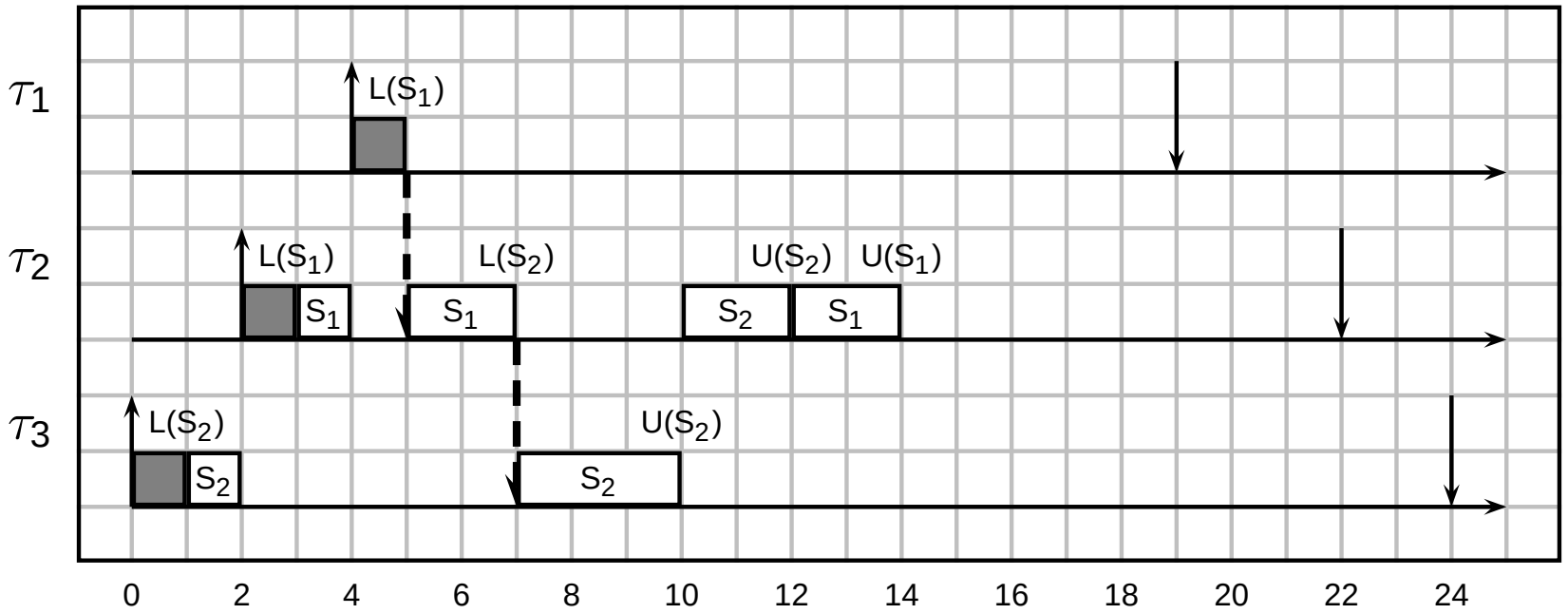

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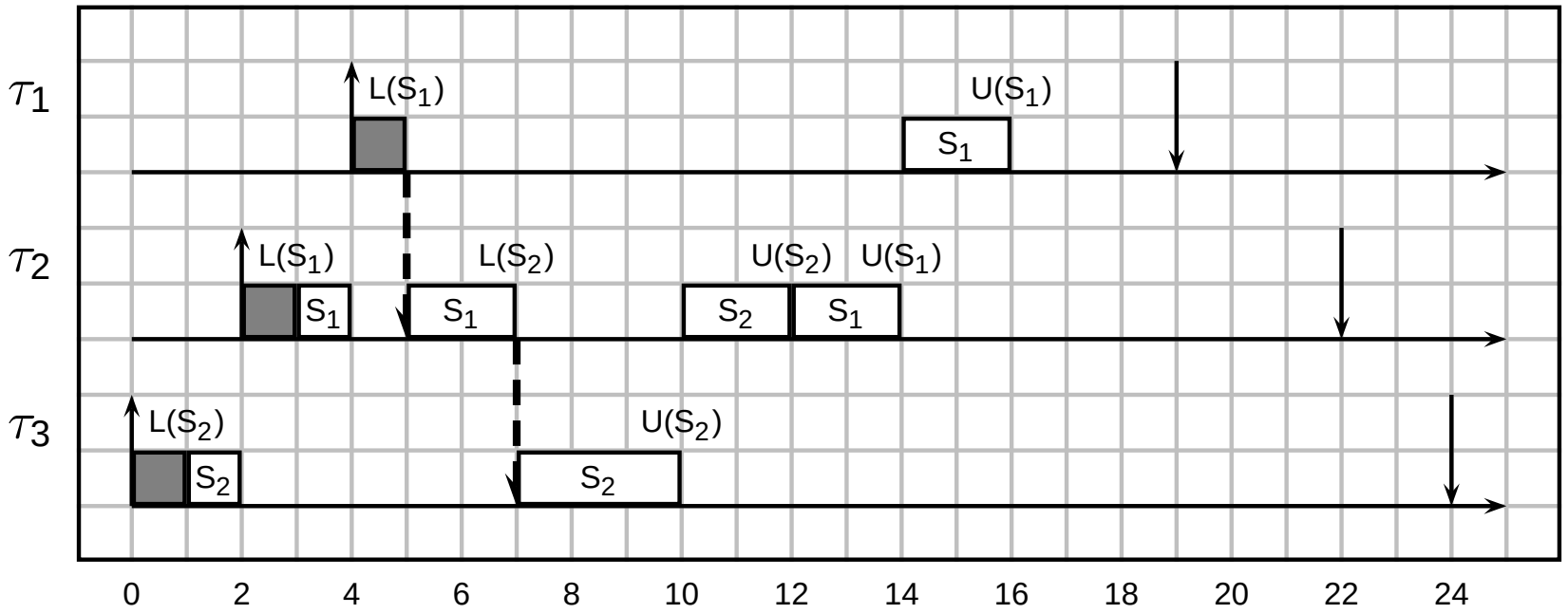

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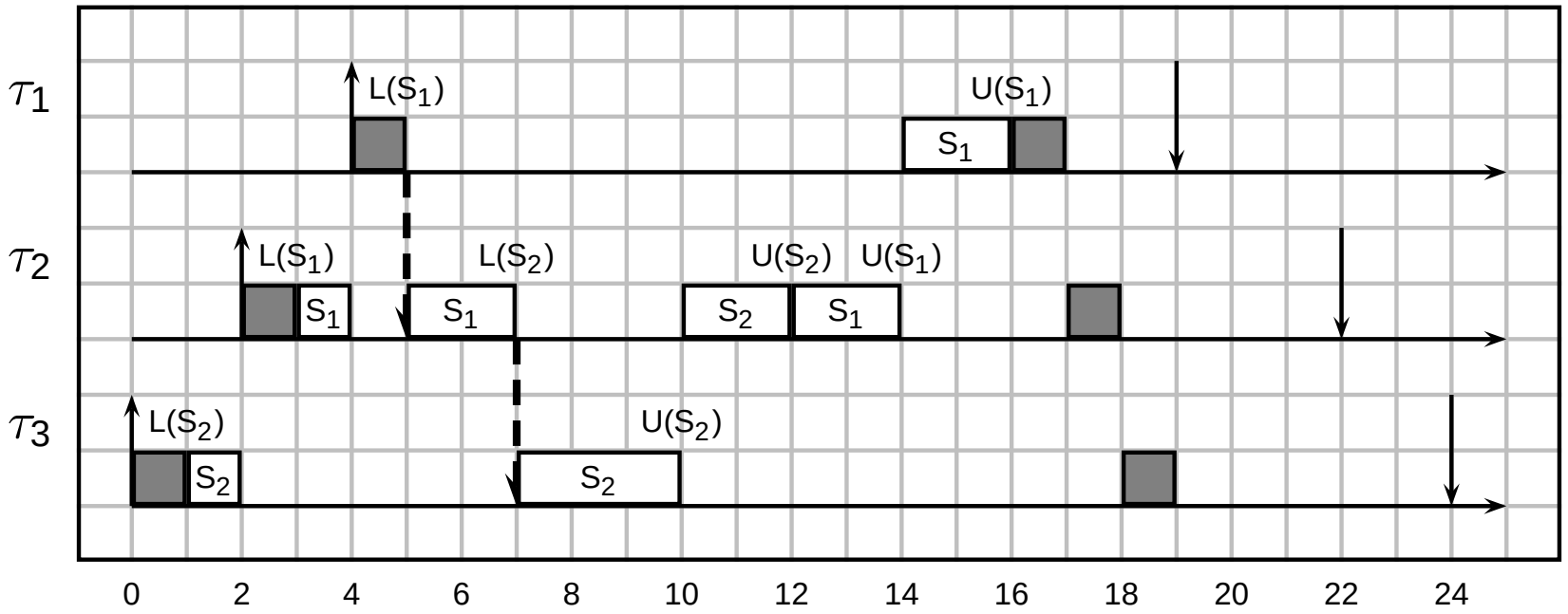

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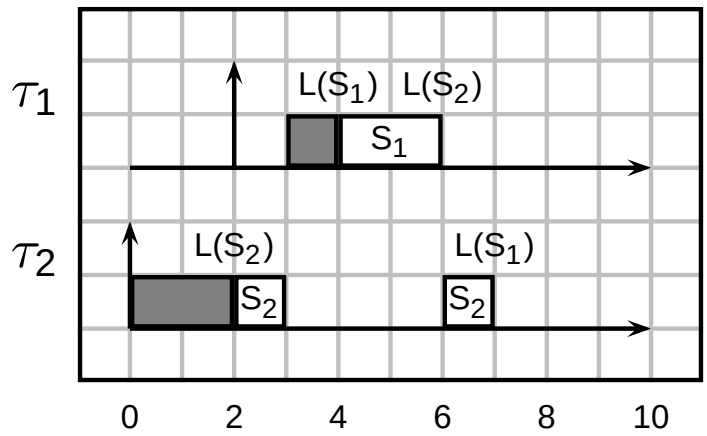
Deadlock problem

- When using nested critical section, the problem of deadlock can occur; i.e. two or more tasks can be blocked waiting for each other.
- The priority inheritance protocol *does not solve* automatically the problem of deadlock, as it is possible to see in the following example.
  - Task τ_1 uses S_2 inside S_1, while task τ_2 uses S_1 inside S_2.



- While τ_1 is blocked on S_2, which is held by τ_2, τ_2 is blocked on S_1 which is held by τ_1: **deadlock!**

Deadlock avoidance

- In the previous example, the priority inheritance protocol does not help.
- To avoid deadlock, it is possible to restrict programming freedom;
  - The problem is due to the fact that resouces are accessed in a random order by τ_1 and τ_2.

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  - If τ_2 accesses the resources in the correct order (S_2 inside S_1, the deadlock is automatically avoided).

The Priority Inheritance Protocol

- Summarising, the main rules are the following;
  - If a task τ_i is blocked on a resource protected by a mutex semaphore S, and the resource is locked by task τ_j, then τ_j *inherits* the priority of τ_i;
  - If τ_j is itself blocked on another semaphore by a task τ_k, then τ_k inherits the priority of τ_i (*multiple inheritance*);
  - If τ_k is blocked, the chain of blocked tasks is followed until a non-blocked task is found that inherits the priority of τ_i.
  - When a task unlocks a semaphore, it returns to the priority it had when locking it.

Outline

Computing the maximum blocking time

- We will compute the maximum blocking time only in the case of non nested critical sections.
  - Even if we avoid the problem of deadlock, when critical sections are nested, the computation of the blocking time becomes very complex due to multiple inheritance.
  - If critical section are not nested, multiple inheritance cannot happen, and the computation of the blocking time becomes simpler.

Theorems

- To compute the blocking time, we must consider the following two important theorems:

Theorem

Under the priority inheritance protocol, a task can be blocked only once on each different semaphore.

Theorem

Under the priority inheritance protocol, a task can be blocked by another lower priority task for at most the duration of one critical section.

- A task can be blocked more than once, but only once per each resource and once by each task.

Blocking time computation

- We must build a *resource usage table*.
  - On each row, we put a task in decreasing order of priority;

- A task can be blocked only by lower priority tasks:
  - Then, for each task (row), we must consider only the rows below (tasks with lower priority).

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- A task can be blocked only by lower priority tasks:
  - Then, for each task (row), we must consider only the rows below (tasks with lower priority).
- A task can be blocked only on resources that it uses directly, or used by higher priority tasks (*indirect blocking*);
  - For each task, we must consider only those column on which it can be blocked (used by itself or by higher priority tasks).

Example of blocking time computation

- let's start from B_1

	S_1	S_2	S_3	B
τ_1	2	0	0	?
τ_2	0	1	0	?
τ_3	0	0	2	?
τ_4	3	3	1	?
τ_5	1	2	1	?

- τ_1 can be blocked only on S_1.
- Therefore, we must consider only the first column, and take the maximum, which is 3.

Example of blocking time computation

- Now τ_2: it can be blocked on S_1 (*indirect blocking*) and on S_2.

	S_1	S_2	S_3	B
τ_1	2	0	0	3
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- Therefore, we must consider the first 2 columns;
- Then, we must consider all cases where two distinct lower priority tasks between τ_3, τ_4 and τ_5 access S_1 and S_2

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- Then, we must consider all cases where two distinct lower priority tasks between τ_3, τ_4 and τ_5 access S_1 and S_2
- The possibilities are:
  - τ_4 on S_1 and τ_5 on S_2: $\rightarrow 5$;
  - τ_4 on S_2 and τ_5 on S_1: $\rightarrow 4$;
- The maximum is $B_2 = 5$.

Example of blocking time computation

- τ_3 can be blocked on all 3 resources

	S_1	S_2	S_3	B
τ_1	2	0	0	3
τ_2	0	1	0	5
τ_3	0	0	2	?
τ_4	3	3	1	?
τ_5	1	2	1	?

- The possibilities are:
  - τ_4 on S_1 and τ_5 on S_2: $\rightarrow 5$;
  - τ_4 on S_2 and τ_5 on S_1 or S_3: $\rightarrow 4$;
  - τ_4 on S_3 and τ_5 on S_1: $\rightarrow 2$;
  - τ_4 on S_3 and τ_5 on S_2 or S_3: $\rightarrow 3$;
- The maximum is $B_3 = 5$.

Example of blocking time computation

- Now:

	S_1	S_2	S_3	B
τ_1	2	0	0	3
τ_2	0	1	0	5
τ_3	0	0	2	5
τ_4	3	3	1	?
τ_5	1	2	1	?

- τ_4 can be blocked on all 3 resources, but only by τ_5.
- The maximum is $B_4 = 2$.
- τ_5 cannot be blocked by any other task (because it is the lower priority task!); $B_5 = 0$;

Example: Final result

- the final result is:

	S_1	S_2	S_3	B
τ_1	2	0	0	3
τ_2	0	1	0	5
τ_3	0	0	2	5
τ_4	3	3	1	2
τ_5	1	2	1	0

Scheduling analysis

- In case of relative deadlines equal to periods, we have:

$$\forall i = 1, \ldots, N \quad \sum_{j=1}^{i} \frac{C_j}{T_j} + \frac{B_i}{T_i} \leq U_{\text{lub}}$$

Example

- In the previous example:

	C	T	U	B
τ_1	4	16	0.25	3
τ_2	3	24	0.125	5
τ_3	4	32	0.125	5
τ_4	5	40	0.125	2
τ_5	4	50	0.08	0

- $U_1 + \frac{B_1}{T_1} = 0.25 + 0.1875 = 0.4375 < 0.743$

Example

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- $U_1 + U_2 + U_3 + U_4 + \frac{B_4}{T_4} = 0.625 + 0.05 = 0.675 < 0.743$

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- $U_1 + U_2 + U_3 + U_4 + U_5 \frac{B_5}{T_5} = 0.705 + 0 < 0.743$

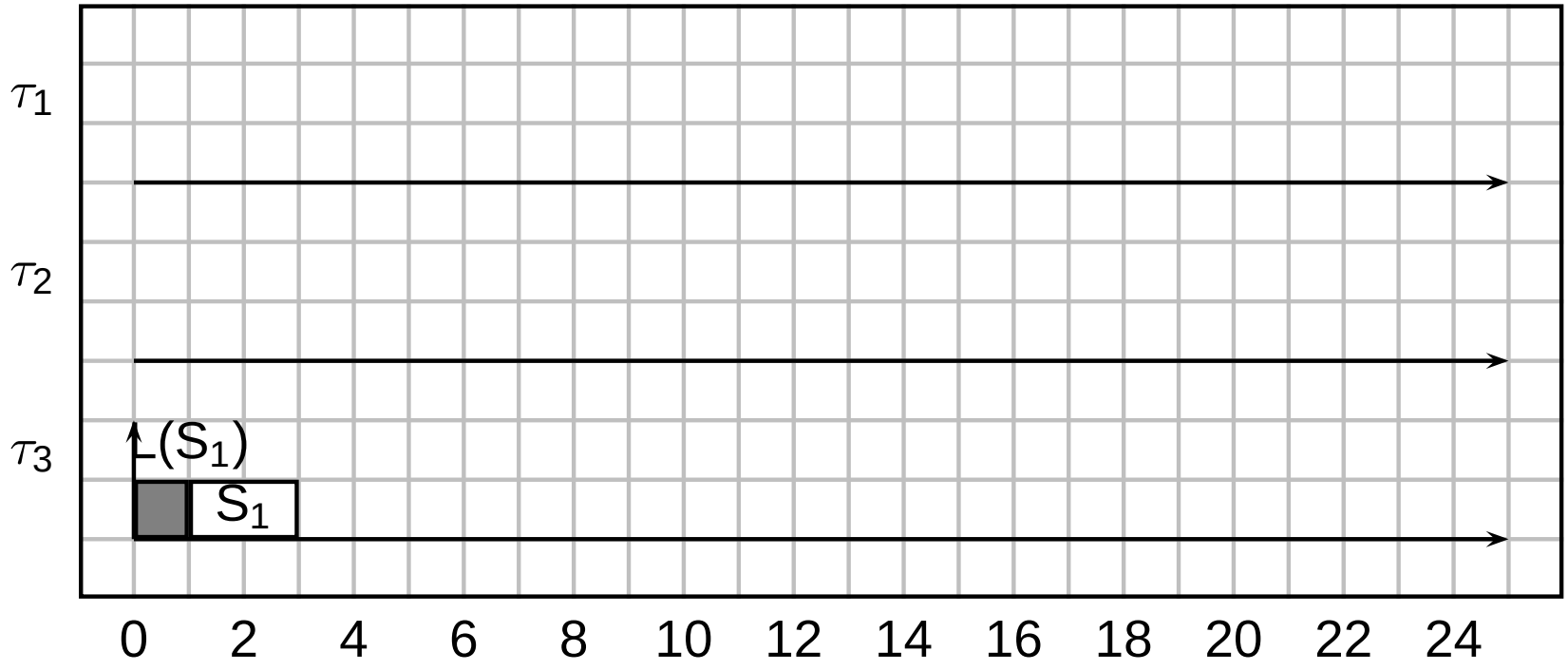
Outline

Problems of Priority inheritance

- Multi blockings
  - A task can be blocked more than once on different semaphores
- Multiple inheritance
  - when considering nested resources, the priority can be inherited multiple times
- Deadlock
  - In case of nested resources, there can be a deadlock

Multiple blocking example

- example:



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Multiple blocking example

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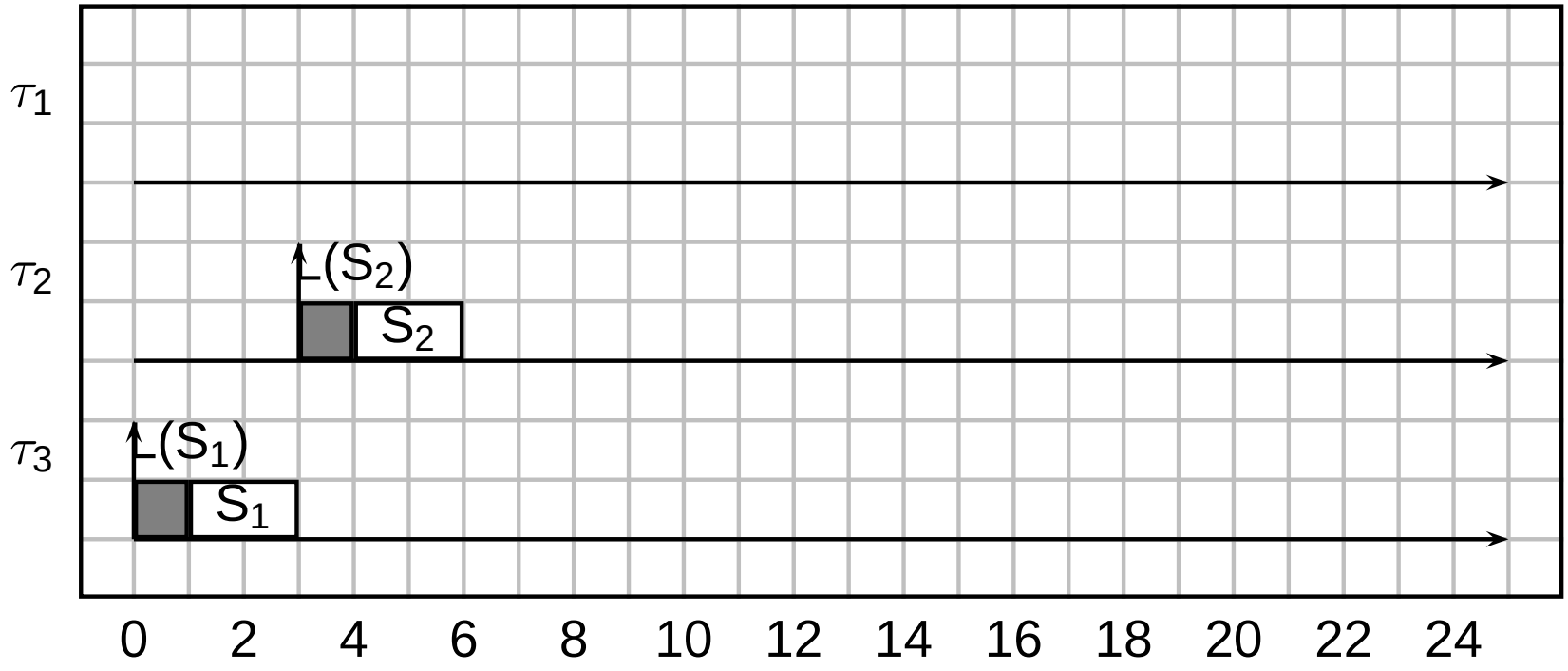

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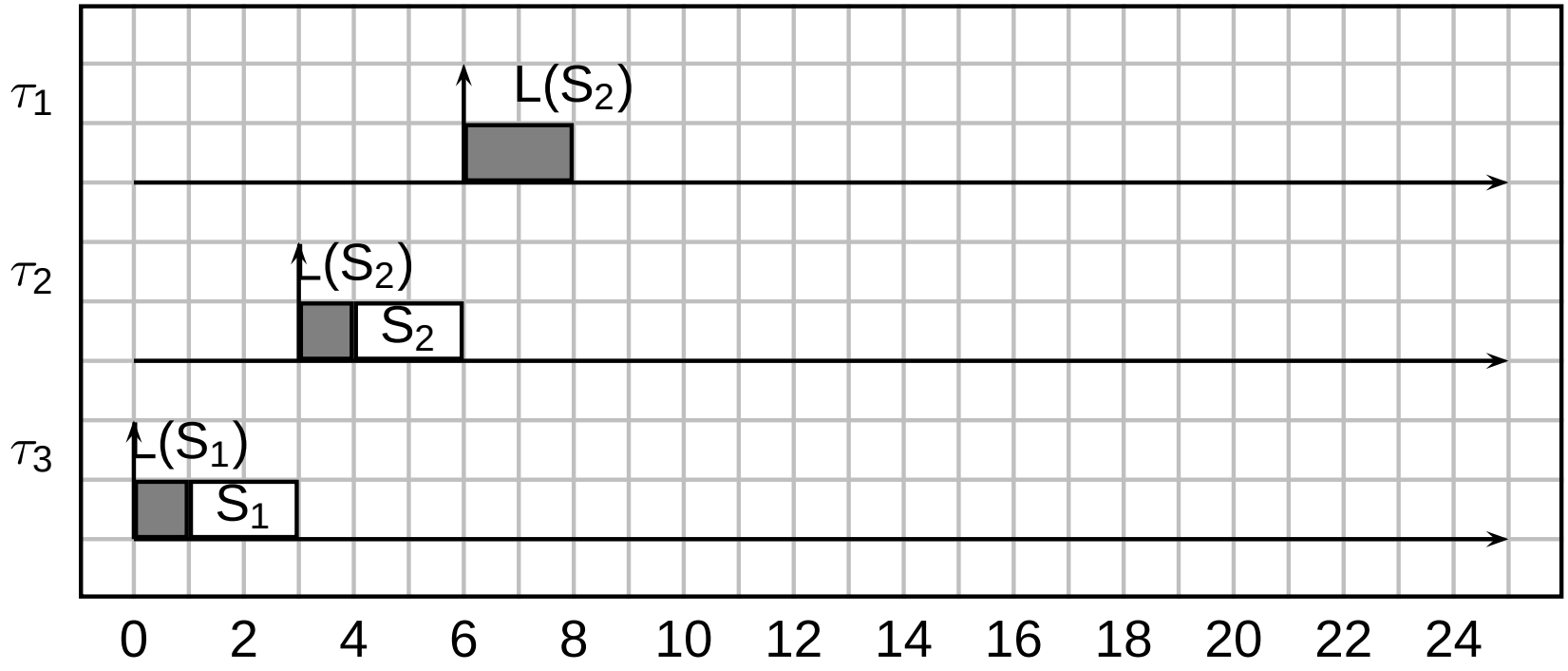

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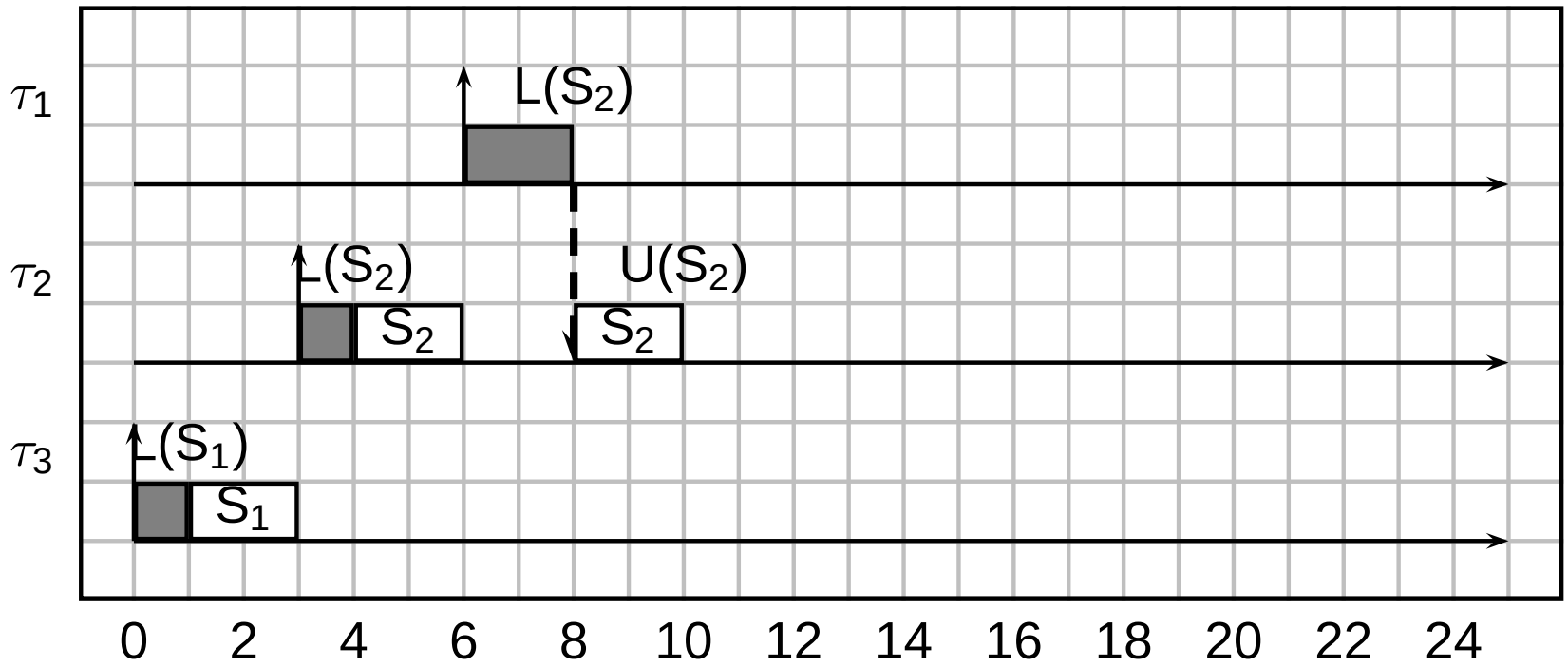

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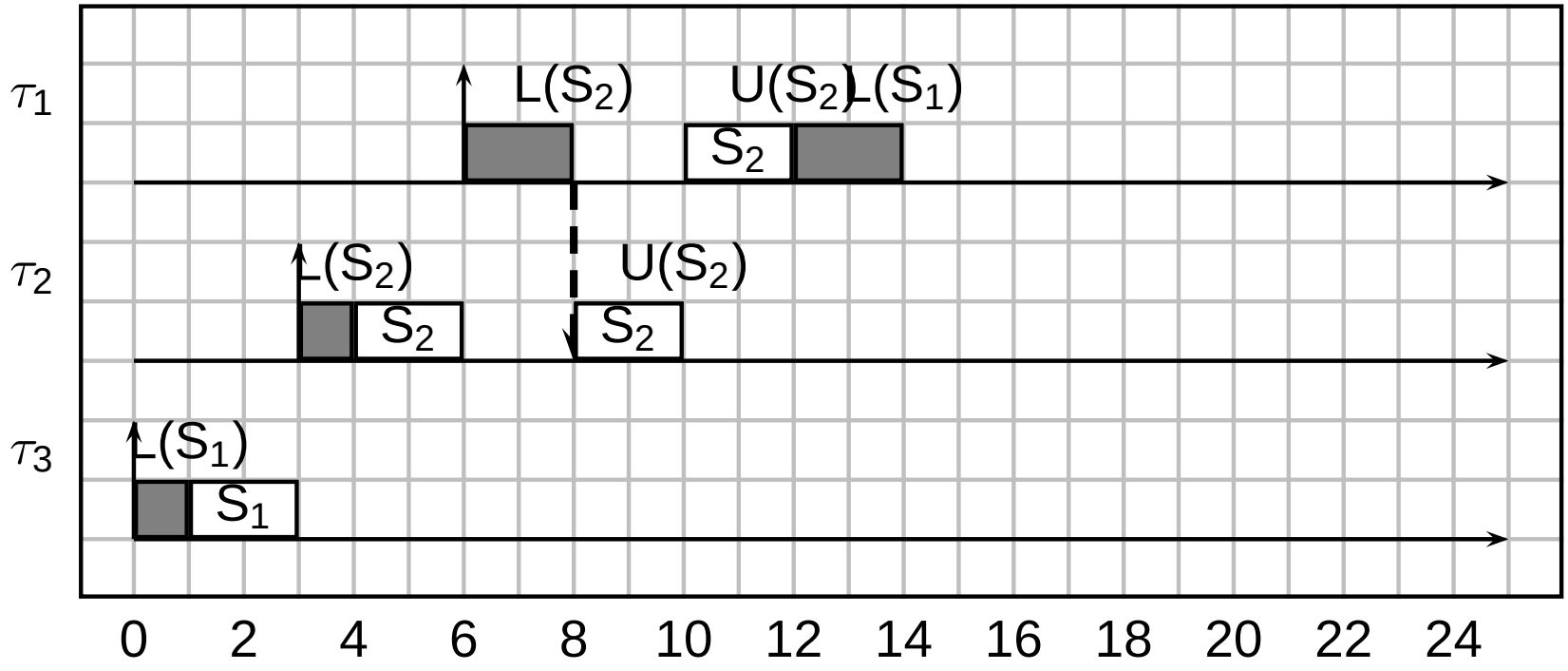

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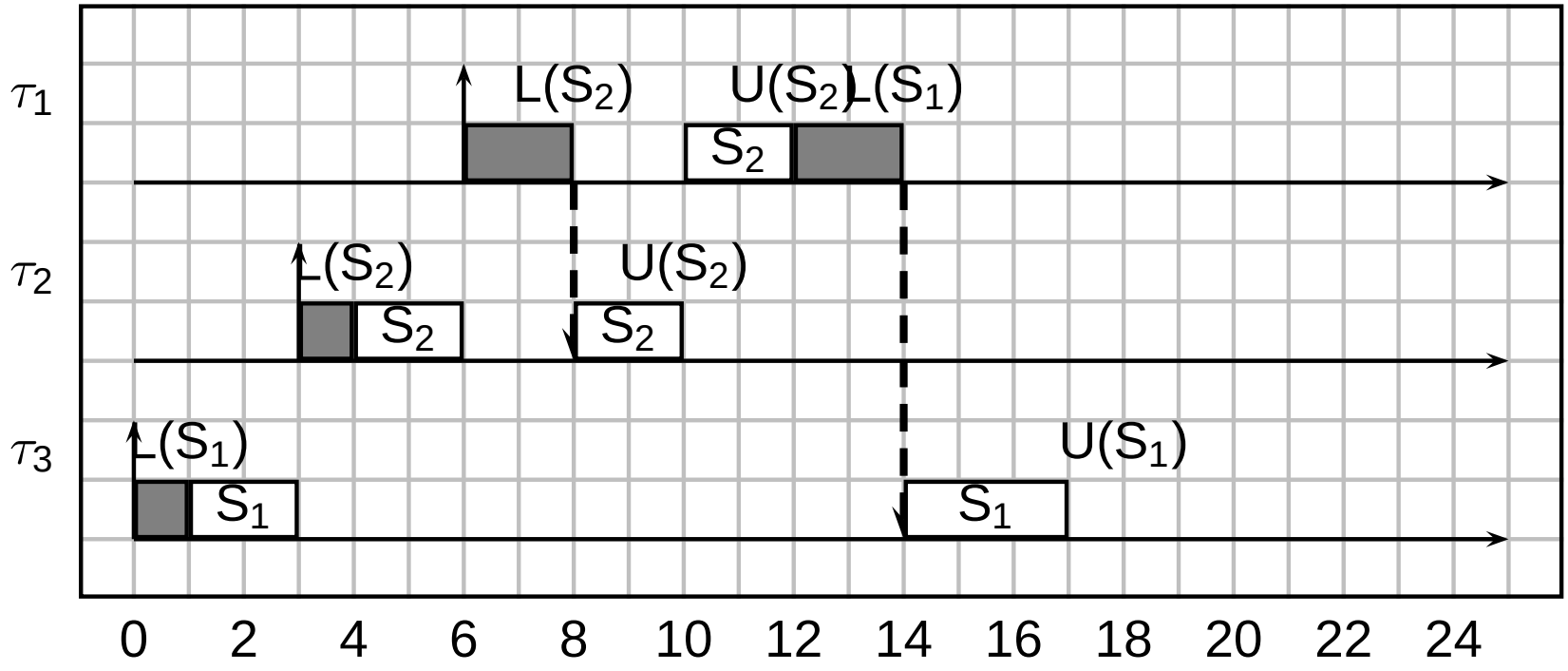

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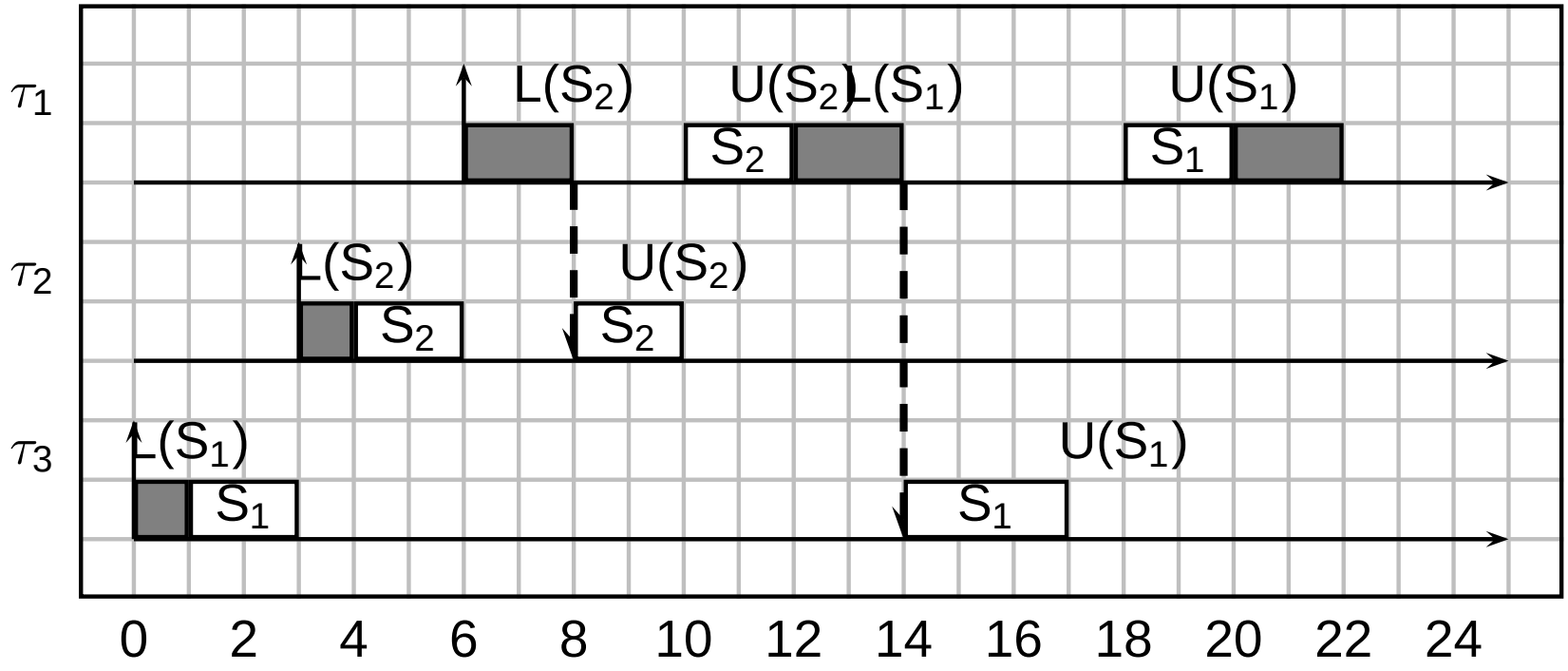

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Possible solution: ceilings

- It is possible to avoid this situation by doing an off-line analysis
- Define the concept of resouce ceiling
- Anticipate the blocking: a task cannot lock a resource if it can potentially block another higher priority task later.

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- Anticipate the blocking: a task cannot lock a resource if it can potentially block another higher priority task later.

Definition

The ceiling of a resource is the priority of the highest priority task that can access it

$$\text{ceil}(S_k) = \max_i \{p_i | \tau_i \text{ uses } S_k\}$$

The priority ceiling protocol

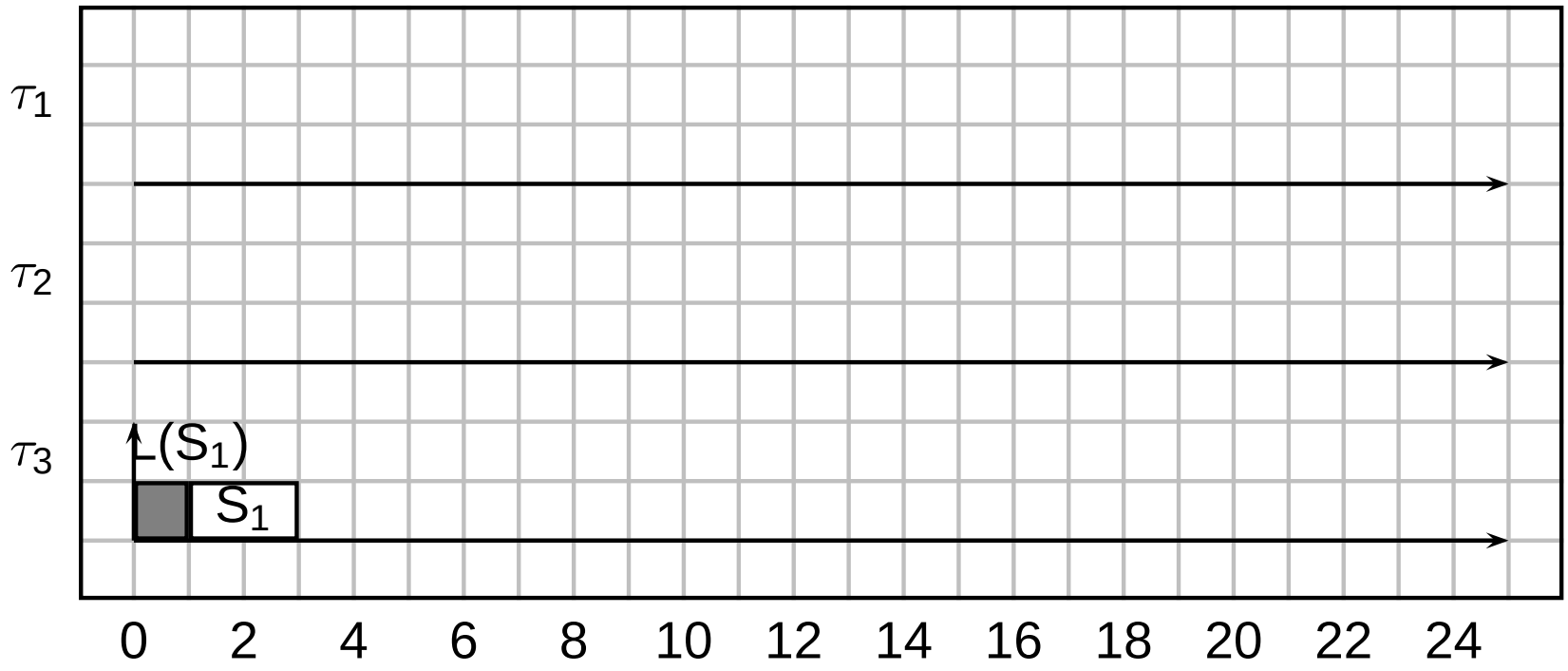
- Basic rules
- When a task τ_i tries to lock a semaphore S_k , the operation is permitted only if the following two conditions hold:
  1. S_k is free and
  2. $p_i > \text{maxceil}_i$ where:

$$\text{maxceil}_i = \max\{\text{ceil}(S_j) | S_j \text{ is locked by a task different from } \tau_i\}$$

- Otherwise, the task is blocked, and the *blocking task* inherits the priority
- The blocking task is the one that holds the lock on the semaphore corresponding to the maximum ceiling maxceil_i.

Example:

- ceil(S_1) $= p_1$ and ceil(S_2) $= p_1$



- task τ_3 acquires the lock

Example:

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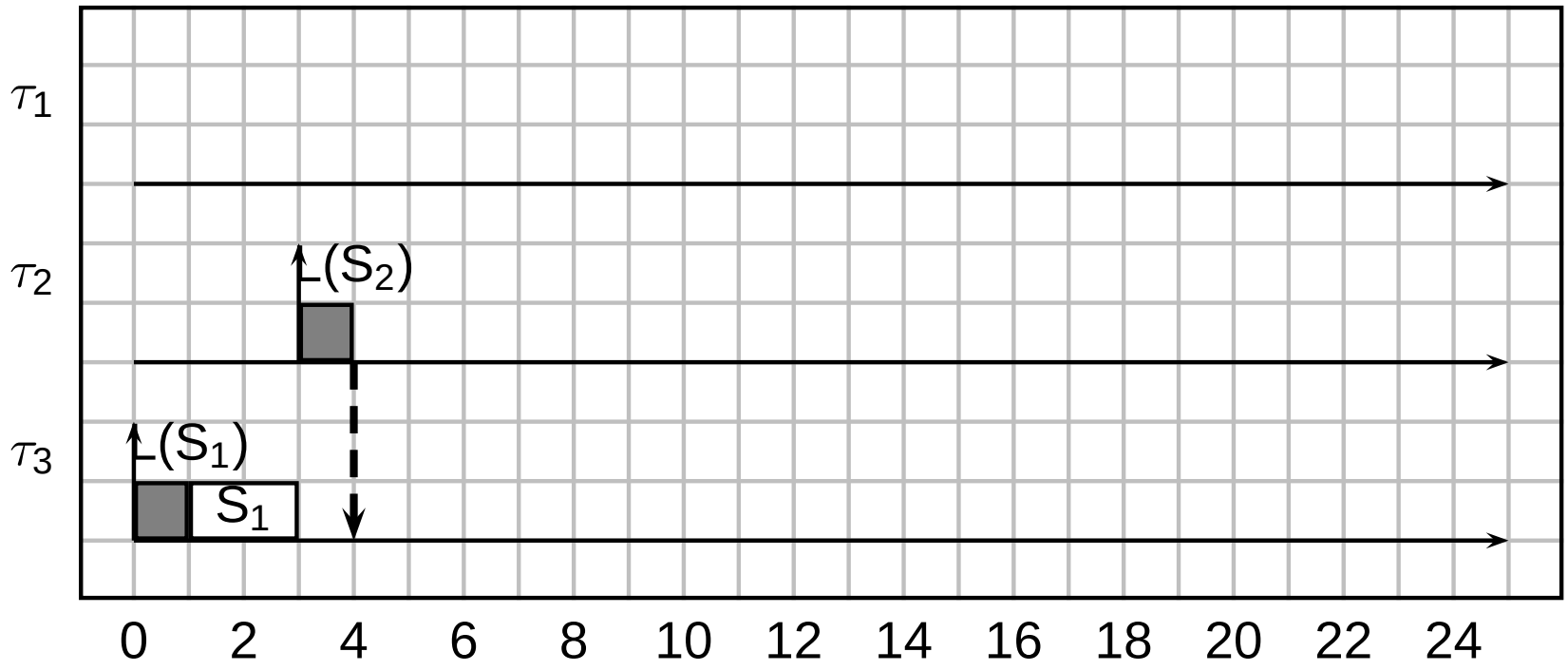

- task τ_3 acquires the lock
- task τ_2 is blocked because $p_2 <$ maxceil $= p_1$
- task τ_3 inherits τ_2's priority

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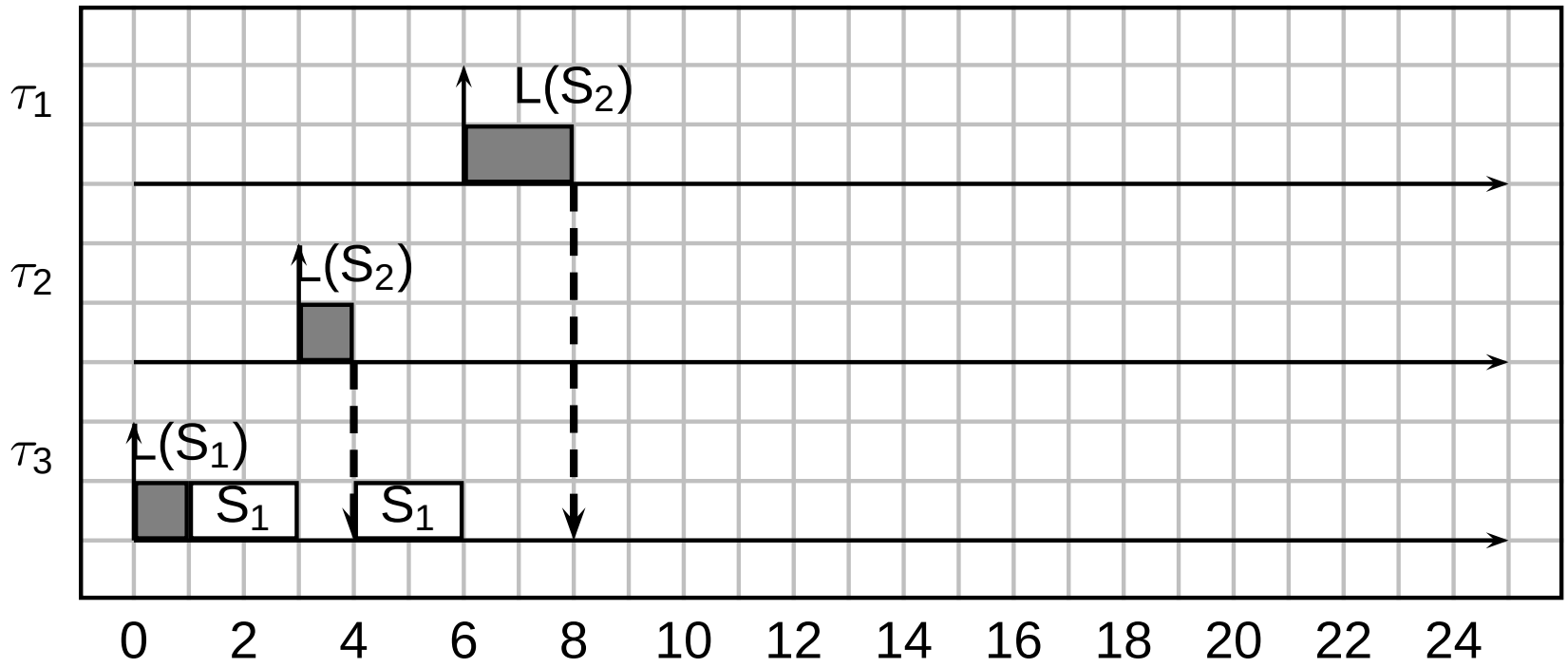

- task τ_3 acquires the lock
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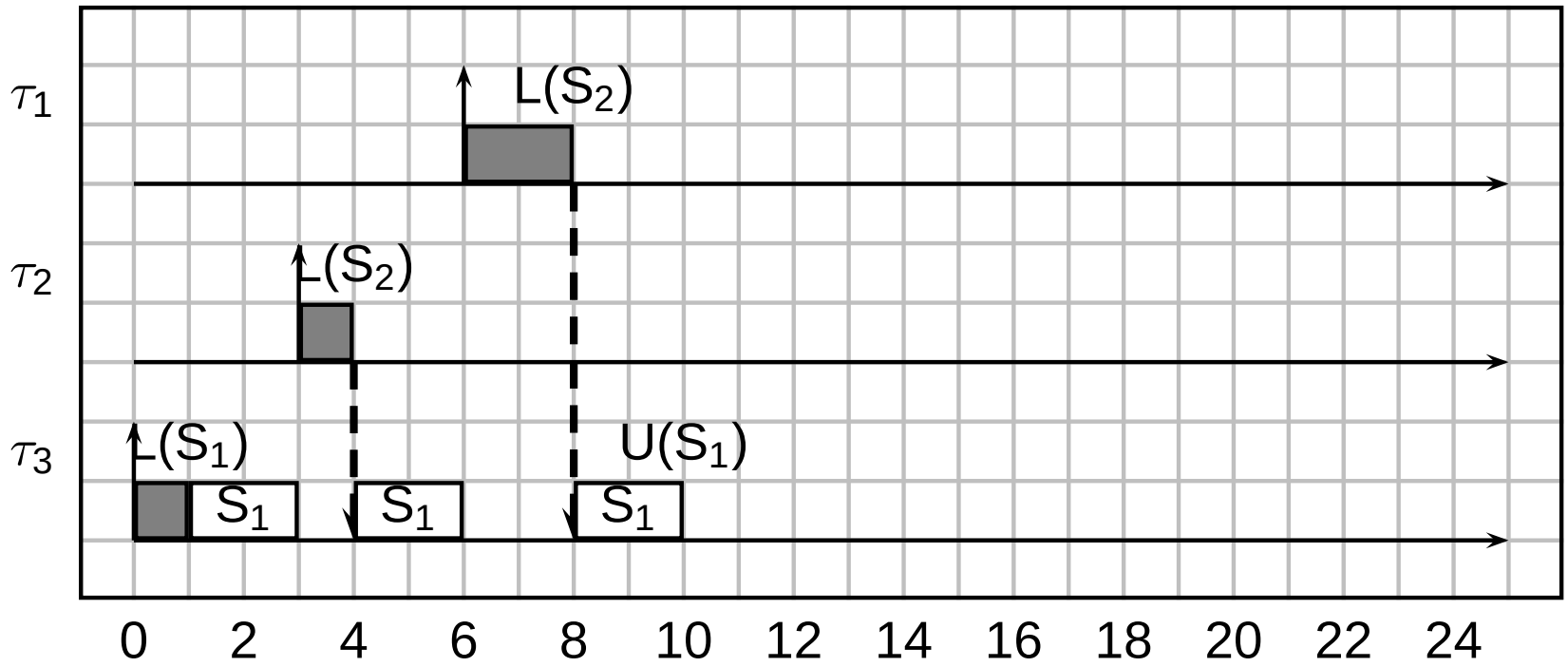

- task τ_3 acquires the lock
- task τ_2 is blocked because $p_2 <$ maxceil $= p_1$
- task τ_3 inherits τ_2's priority
- task τ_1 is blocked for the same reason
- task τ_3 inherits τ_1's priority
- task τ_3 returns to its original priority, since it is not blocking anyone

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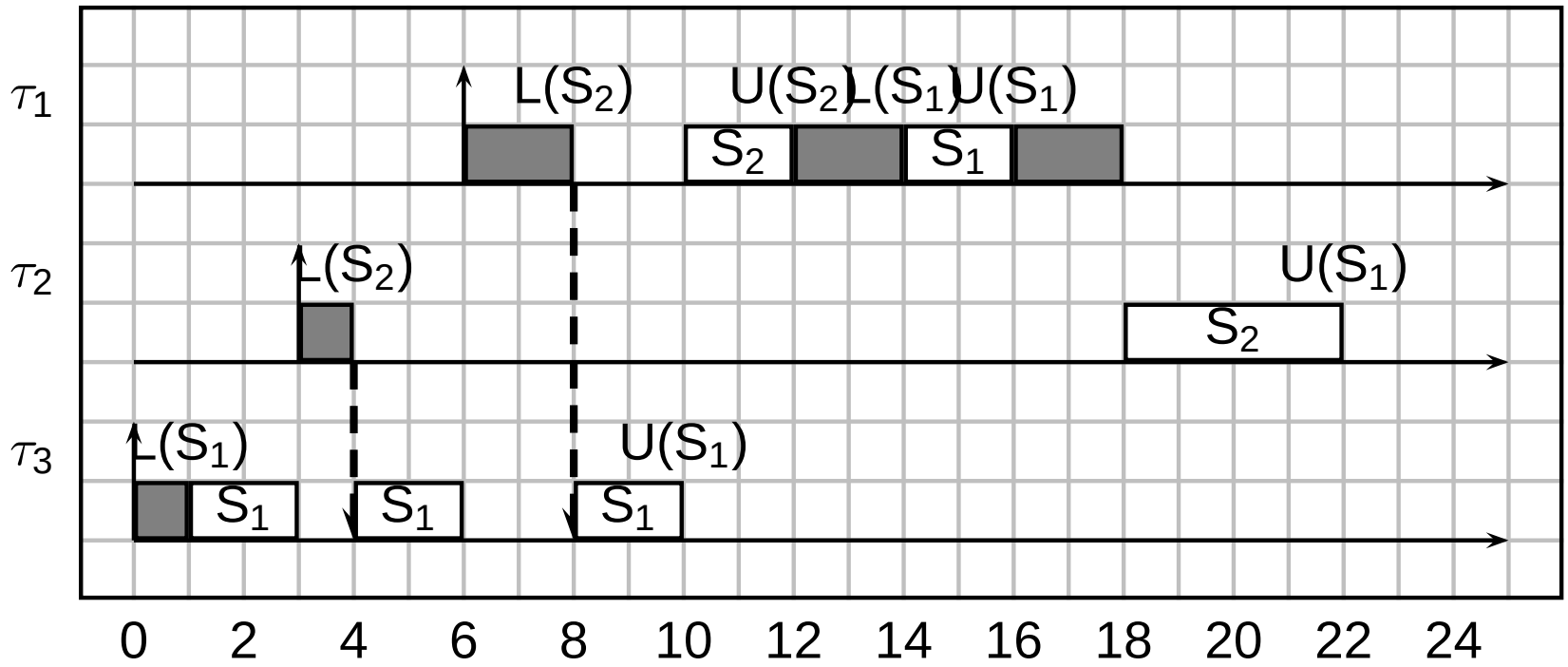

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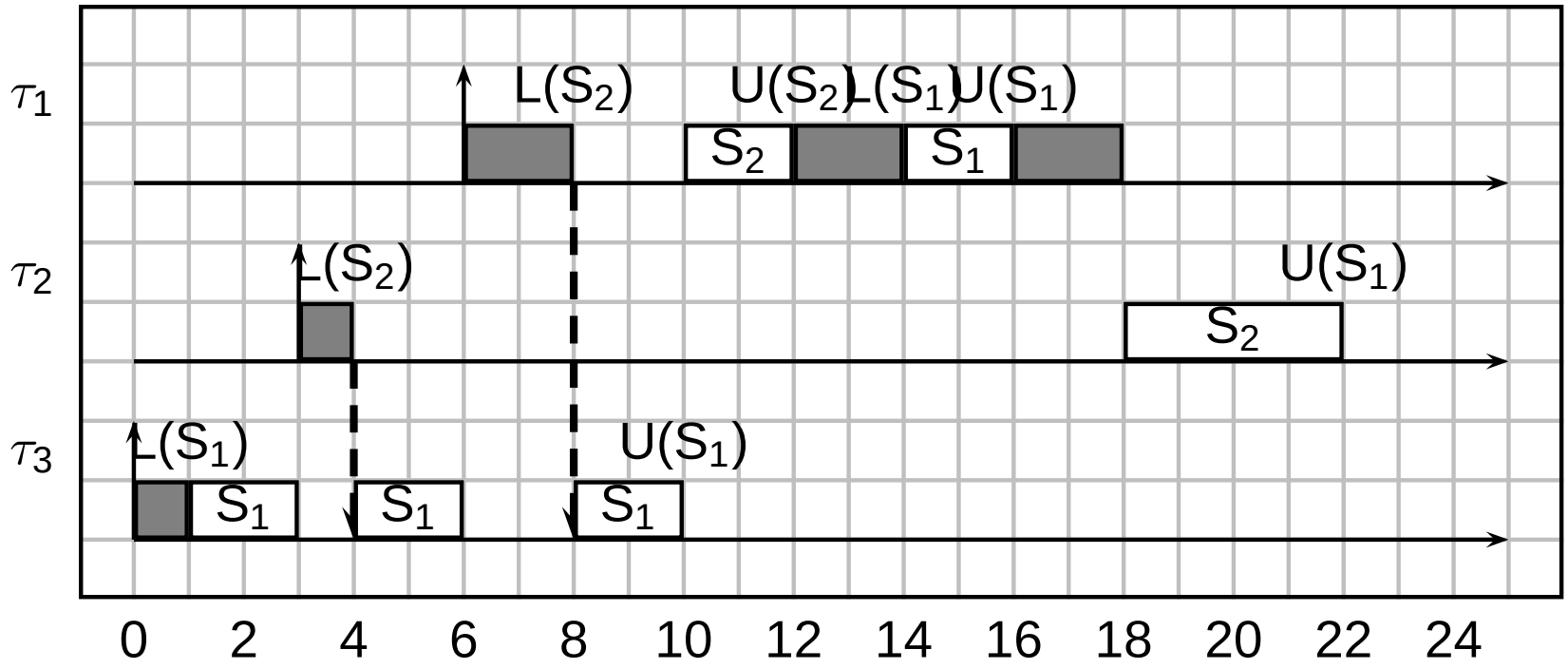
Blocking

- In the previous example:



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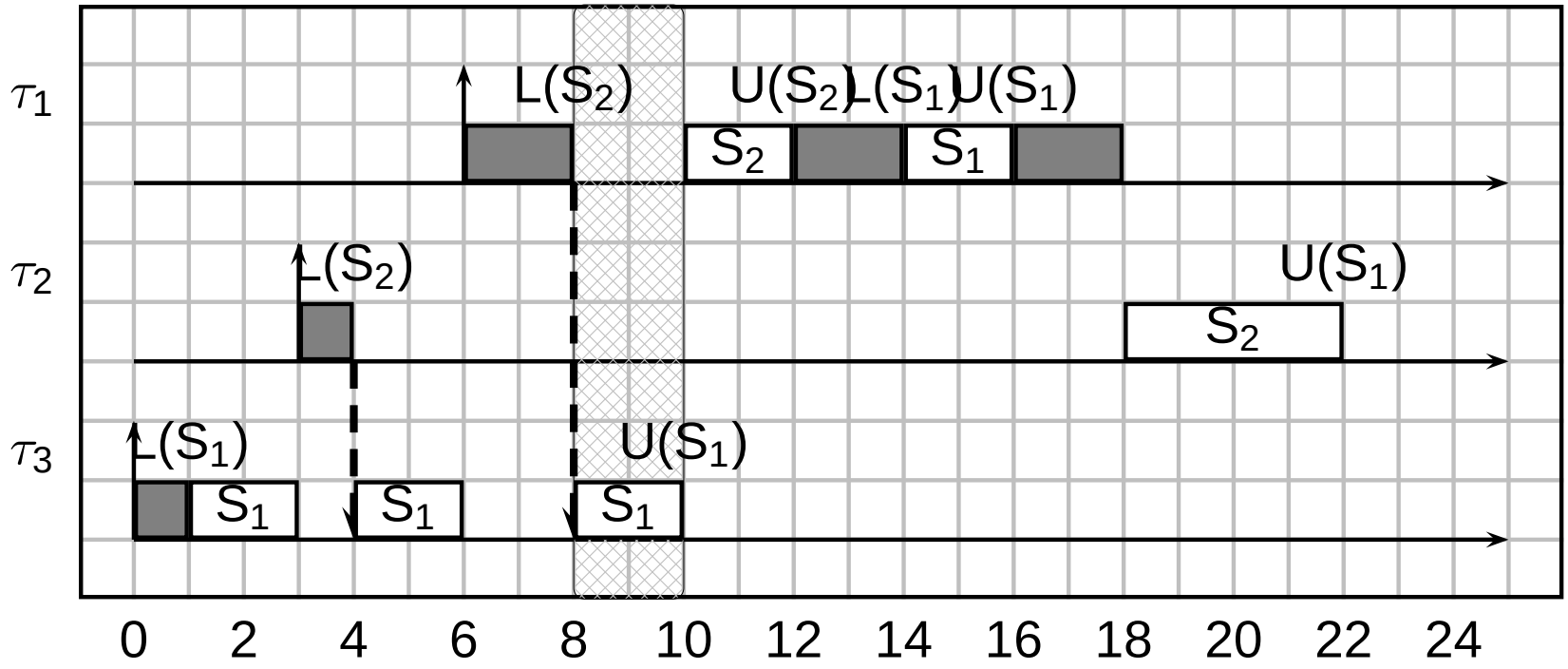

- Blocking time for τ_1: 2

Blocking

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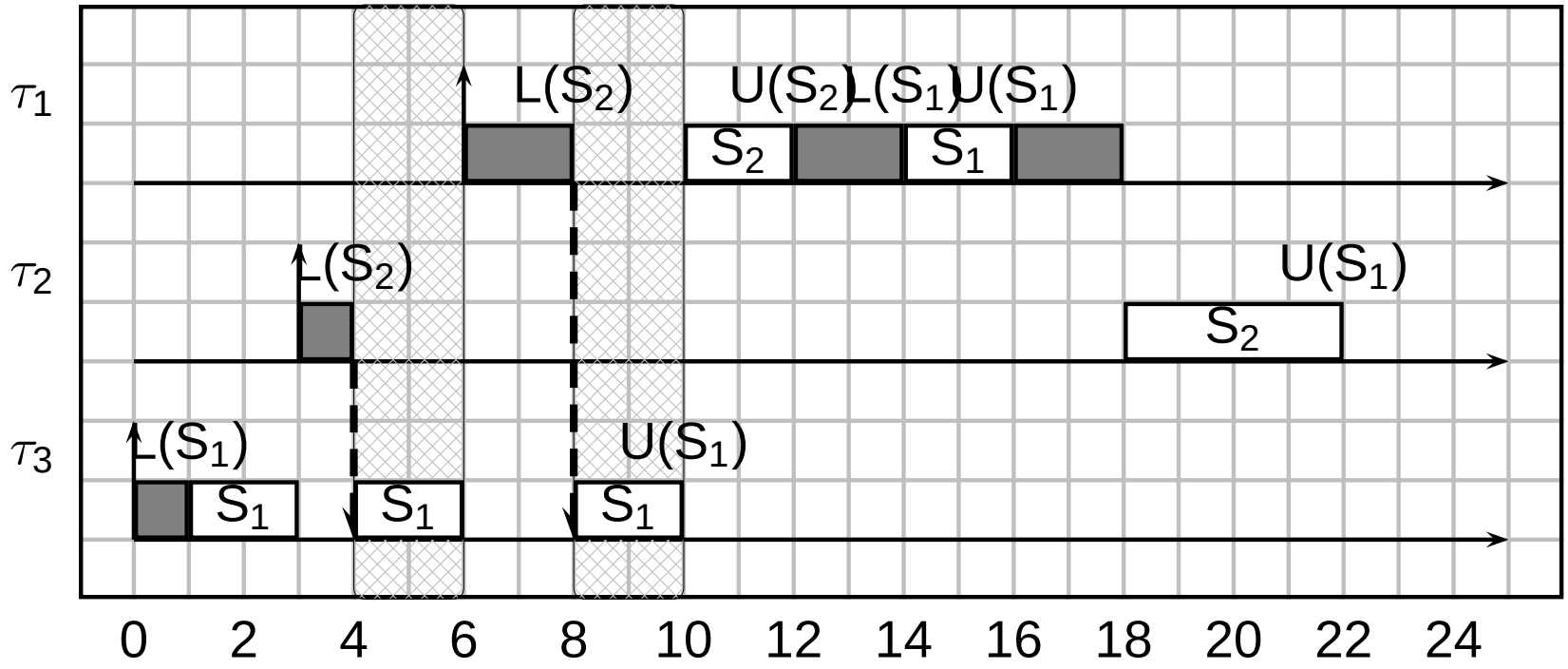

- Blocking time for τ_1: 2
- Blocking time for τ_2: 4

Blocking

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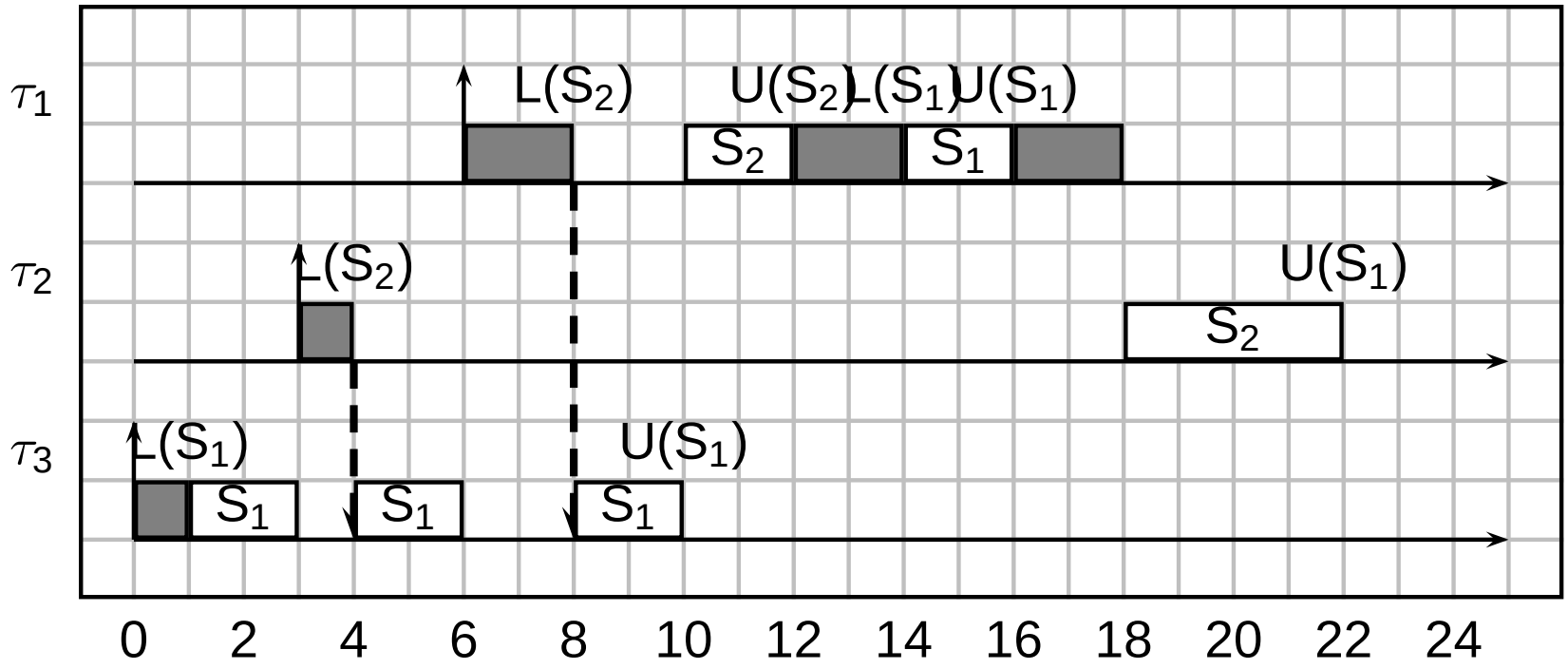

- Blocking time for τ_1: 2
- Blocking time for τ_2: 4
- No multiple blockings!

Properties of PCP

Theorem

A task can be blocked at most once by any resource or task.

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The Priority Ceiling Protocol prevents deadlock

- Therefore, we can nest critical sections safely

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The Priority Ceiling Protocol prevents deadlock

- Therefore, we can nest critical sections safely

Corollary

The maximum blocking time for a task is at most the length of one critical section

- It follows that in the resource table, we have to consider only one cell

Example of blocking time computation – PCP

- let's start from B_1

	S_1	S_2	S_3	B
τ_1	2	0	0	?
τ_2	0	1	0	?
τ_3	0	0	2	?
τ_4	3	3	1	?
τ_5	1	2	1	?

- τ_1 can be blocked only on S_1.
- Therefore, we must consider only the first column, and take the maximum, which is 3.

Example of blocking time computation – PCP

- Now τ_2: it can be blocked on S_1 (*indirect blocking*) and on S_2.

	S_1	S_2	S_3	B
τ_1	2	0	0	3
τ_2	0	1	0	?
τ_3	0	0	2	?
τ_4	3	3	1	?
τ_5	1	2	1	?

- Therefore, we must consider the first 2 columns;
- The maximum is $B_2 = 3$.

Example of blocking time computation – PCP

- τ_3 can be blocked on all 3 resources

	S_1	S_2	S_3	B
τ_1	2	0	0	3
τ_2	0	1	0	3
τ_3	0	0	2	?
τ_4	3	3	1	?
τ_5	1	2	1	?

- The maximum is $B_3 = 3$.

Example of blocking time computation – PCP

- Now :

	S_1	S_2	S_3	B
τ_1	2	0	0	3
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- τ_4 can be blocked on all 3 resources,
- The maximum is $B_4 = 2$.
- τ_5 cannot be blocked by any other task (because it is the lower priority task!); $B_5 = 0$;

Example: Final result – PCP

- Final result:

	S_1	S_2	S_3	B
τ_1	2	0	0	3
τ_2	0	1	0	3
τ_3	0	0	2	3
τ_4	3	3	1	2
τ_5	1	2	1	0

PCP – problems

- The PCP has some disadvantages
- The implementation is very complex, even more than PI
  - Very little known implementations,
  - difficult to prove correctness of implementation
- The PCP causes still many context switches
- we need something simpler to be implemented!

Outline

Stack Resource Policy

- This protocol is also known with the name of Immediate Priority Ceiling Protocol (IPCP).
  The basic ideas are the following:
  - We anticipate the blocking even more
  - the task cannot even start executing if it is not guaranteed to take all resources
- Properties:
  - Very simple implementation
  - A task blocks at most once before starting execution
  - The execution order is like a “stack”.

Preemption levels and rules

- The preemption level π_i of a task is a generalization of the concept of priority for preemptive scheduling;
- In Fixed Priority, the preemption level of a task is defined as its priority: $\pi_i = p_i$.
- In EDF the preemption level will be defined as the inverse of the relative deadline of a task.

Definition

The ceiling of a resource is the preemption level of the task with the highest preemption level among those that can access the resource

$$\text{ceil}(S_k) = \max_i \{\pi_i | \tau_i \text{ uses } S_k\}$$

Definition

The system ceiling at any instant of time is the maximum ceiling among all locked resources

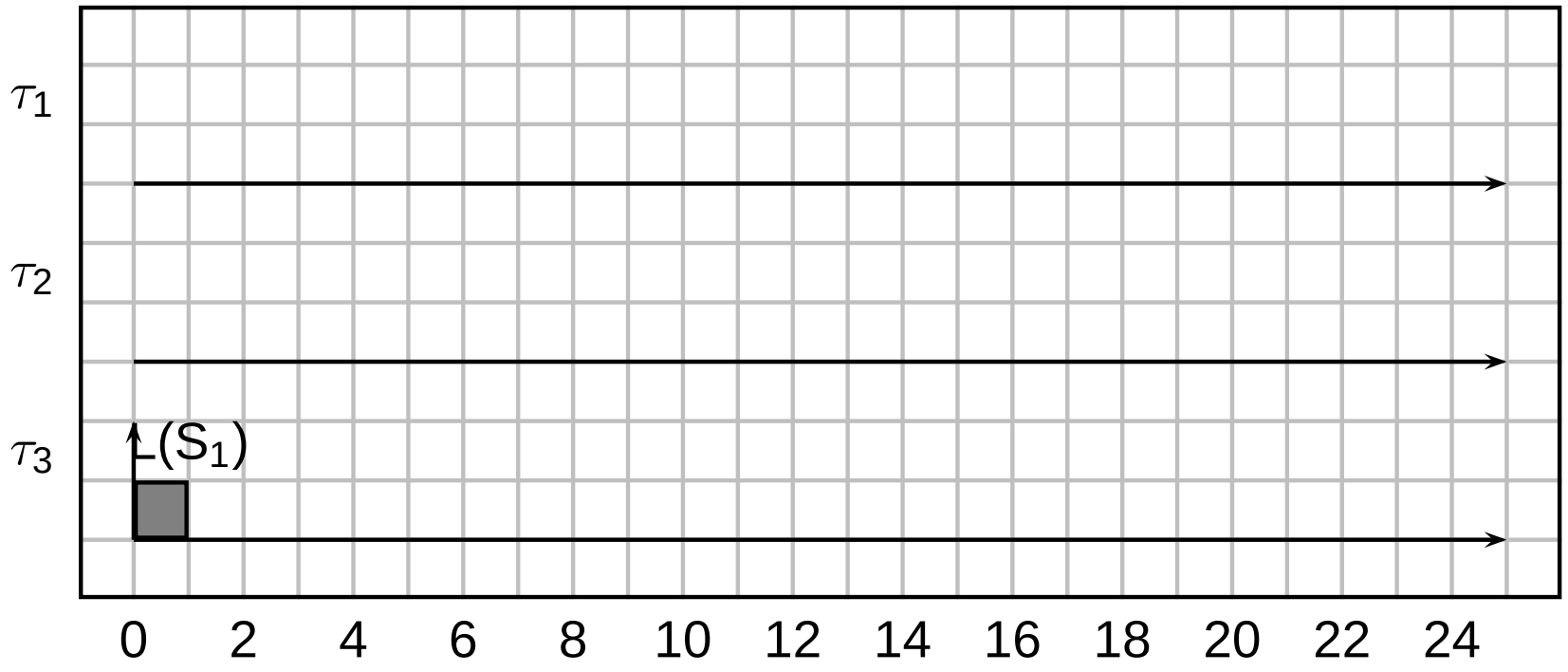
$$\Pi_s(t) = \max\{ceil(S) | S \text{is locked at} t\}$$

SRP rule

- The protocol rule is:
- A task that arrives at time t can start executing only if:
  1. it is the highest priority task
  2. its preemption level is greater than current system ceiling: $\pi_i > \Pi_s(t)$

Example

- Example



- Task τ_3 raises the sys ceiling to p_1

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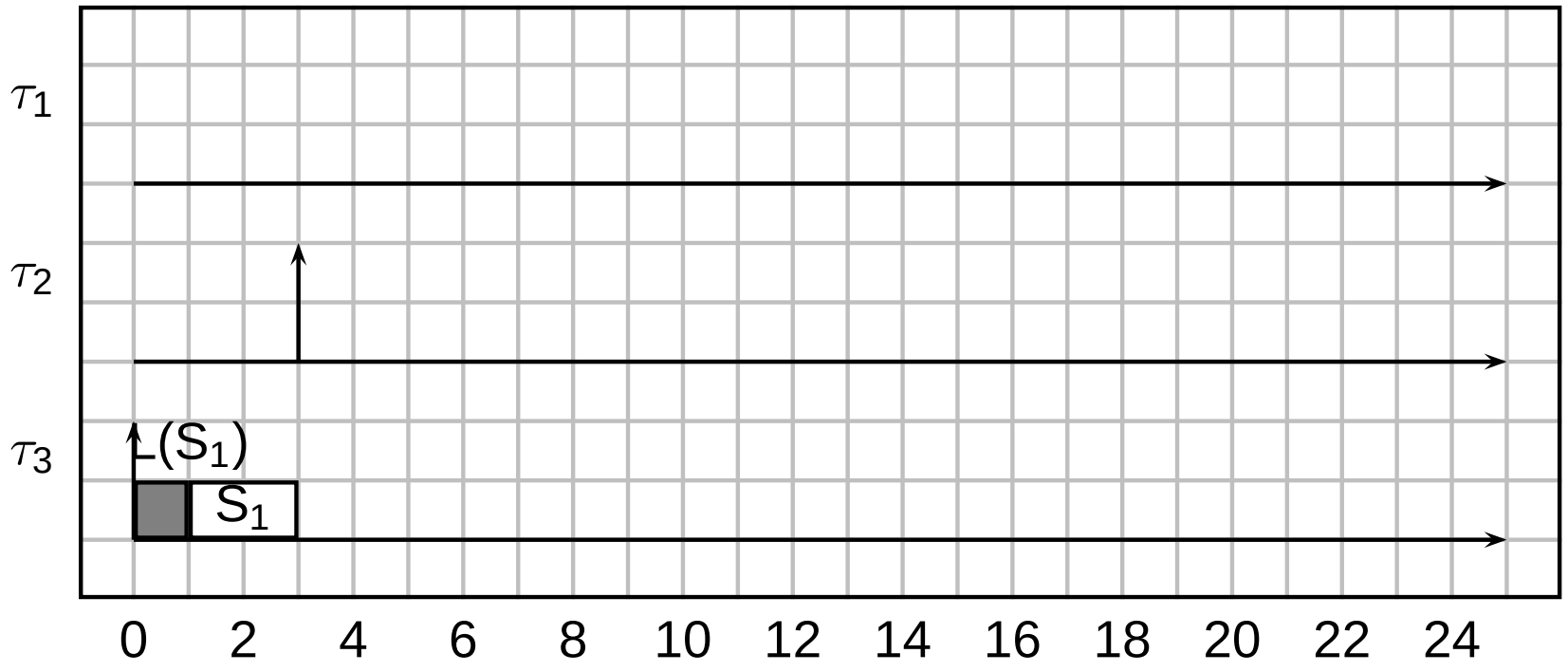

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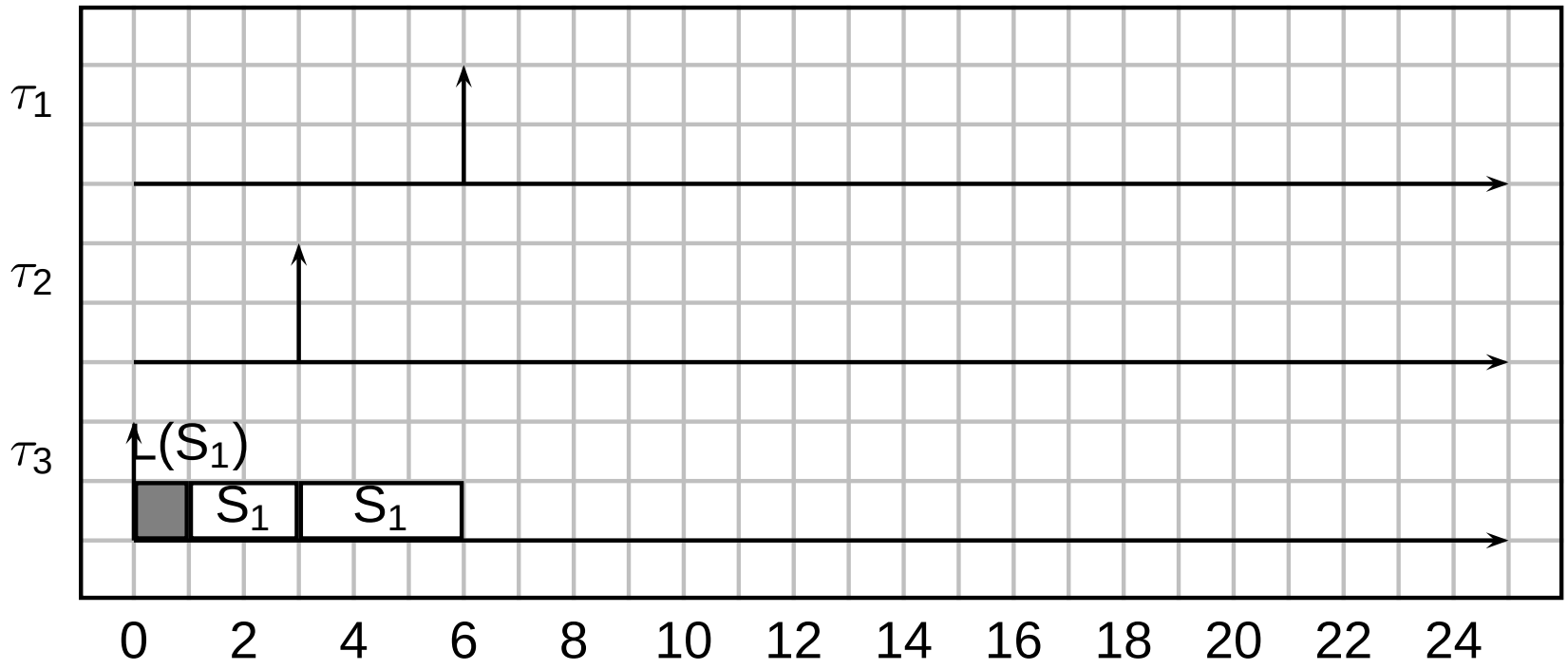

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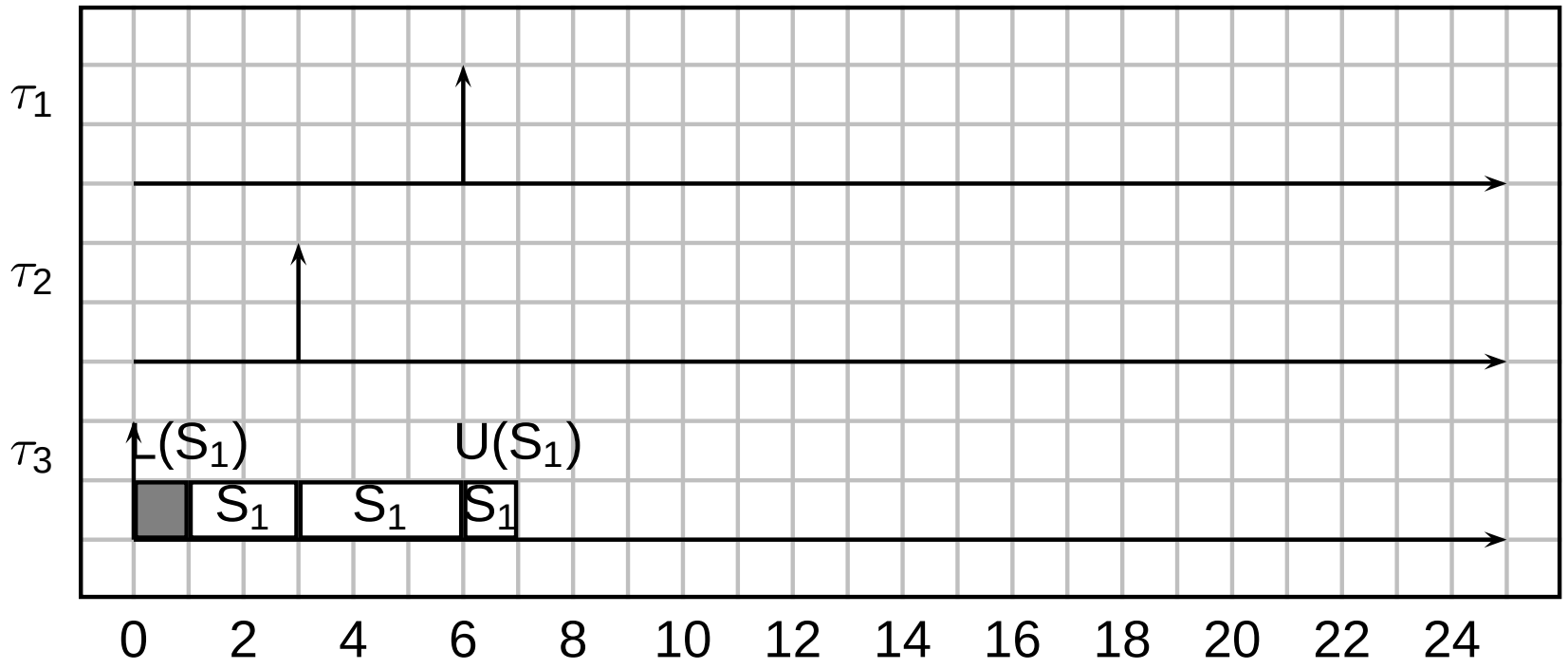

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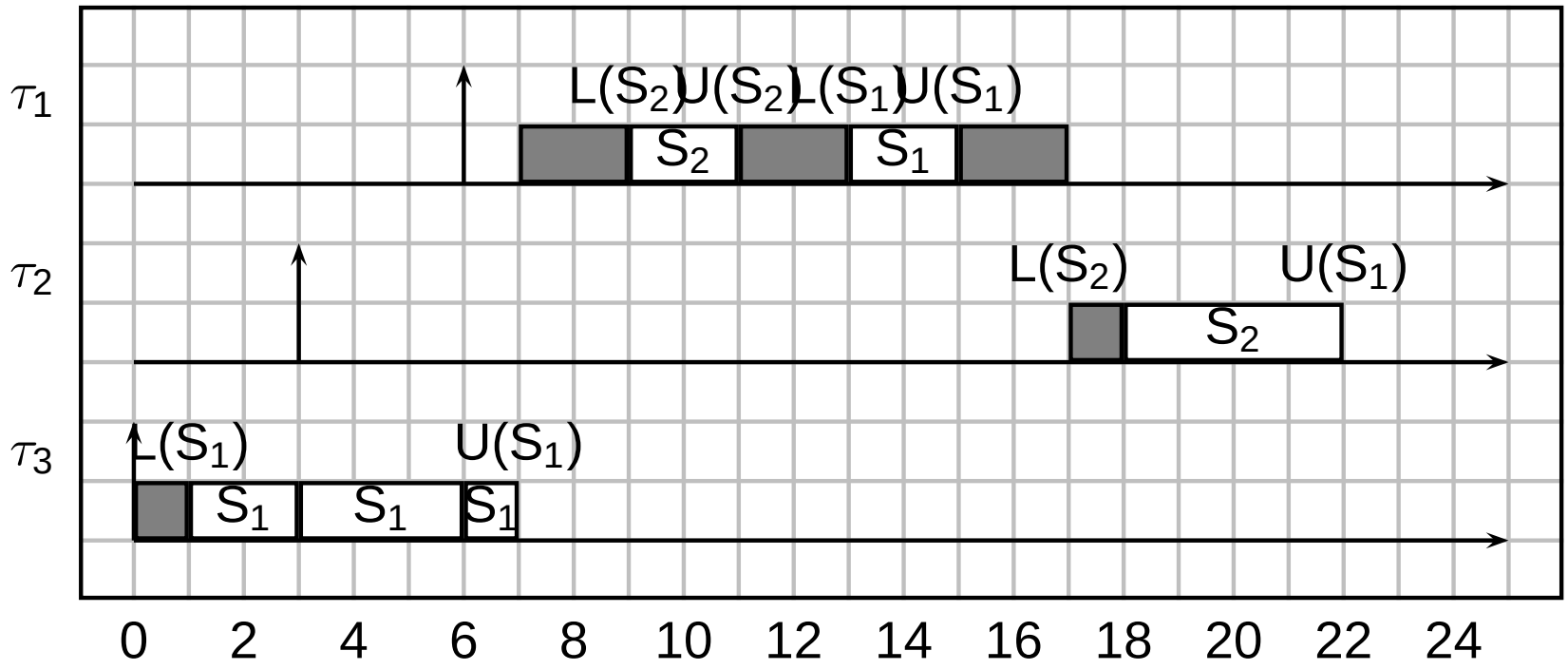

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The maximum blocking time for a task is at most the length of one critical section

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Properties

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Therefore, we can nest critical sections safely

Corollary

The maximum blocking time for a task is at most the length of one critical section

Therefore, the blocking time is the same as PCP.
It can be proven that this is the minimal possible blocking time

SRP vs. PCP

- SRP reduces the number of preemptions
- SRP is very easy to be implemented
  - No need to do inheritance
  - No need to block tasks in semaphore queues
  - It makes it possible for all tasks to share the same stack

Non preemptive scheduling

- Using SRP is equivalent to selectively disable preemption for a limited amount of time
- We can disable preemption only for some group of tasks
- The SRP is a generalization of the preemption-threshold mechanism

Outline

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Synchronization protocols with EDF

- Both the Priority inheritance Protocol and the Stack Resource Policy can be used under EDF without any modification.
- Let's first consider PI.
  - When a higher priority *job* is blocked by a lower priority job on a shared mutex sempahore, then the lower priority job *inherits* the priority of the blocked job.

Synchronization protocols with EDF

- Both the Priority inheritance Protocol and the Stack Resource Policy can be used under EDF without any modification.
- Let's first consider PI.
  - When a higher priority *job* is blocked by a lower priority job on a shared mutex sempahore, then the lower priority job *inherits* the priority of the blocked job.
  - In EDF, the priority of a job is inversely proportional to its absolute deadline.
  - Here, you should substiture *higher priority job* with job with an early deadline and *inherits the priority* with inherits the absolute deadline.

Preemption levels

- To compute the blocking time, we must first order the tasks based on their *preemption levels*.

Definition

Every task τ_i is assigned a preemption level π_i such that it can preempt a task τ_j if and only if $\pi_i > \pi_j$.

- In fixed priority, the preemption level is the same as the priority.
- In EDF, the preemption level is defined as $\pi_i = \frac{1}{D_i}$.

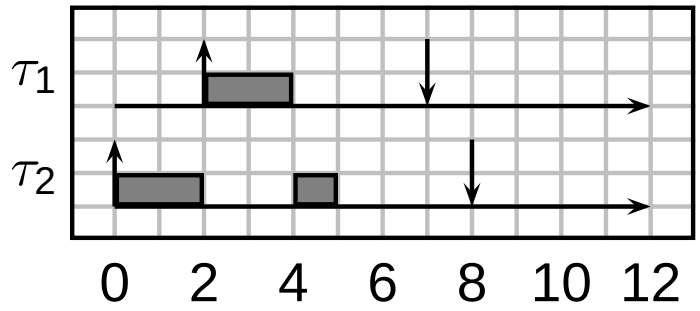
Preemption Levels - II

- If τ_i can preempt τ_j, then the following two conditions must hold:
- τ_i arrives after τ_j has started to execute and hence $a_i > a_j$,
- the absolute deadline of τ_i is shorter than the absolute deadline of τ_j ($d_i \leq d_j$).
- It follows that

$$\begin{aligned} d_i &= a_i + D_i \leq d_j = a_j + D_j \Rightarrow \\ D_i - D_j &\leq a_j - a_i < 0 \Rightarrow \\ D_i &< D_j \Rightarrow \\ \pi_i &> \pi_j \end{aligned}$$

Preemption levels

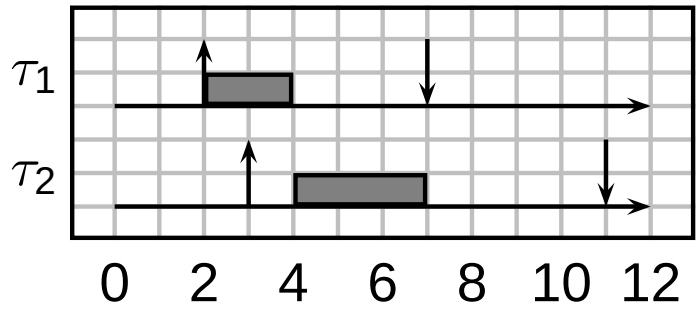
- With a graphical example:



- Notice that $\pi_1 > \pi_2$;
- In this case, τ_1 preempts τ_2.

Preemption levels

- With a graphical example:



- Notice that $\pi_1 > \pi_2$;

- τ_2 cannot preempt τ_1 (because its relative deadline is greater than τ_1).

Computing the blocking time

- To compute the blocking time for EDF + PI, we use the same algorithms as for FP + PI. In particular, the fundamental theorem for PI is still valid:

Theorem

Each task can be blocked only once per each resource, and only for the length of one critical section per each task.

Computing the blocking time - II

- In case on non-nested critical sections, build a *resource usage table*
  - At each row put a task, ordered by decreasing preemption levels
  - At each column, put a resource
  - In each cell, put the worst case duration ξ_{ij} of any critical section of task τ_i on resource S_j

Computing the blocking time - II

- In case on non-nested critical sections, build a *resource usage table*
  - At each row put a task, ordered by decreasing preemption levels
  - At each column, put a resource
  - In each cell, put the worst case duration ξ_{ij} of any critical section of task τ_i on resource S_j
- The algorithm for the blocking time for task τ_i is the same:
  - Select the rows below the i-th;
  - we must consider only those column on which it can be blocked (used by itself or by higher priority tasks)
  - Select the maximum sum of the $\xi_{k,j}$ with the limitation of at most one $\xi_{k,j}$ for each k and for each j.

Schedulability formula

- In case of relative deadlines equal to periods, we have:

$$\forall i = 1, \ldots, N \quad \sum_{j=1}^{i} \frac{C_j}{T_j} + \frac{B_i}{T_i} \leq 1$$

- In case of relative deadlines less than the periods:

$$\forall i = 1, \ldots, N \quad \forall L < L^*$$

$$\sum_{j=1}^{N} \left(\left\lfloor \frac{L - D_j}{T_j} \right\rfloor + 1 \right) C_j + B_i \leq L$$

$$L^* = \frac{U}{1 - U} \max_i (T_i - D_i)$$

Complete example

- Here we analyze a complete example, from the parameters of the tasks, and from the resource usage table, we compute the B_is, and test schedulability.

	C_i	T_i	U_i	R_1	R_2	B_i
τ_1	2	10	.2	1	0	?
τ_2	5	15	.33	2	1	?
τ_3	4	20	.2	0	2	?
τ_4	9	45	.2	3	4	?

Complete example: blocking times

- Blocking time for τ_1:

	C_i	T_i	U_i	R_1	R_2	B_i
τ_1	2	10	.2	1	0	3
τ_2	5	15	.33	2	1	?
τ_3	4	20	.2	0	2	?
τ_4	9	45	.2	3	4	?

Complete example: blocking times

- Blocking time for τ_2:

	C_i	T_i	U_i	R_1	R_2	B_i
τ_1	2	10	.2	1	0	3
τ_2	5	15	.33	2	1	5
τ_3	4	20	.2	0	2	?
τ_4	9	45	.2	3	4	?

Complete example: blocking times

- Blocking time for τ_3:

	C_i	T_i	U_i	R_1	R_2	B_i
τ_1	2	10	.2	1	0	3
τ_2	5	15	.33	2	1	5
τ_3	4	20	.2	0	2	4
τ_4	9	45	.2	3	4	?

Complete example: blocking times

- Blocking time for τ_4:

	C_i	T_i	U_i	R_1	R_2	B_i
τ_1	2	10	.2	1	0	3
τ_2	5	15	.33	2	1	5
τ_3	4	20	.2	0	2	4
τ_4	9	45	.2	3	4	0

Complete Example: schedulability test

- General formula:

$$\forall i = 1, \ldots, 4 \quad \sum_{j=1}^{i} \frac{C_j}{T_j} + \frac{B_i}{T_i} \leq 1$$

- Task τ_1:

$$\frac{C_1}{T_1} + \frac{B_1}{T_1} = .2 + .3 = .5 \leq 1$$

Complete Example: schedulability test

- General formula:

$$\forall i = 1, \ldots, 4 \quad \sum_{j=1}^{i} \frac{C_j}{T_j} + \frac{B_i}{T_i} \leq 1$$

- Task τ_2:

$$\frac{C_1}{T_1} + \frac{C_2}{T_2} + \frac{B_2}{T_2} = .5333 + .3333 = .8666 \leq 1$$

Complete Example: schedulability test

- General formula:

$$\forall i = 1, \ldots, 4 \quad \sum_{j=1}^{i} \frac{C_j}{T_j} + \frac{B_i}{T_i} \leq 1$$

- Task τ_3:

$$\frac{C_1}{T_1} + \frac{C_2}{T_2} + \frac{C_3}{T_3} + \frac{B_3}{T_3} = .2 + .333 + .2 + .2 = 0.9333 \leq 1$$

Complete Example: schedulability test

- General formula:

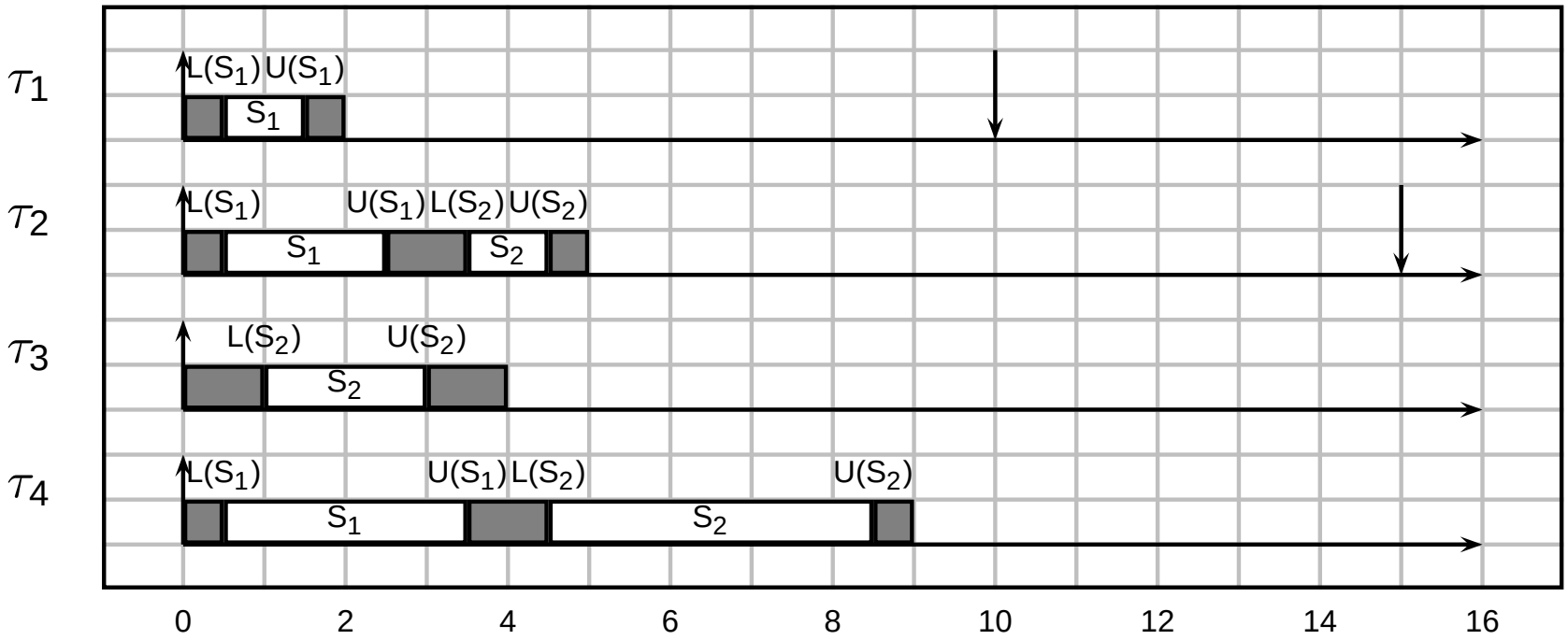
$$\forall i = 1, \ldots, 4 \quad \sum_{j=1}^{i} \frac{C_j}{T_j} + \frac{B_i}{T_i} \leq 1$$

- Task τ_4:

$$\frac{C_1}{T_1} + \frac{C_2}{T_2} + \frac{C_3}{T_3} + \frac{C_4}{T_4} + \frac{B_4}{T_4} = .2 + .3333 + .2 + .2 + 0 = .9333 \leq 1$$

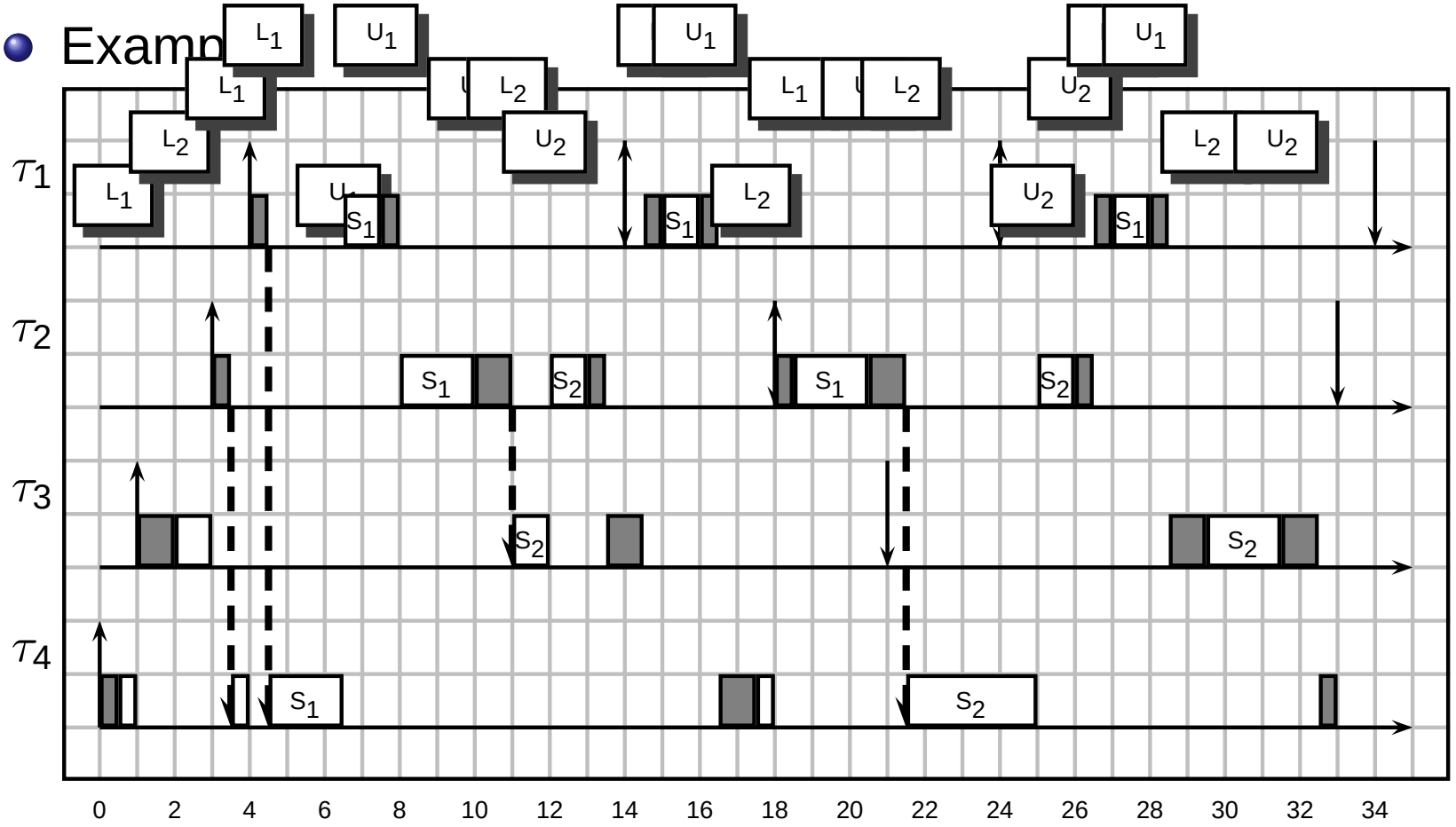
Complete example: scheduling

- Now we do an example of possible schedule.
- We assume that the task access the resources as follows:



Complete example: schedule

- Examp



- In the graph, $L_1 = \text{Lock}(S_1)$, $U_1 = \text{Unlock}(S_1)$, $L_2 = \text{Lock}(S_2)$, $U_2 = \text{Unlock}(S_2)$.
- The tasks start with an offset, because in the example we want to highlight the blocking times at the beginning.

Outline

Stack Resource Policy

- Once we have defined the preemption levels, it is easy to extend the stack resource policy to EDF.
- The main rule is the following:
  - The *ceiling* of a resource is defined as the highest preemption level among the ones of all tasks that access it;
  - At each instant, the *system ceiling* is the highest among the ceilings of the locked resources;
  - A task is not allowed to start executing until its deadline is the shortest one and its preemption level is strictly greater than the system ceiling;

Complete Example

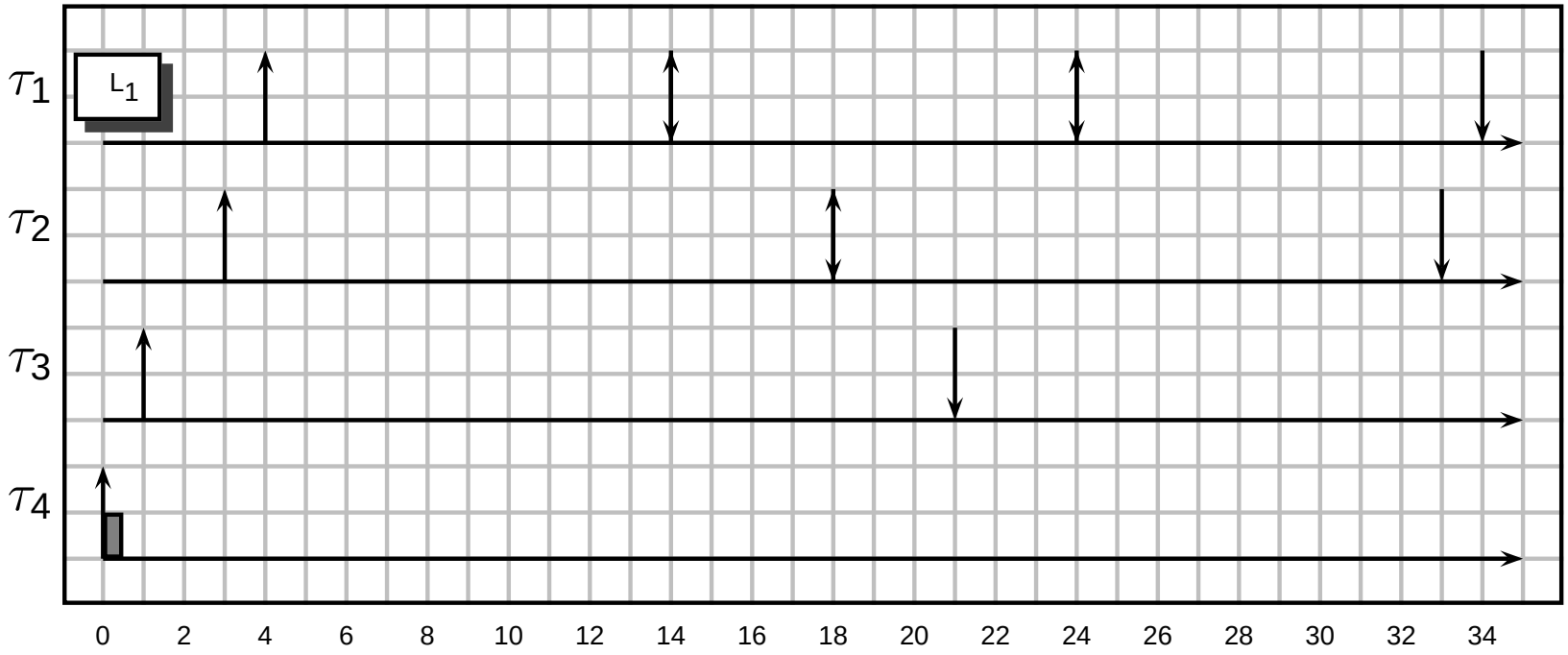
- Now we analyze the previous example, assuming EDF+SRP.

	C_i	T_i	U_i	R_1	R_2	B_i
τ_1	2	10	.2	1	0	?
τ_2	5	15	.33	2	1	?
τ_3	4	20	.2	0	2	?
τ_4	9	45	.2	3	4	?

- Let us first assign the preemption levels.
  - The actual value of the preemption levels is not important, as long as they are assigned in the right order.
  - To make calcumations easy, we set $\pi_1 = 4$, $\pi_2 = 3$, $\pi_3 = 2$, $\pi_4 = 1$.
- Then the resource ceilings:
  - $\text{ceil}(R_1) = \pi_1 = 4$, $\text{ceil}(R_2) = \pi_2 = 3$.

Schedule

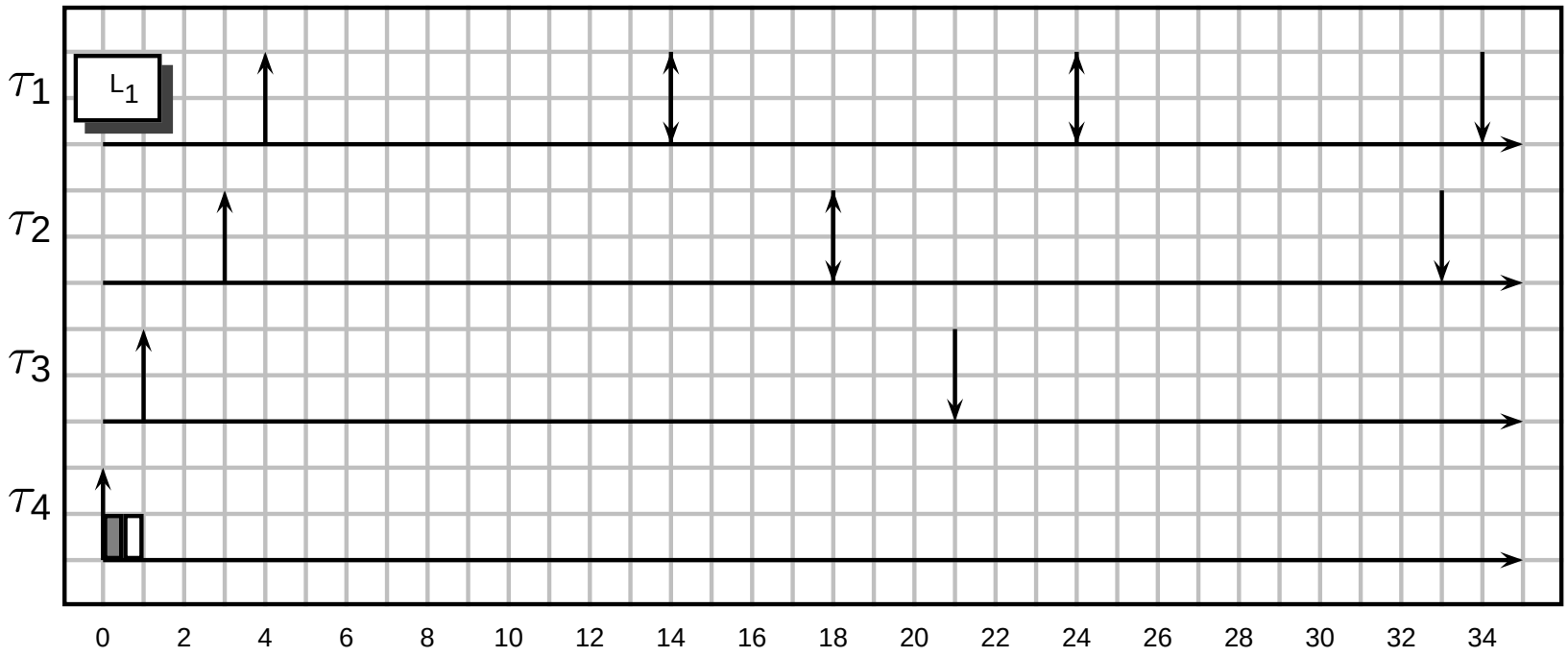
- Schedule



- At this point, the system ceiling is raised to π_1 (the ceiling of R_1).

Schedule

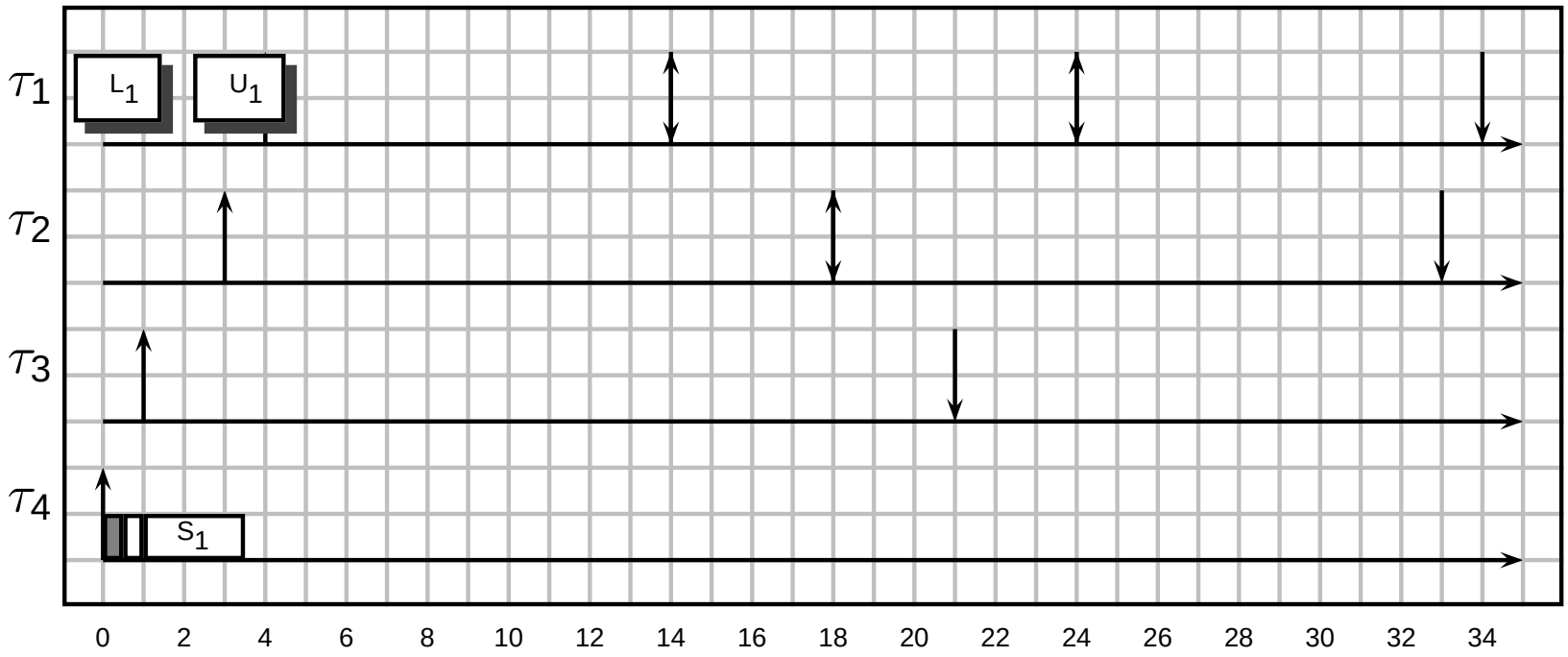
- Schedule



- At this point, the system ceiling is raised to π_1 (the ceiling of R_1). Task τ_3 cannot start executing, because $\pi_3 < \pi_1$. Same for τ_2.

Schedule

- Schedule



- At this point, the system ceiling is raised to π_1 (the ceiling of R_1). Task τ_3 cannot start executing, because $\pi_3 < \pi_1$. Same for τ_2.
- The system ceiling goes back to 0. Now τ_2 can start.

Schedule

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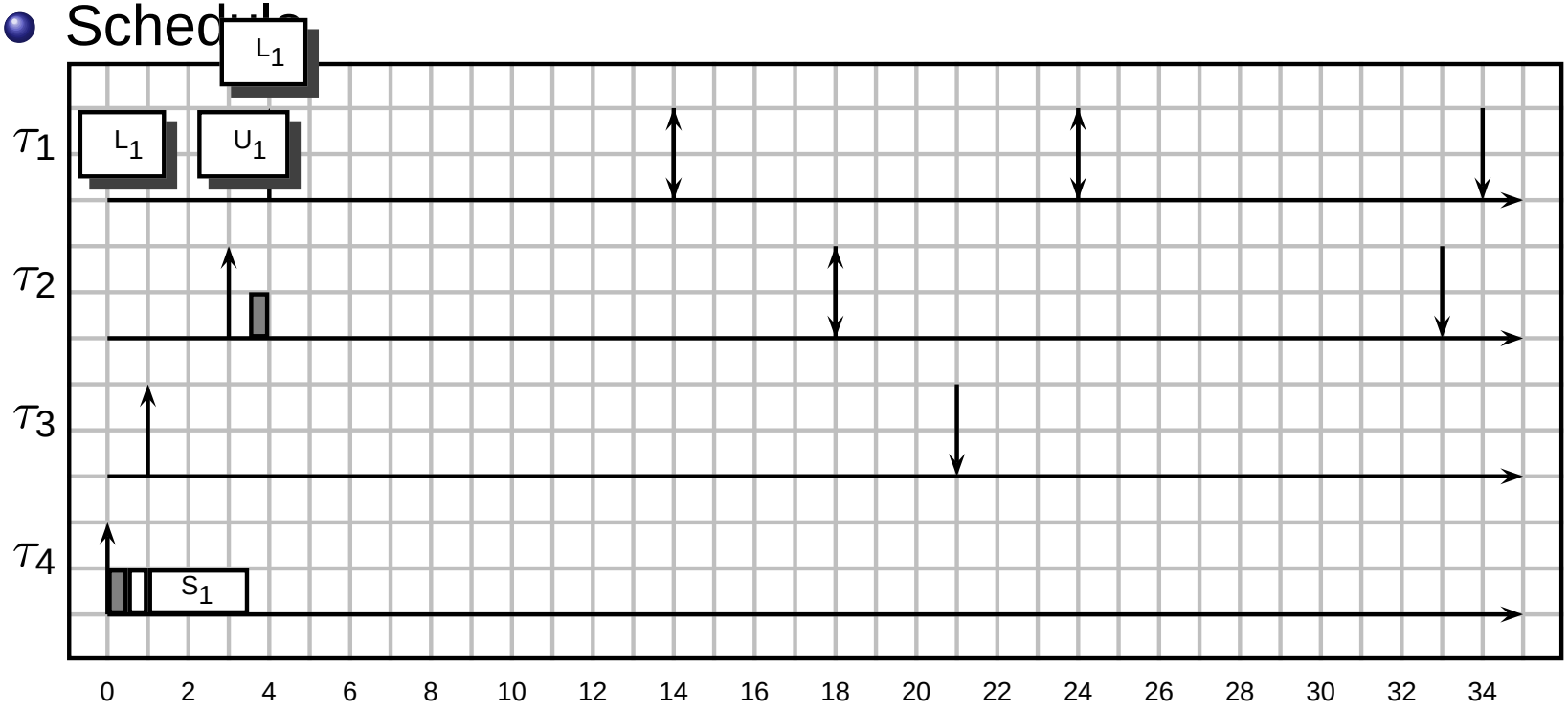

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- in this example, we assume that τ_2 locks R_1 just before τ_1 arrives. Then, sys ceil = π_1 and τ_1 cannot preempt.

Schedule

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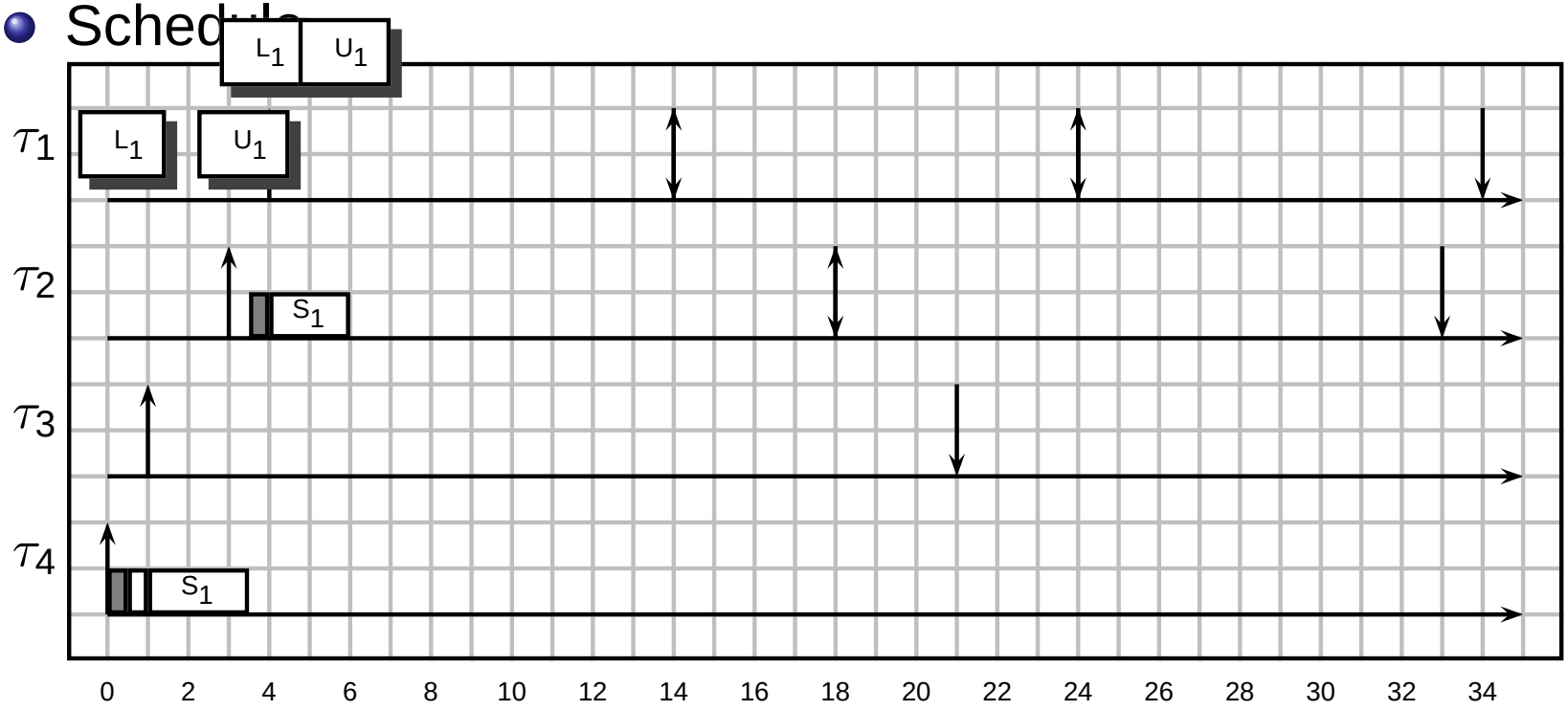

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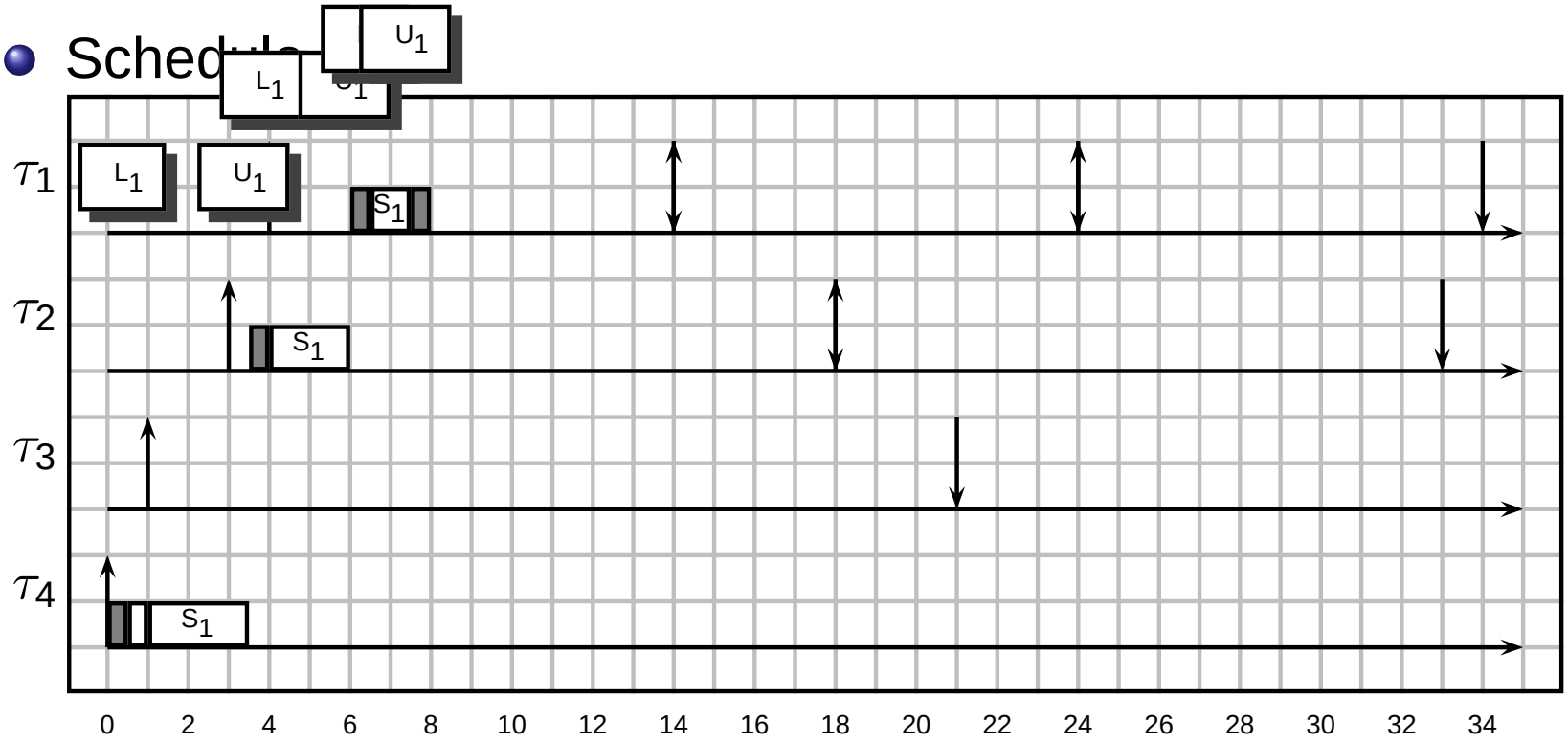

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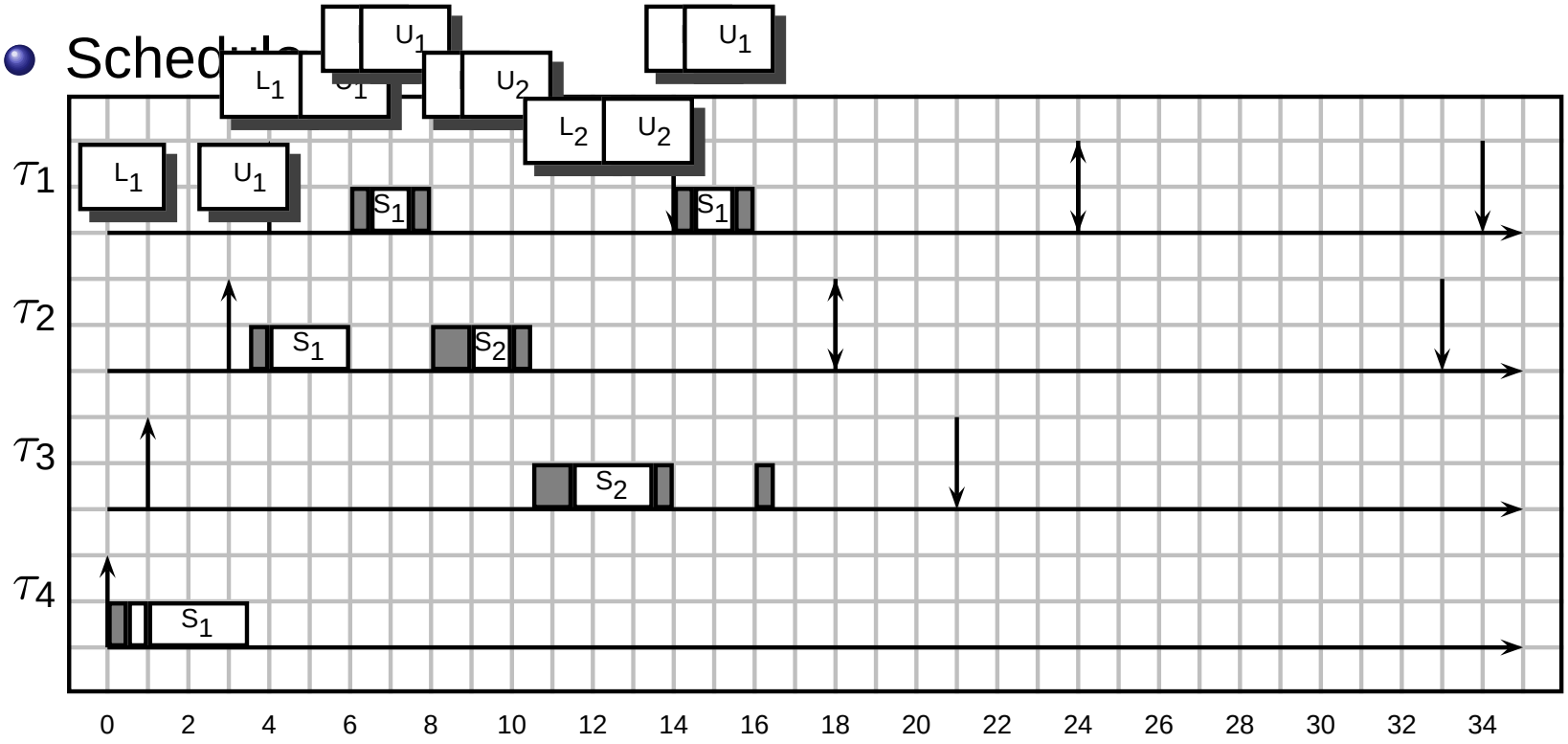

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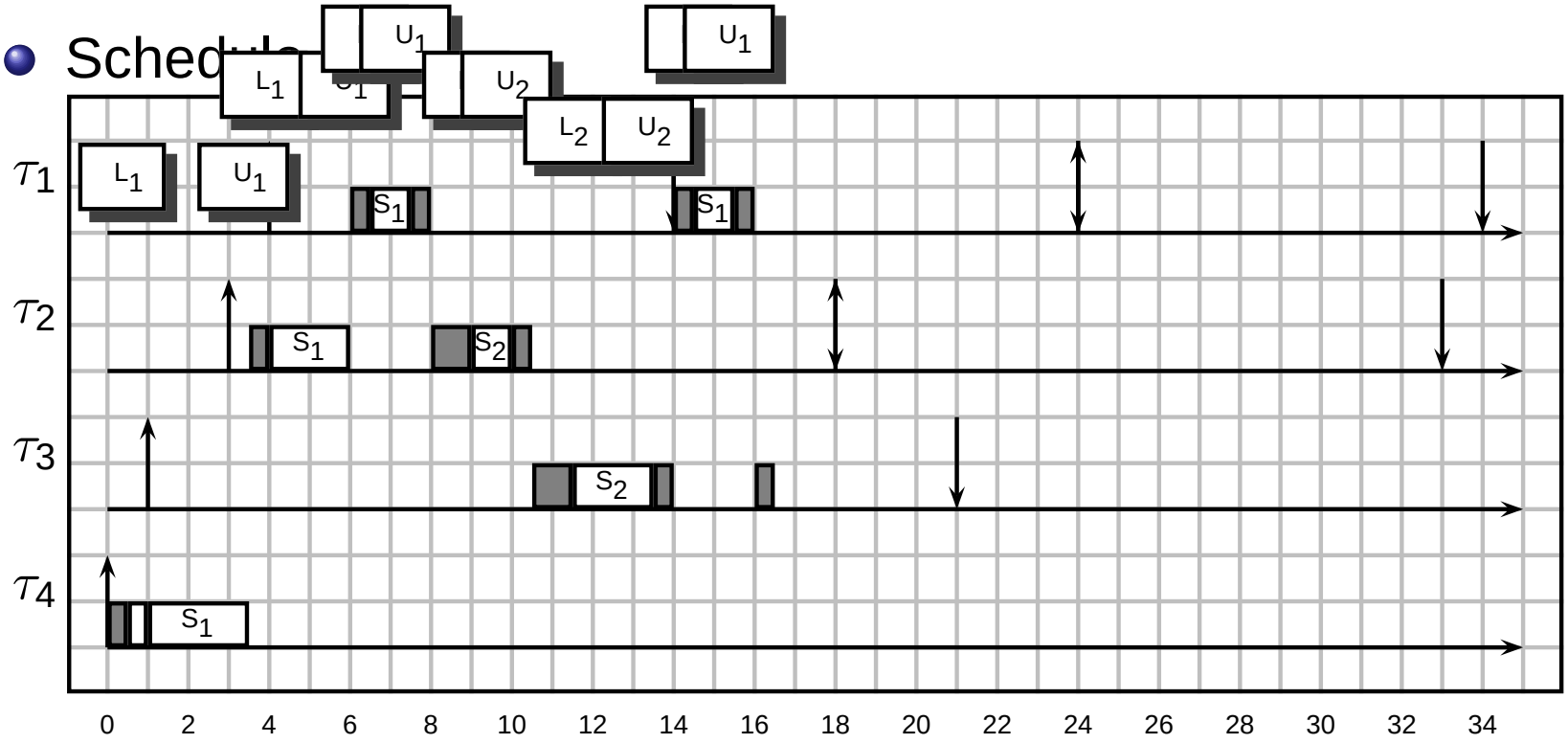

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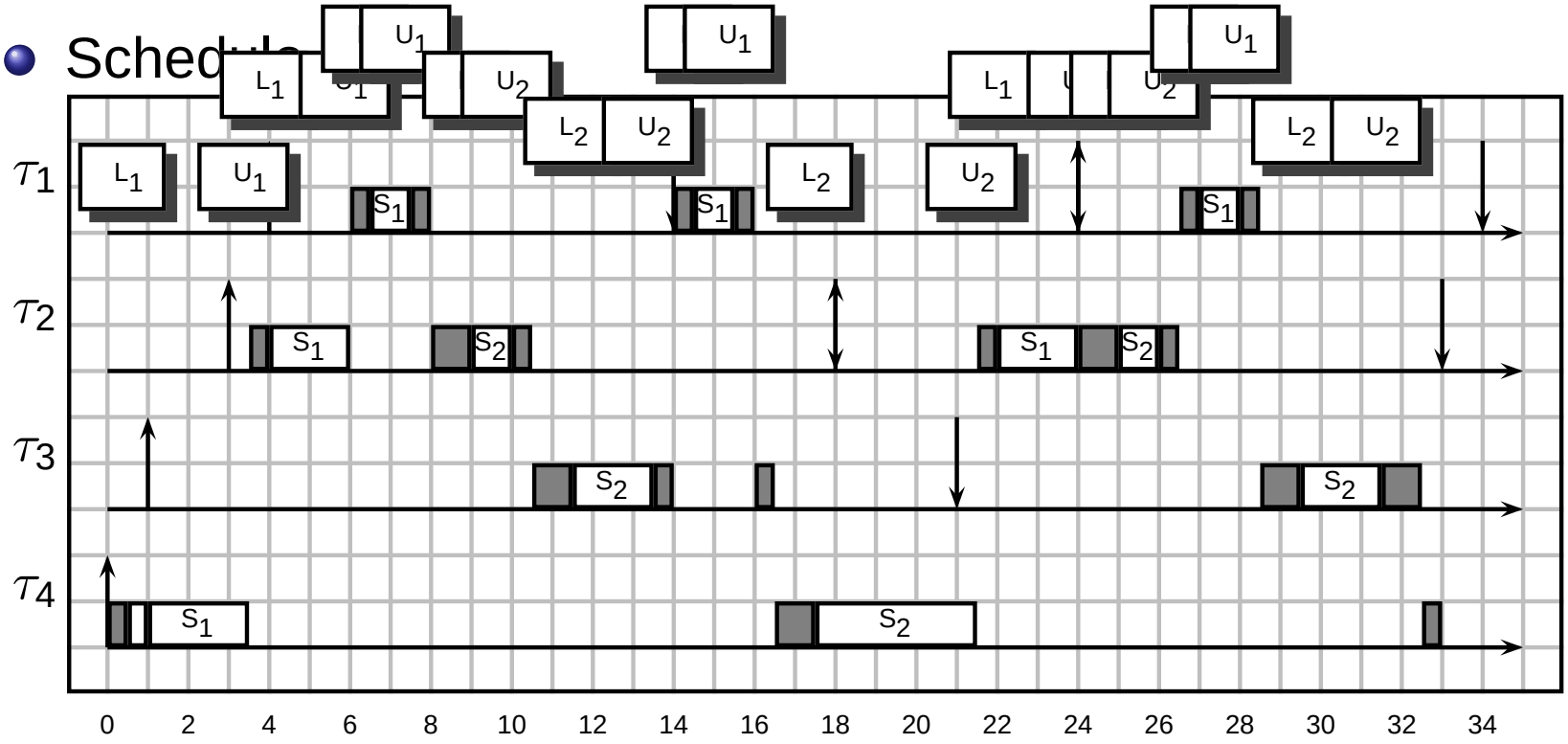

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Blocking time computation

- The computation of the blocking time is the same as in the case of FP + SRP;
- The only difference is that, when the resource access table is built, tasks are ordered by decreasing preemption level, instead than by priority.
- In the previous example:

	C_i	T_i	U_i	R_1	R_2	B_i
τ_1	2	10	.2	1	0	3
τ_2	5	15	.33	2	1	4
τ_3	4	20	.2	0	2	4
τ_4	9	45	.2	3	4	0

- Notice that, since the blocking times in this case are less than in the case of Priority Inheritance, then the system is schedulable. As an exercise, check that the schedulability condition holds.