Model-based testing

Marco Di Natale

Scuola Superiore S. Anna- Pisa, Italy

Taken from

"Model-Based Testing of Reactive Systems" by M. Broy, B. Jonsson, J. Katoen, M. Leucker and A. Pretschner editors, Springer Verlag

Chapter 4: Conformance Testing by Angelo Gargantini

Chapter 1: Homing and Synchronizing Sequences, by Sven Sandberg

Purpose of this Lesson

- Learn methods for checking correctness in the implementation of an FSM

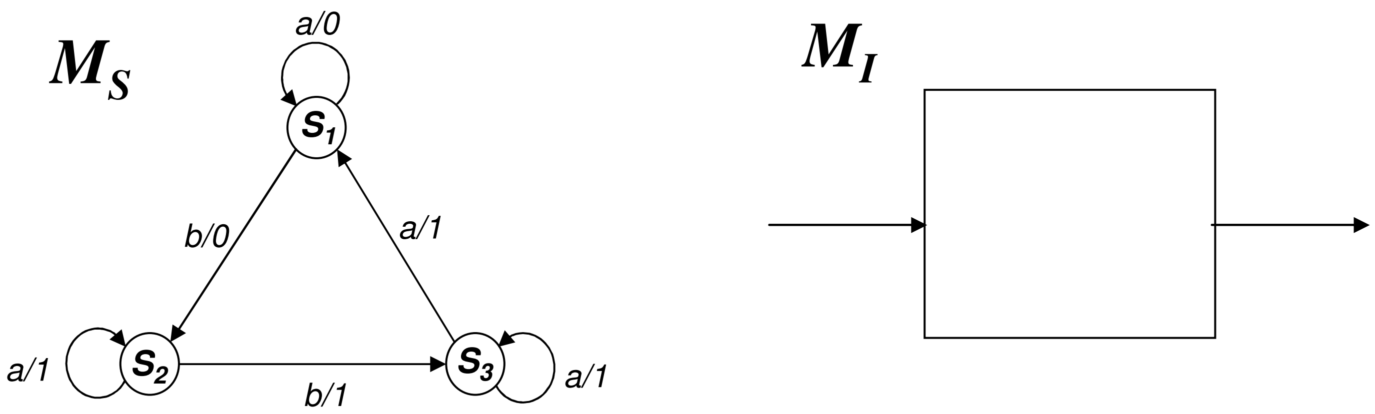
Model-based testing

Conformance testing between FSMs

- typically a model and its implementation

Given a FSM specification M_S, for which we know the transition diagram, and another FSM M_I, which is the implementation and for which we can only observe the behavior, we want to know if M_I correctly implements M_S.

Also called *fault detection* or *machine verification*

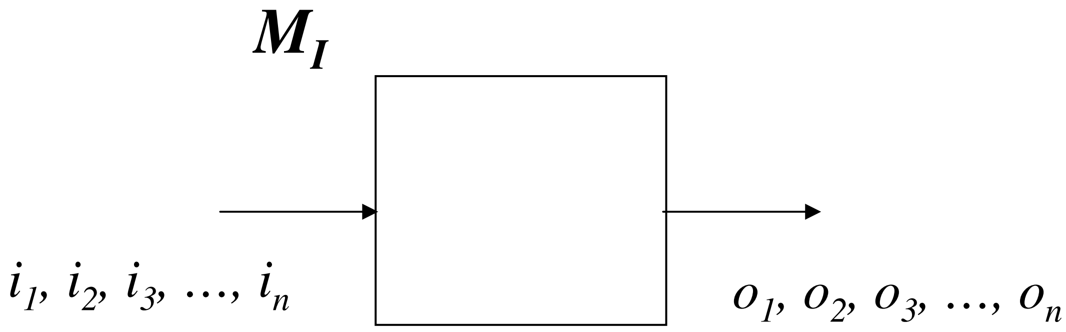


Conformance testing

M_I conforms to M_S *if and only if their initial states are equivalent and they will produce the same output sequence for any possible input sequence*.

To prove this we need to find a set of input sequences that we can apply to M_I to prove its equivalence. Note that applying all input sequences is equivalent to applying the concatenation of all the input sequences. This concatenation is called *checking sequence*

A checking sequence for M_S is an input sequence that distinguishes the class of machines equivalent to M_S from other machines



Conformance testing

Checking sequences differ for the cost to be produced, the size of the test suite (their total length) and their fault detection capability.

- They should be rather short to be practically applicable
- They should cover the implementation as much as possible and detect as many faults as possible

Model-based testing

Assumptions (requirements)

- M_S is reduced or minimal

  Q1: How to compute a minimal FSM given a specification?

- M_S is deterministic and completely specified: the state transition and the output function are defined for every state and every input symbol
- M_S is strongly connected. Every state is reachable from every other state via one or more transitions
  - At least all states must be reachable from the initial one, if a ***reset transition*** is available, allowing machines with deadlocks
- M_I does not change during testing and it has the same set of inputs and outputs as M_S.
  - Implications here (data inconsistency for incorrect concurrent implementations)

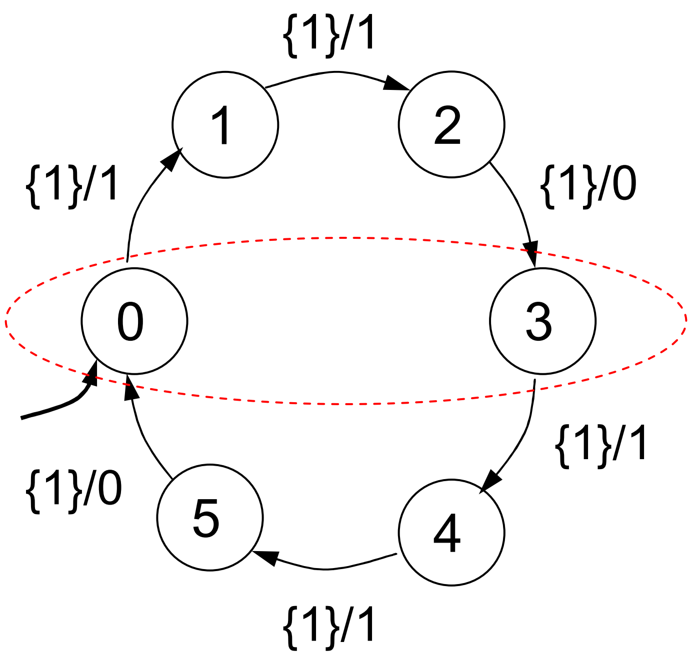
Notation

$\lambda(s,x)$ = output function: s is the state, x the input

$\sigma(s,x)$ = state function: s is the state, x the input

Model-based testing

- How to compute a minimal FSM given a specification



Two states s and t are equivalent iff $\lambda(s,x)=\lambda(t,x)$ for each possible input sequence $x \in I^*$.

That is, for each input sequence, the machine starting in s will produce the same output as the machine starting in t.

- Possibly checked using the simulation relation
- *But there is a better way …*

If states s and t are equivalent, then the machine obtained by merging the two states is equivalent to the original one.

For each machine there is an equivalent one with a minimum number of states, called reduced or minimized machine.

Model-based testing

Given a machine M, the minimized machine equivalent to M can be obtained by a partition refinement procedure.

A partition of S is a set $\{B_1, B_2, \ldots, B_n\}$ of subsets of S (also called blocks), such that $\cup B_i = S$ and $B_i \cap B_j = \varnothing$.

Given a mealy machine, the states of the equivalent minimized machine are the coarsest (with minimum number of elements) partition of S such that, whenever s and t are in the same block, then

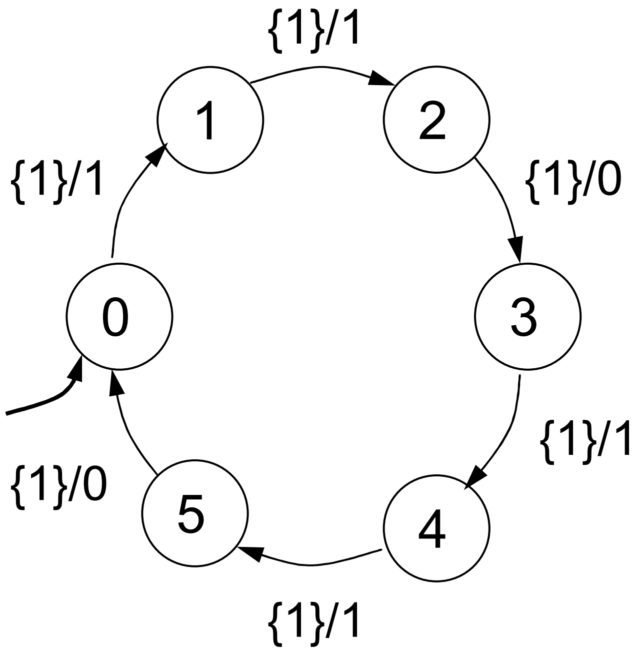
- $\lambda(s,a)=\lambda(t,a)$ for each input a and
- $\sigma(s,a)$ and $\sigma(t,a)$ are in the same block for each a

The coarsest partition can be found starting from an initial partition of S where s and t are in the same block iff $\lambda(s,a)=\lambda(t,a)$

Then, the initial partition is iteratively refined:

- Take a block B_i
- Examine $\sigma(s,a)$ for each $s \in B_i$ and $a \in I$. Partition B_i so that s and t stay in the same block iff $\sigma(s,a)$ and $\sigma(t,a)$ are in the same block of the current partition. (repeated until refinements are possible)

Model-based testing



Initial partition (based on output)

$B_1=\{2,5\}$ $B_2=\{0,1,3,4\}$

Consider B_1

$\sigma(2,1)=3$ $\sigma(5,1)=0$ 0 and 3 are in the same partition

Consider B_2

$\sigma(0,1)=1$ $\sigma(1,1)=2$ $\sigma(3,1)=4$ $\sigma(4,1)=5$

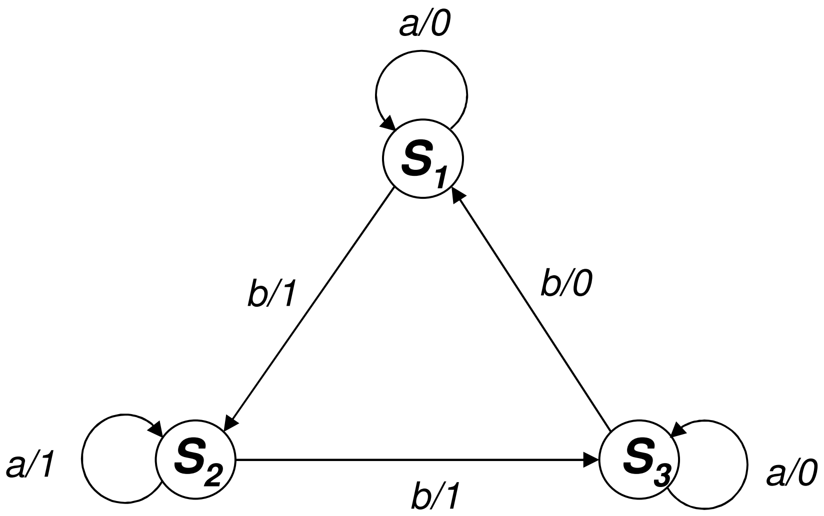
Refined in $B_2=\{1,4\}$ $B_2=\{0,3\}$

Model-based testing

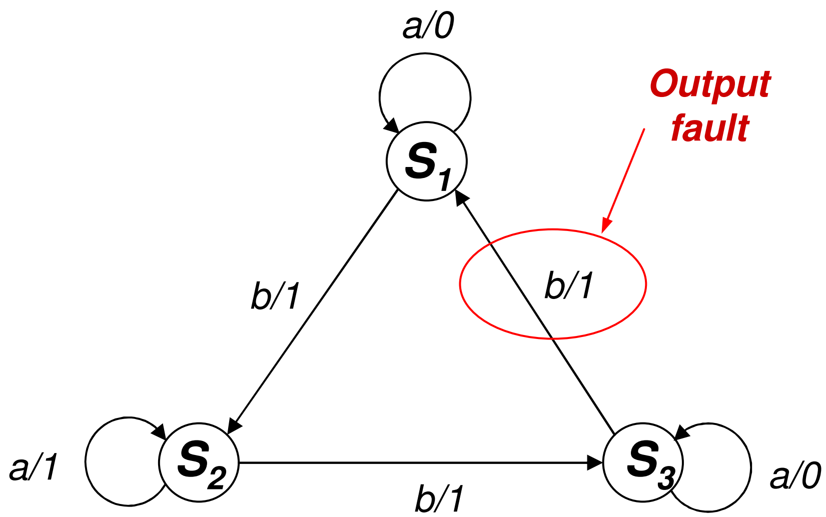
Assumptions (not essential)

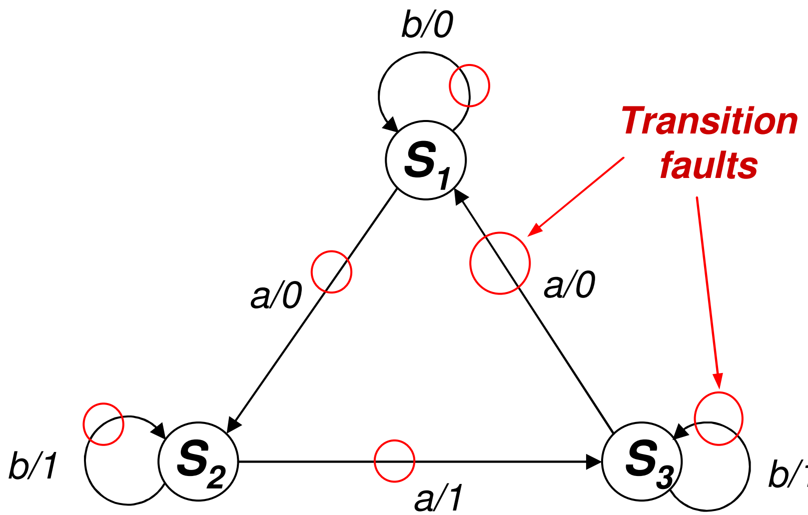
- M_S and M_I have an initial state and M_I is in its initial state before we conduct a conformance test.
  - If not, we can apply a *homing sequence* to M_I. The initial state is s_1.
- M_I has the same number of state as M_S.
  - Faults do not increase the number of states
    - Not included faults that create inconsistent states, such as, race conditions
  - Possible faults then can only be of two types
    - **Output faults**: the transition produces the wrong output
    - **Transfer faults**: the implementation goes to a wrong state.

Example



Faulty implementations





Model-based testing

Assumptions (not essential) continues …

- M_S and M_I have a special input *reset* that brings them back to the initial state without producing any output.
  - This assumption will be relaxed
- M_S and M_I have a special input *status* to which they respond with an output that uniquely identifies the state in which they are. The state is not changed
  - (If in s_i, the output is s_i)
  - This assumption will be relaxed
- M_S and M_I have a special set of inputs set(s_j), such that when set(s_j) is received in the initial state, the machines move to s_j without producing any output.
  - This assumption will be relaxed

Algorithm for conformance test

Under these assumptions, this is a conformance test

For all $s \in S$, $a \in I$:

1. *Apply a reset message to bring* M_I *to the initial state*
2. *Apply set(s) message to transfer* M_I *to state s*
3. *Apply the input value a*
4. *Verify that the output received conforms to* $\lambda_S(s,a)$
5. *Apply the status message to verify that the final state conforms to* $\delta_S(s,a)$

Output fault detection

Transition fault detection

Algorithm for conformance test

- The algorithm should also test the behavior for set, reset and status
- To test *status*, simply apply it twice in every state s_i after set(s_i) (first to test that it returns s_i, then to check that it does not change the state)
- Once *status* is tested, we can test *set* and *reset* by applying them in every state and verifying the result with *status*.

- The algorithm is the concatenation of reset, set(s), a and status $\forall$ s$\in S$ and $\forall$ a$\in I$.
- The length of the sequence is 4*pn* where p= $|I|$ and n= $|S|$

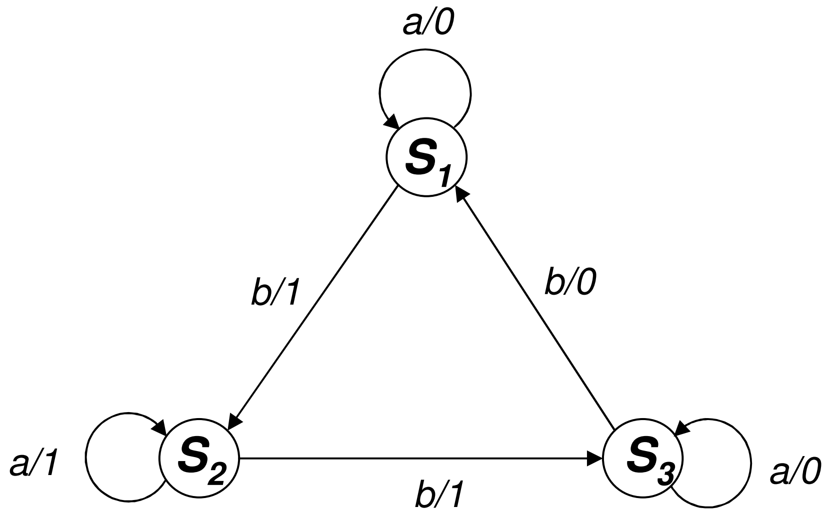
Algorithm for conformance test

- The main problem of the algorithm is the need for the set(s) input, which is typically not available.
- There is a sequence that avoids the need for set and possibly shortens the test run.
- We need a sequence that traverses every state and every transition, without restarting from the initial state after each test (and without using a set). Such a sequence is called Transition Tour (TT)
- A Transition Tour is an input sequence $a_1, a_2, a_3, \ldots, a_n$ that takes the machine to a sequence of states $z_1, z_2, z_3, \ldots, z_n$ such that,
  - for all $s \in S$, there exists $z_j = s$ and, (every state is visited)
  - for all $i \in I$ and $s \in S$, there exists j such that $z_j = s$ and $a_j = i$ (every transition out of every state is taken)

Algorithm for conformance test

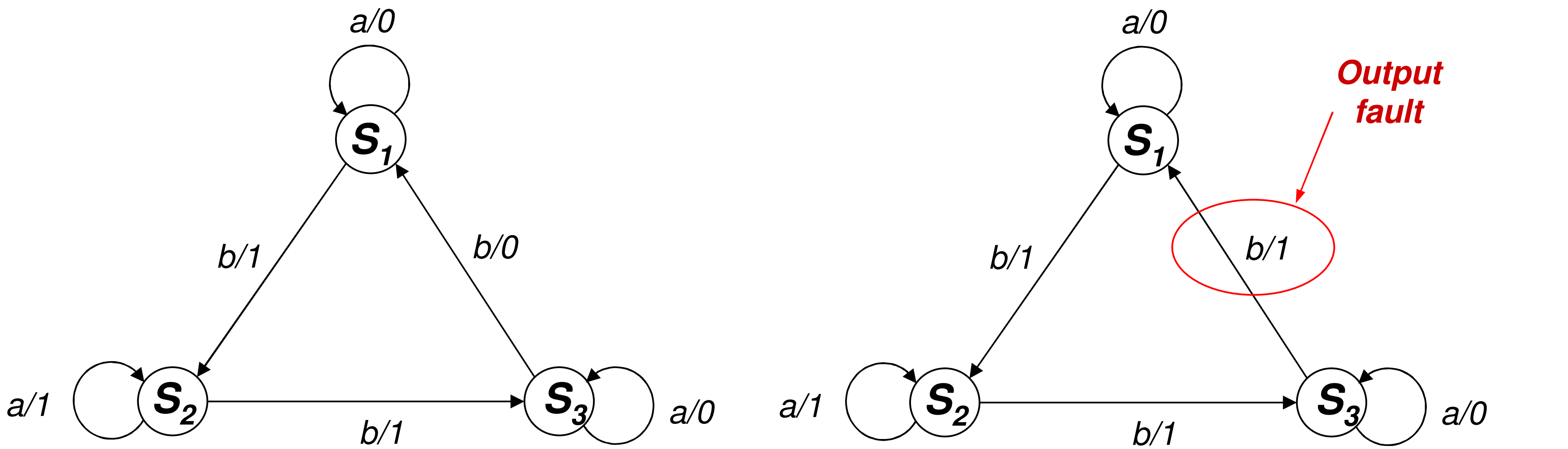
- If a Transition Tour is available, simply perform the input sequence a_1, *status*, a_2, *status*, a_3, …*status*, a_n to test conformance.
- The length of the TT sequence is at least 2*p*n.
- The shortest path that traverses each transition exactly once is called Euler Tour
- For connected FSMs an Euler Tour exists if they are also symmetric (every state is the source and destination of the same number of transitions)
  – And can be found in time linear in the number of transitions
- For non-symmetric FSMs, finding the shortest tour is another graph theory well-known problem (Chinese postman problem) that can be solved in polynomial time

Example



Checking sequence	*b*	*s*	*a*	*s*	*b*	*s*	*a*	*s*	*b*	*s*	*a*	*s*
Start state	1	2	2	2	2	3	3	3	3	1	1	1
Output	***1***	2	***1***	2	***1***	3	***0***	3	***0***	1	***0***	1
End state	2	2	2	2	3	3	3	3	1	1	1	1

Example

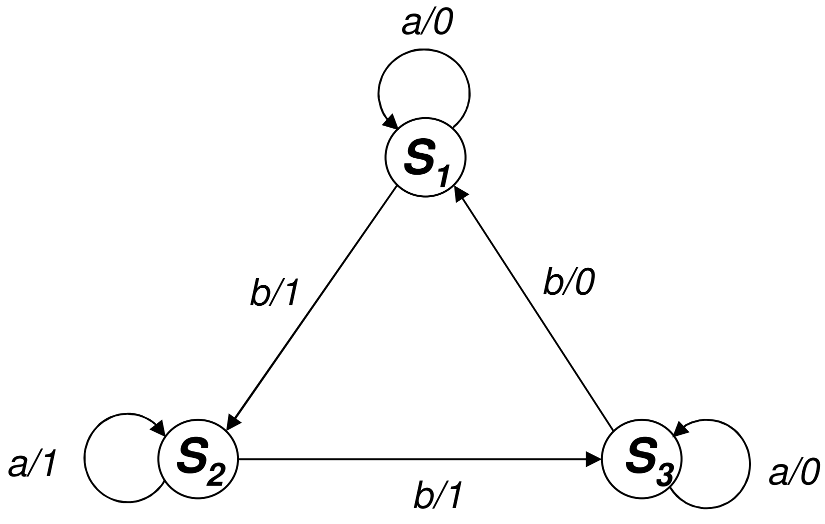


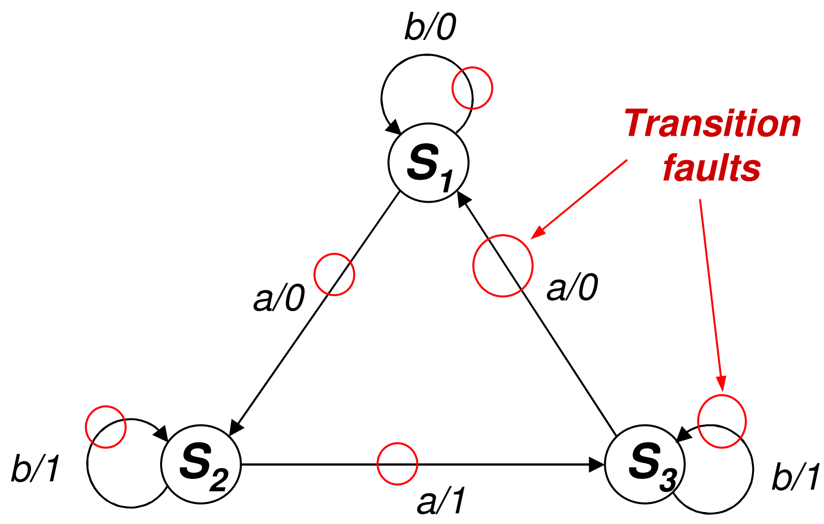
Checking sequence	b	s	a	s	b	s	a	s	b	s	a	s
Start state	1	2	2	2	2	3	3	3	3	1	1	1
Output	1	2	1	2	1	3	0	3	1 (0)	1	0	1
End state	2	2	2	2	3	3	3	3	1	1	1	1

The Transition Tour method

- The TT method without the status message achieves only transition coverage (not status coverage)
- A test that visits all the states but not all transitions is a state tour and obtains state coverage
  - Simple transition coverage is not enough to test correctness!

Example





Check. seq.	a	b	a	b	a	b
Start state	1	2	2	3	3	1
Output	***0***	***1***	***1***	***1***	***0***	***0***
End state	2	2	3	3	1	1

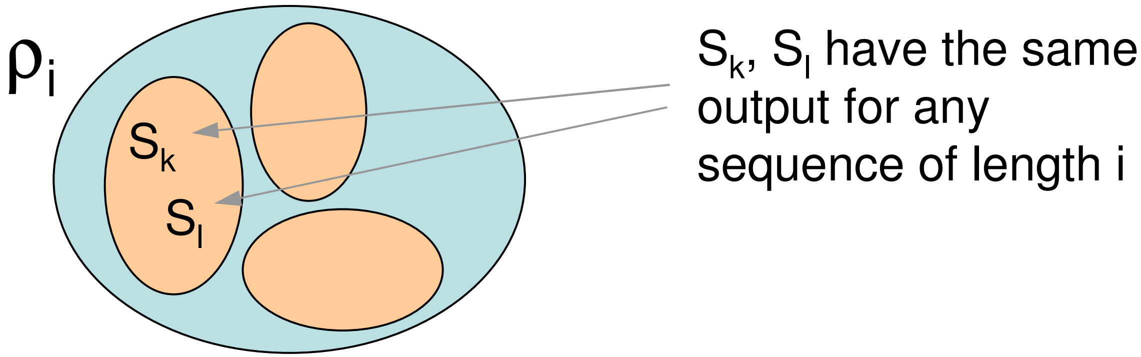
Check. seq.	b	a	b	a	b	a
Start state	1	1(2)	2	2(3)	3	3(1)
Output	***0(1)***	***0(1)***	***1***	***1(0)***	***1(0)***	***0***
End state	1(2)	2	2(3)	3	3(1)	1

Using Separating Sequences instead of status

- The status message is replaced by a (set of) sequences called separating sequences.
- Since M_S is minimal, for every pair of states s_i, s_j, there exists an input sequence x that distinguishes between them by creating different outputs: $\lambda(s_i, x) \neq \lambda(s_j, x)$
- That is, we need a “signature” that characterizes each state. This “signature” is a behavior starting from the state.
- Let’s reason about those “signatures” … how long should they be?

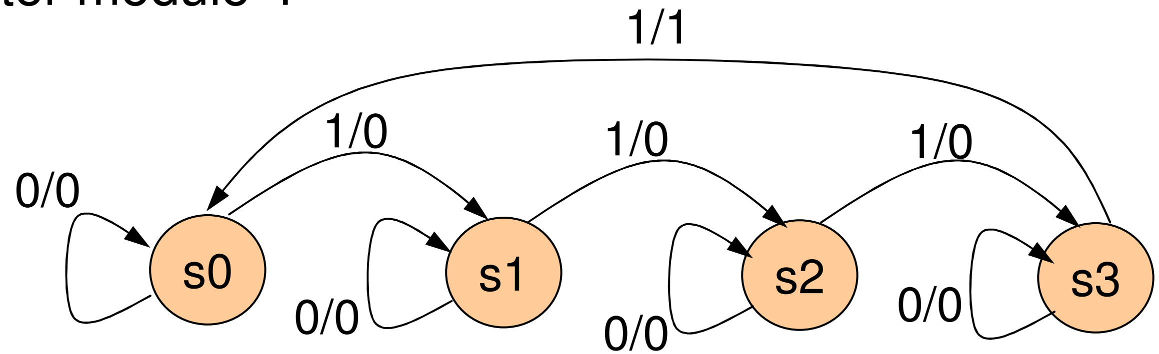
Separating sequences

- Define a sequence ρ_0, ρ_1, of partitions, so that two states are in the same class of ρ_i if and only if they do not have any separating sequence of length i
- $\rho_0 = \{S\}$
- ρ_{i+1} is a refinement of ρ_i
  - Lemma: if $\rho_{i+1} = \rho_i$ for some i, then the rest of the sequence of partitions is constant, $\rho_j=\rho_i$ for all j>i.
- Since partitions can be refined at most n times, the sequence is constant after at most n steps.
- Since the machine is minimized, at this point each partition is a singleton



Separating sequences: example

- Counter modulo 4



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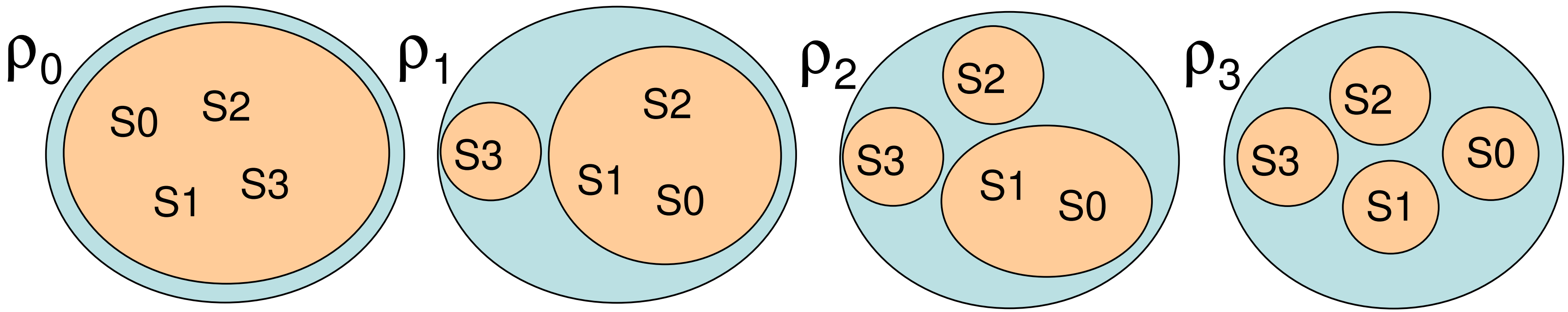


Separating sequences

- Step1: build the partitions ρ

  Start from ρ_1

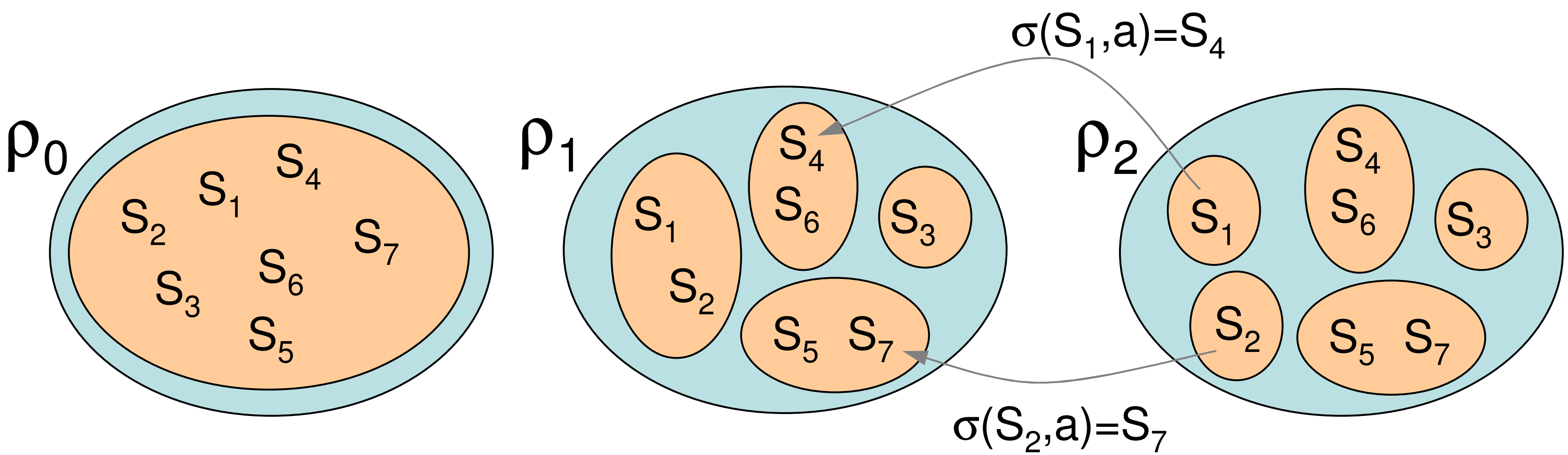
  – Two states s and t belong to different partitions of ρ_1 iff $\exists a \in I$ such that $\lambda(s,a) \neq \lambda(t,a)$

  – ρ_1 can be computed according to the definition

    - Try all possible input symbols

  Iteratively ….

  – Two states s and t belong to different partitions of ρ_i with i>1 iff $\exists a \in I$ such that $\sigma(s,a)$ and $\sigma(t,a)$ belong to different sets of ρ_{i-1} and to the same set of ρ_{i-2}



Separating sequences

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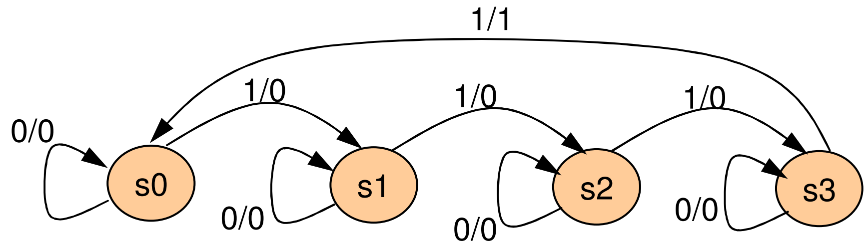
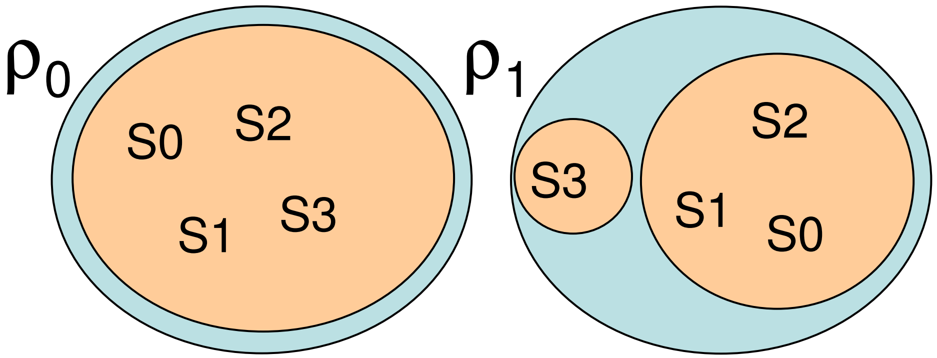
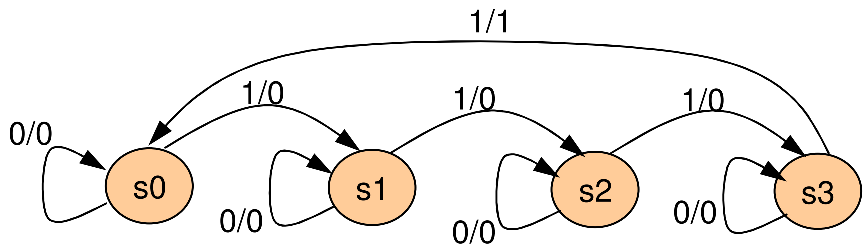


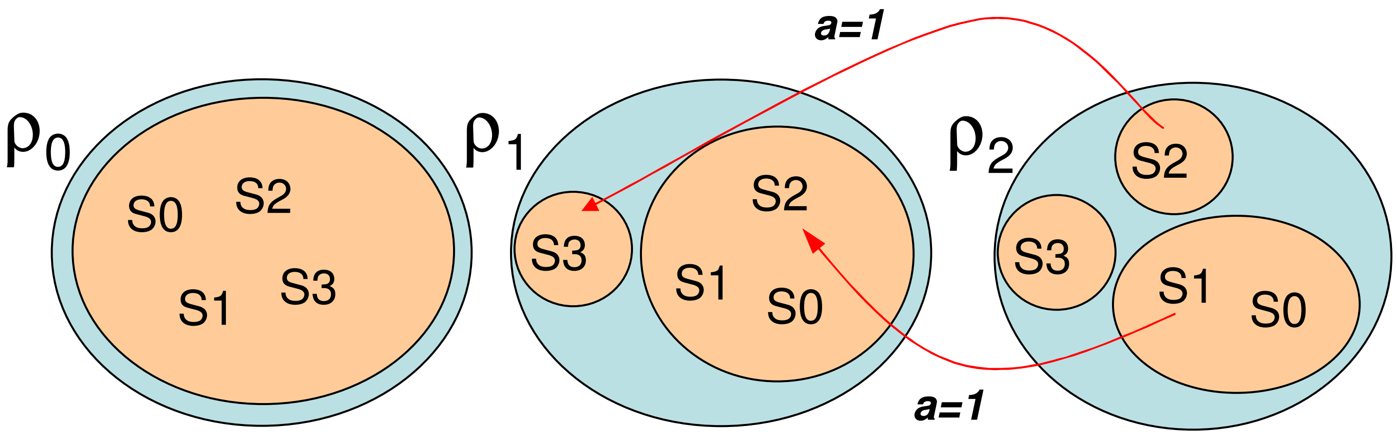


Separating sequences

Iteratively ….

– Two states s and t belong to different partitions of ρ_i with i>1 iff $\exists a \in I$ such that $\sigma(s,a)$ and $\sigma(t,a)$ belong to different sets of ρ_{i-1} and to the same set of ρ_{i-2}





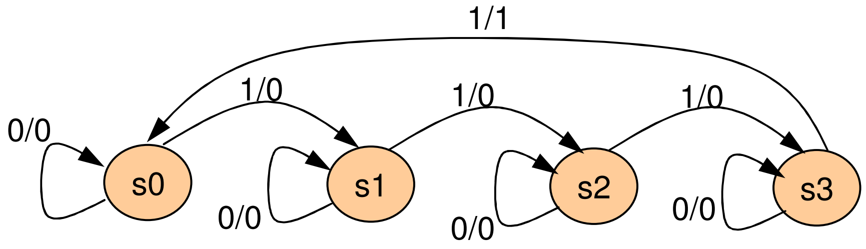
Separating sequences

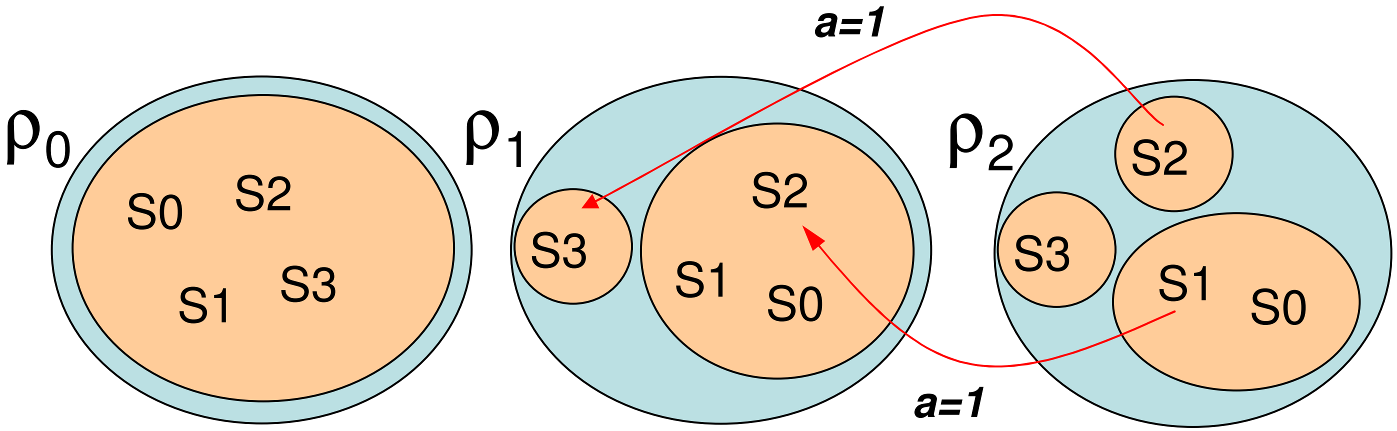
- Step 2: find the separating sequence for $s,t \in S$
  - Find the smallest index j such that s and t belong to different sets of ρ_j
  - Recursively, the separating sequence has the form ax, where x is the shortest separating sequence for the pair $\sigma(s,a)$ and $\sigma(t,a)$
  - Thus, we need to find the input a that takes s and t to different sets of ρ_{j-1} and repeat the process until we reach ρ_0
  - The concatenation of all the inputs is the separating sequence

  - (the algorithm needs O(n) memory)

Separating sequences: example

- Step 2: find the separating sequence for S1, S2 $\in$S
  - Find the smallest index j such that s and t belong to different sets of ρ_j (**2**)
  - Thus, we need to find the input a that takes s and t to different sets of ρ_{j-1} and repeat the process until we reach ρ_0 (**a=1**)
  - Recursively, the separating sequence has the form ax, where x is the shortest separating sequence for the pair σ(s,a) and σ(t,a) (x is separating sequence for S2,S3) (**x=1**)
  - ax = 1,1





Transition cover set

- The **transition cover set** of M_s is a set **P** of input sequences such that, for every $s \in S$ and $a \in I$ there exists a sequence $x \in P$ ending with the transition that applies a to s
- P is a set closed under prefix selection
  - If $x \in P$ then prefix(x) in P (the empty sequence ε is assumed to be part of any P)
- One way of constructing P:
  - Build a testing tree T of M_s (next algorithm) and then take the input sequences of all the partial paths of T

Building a test tree

1. The initial state of M_s is the root (level 1) of T

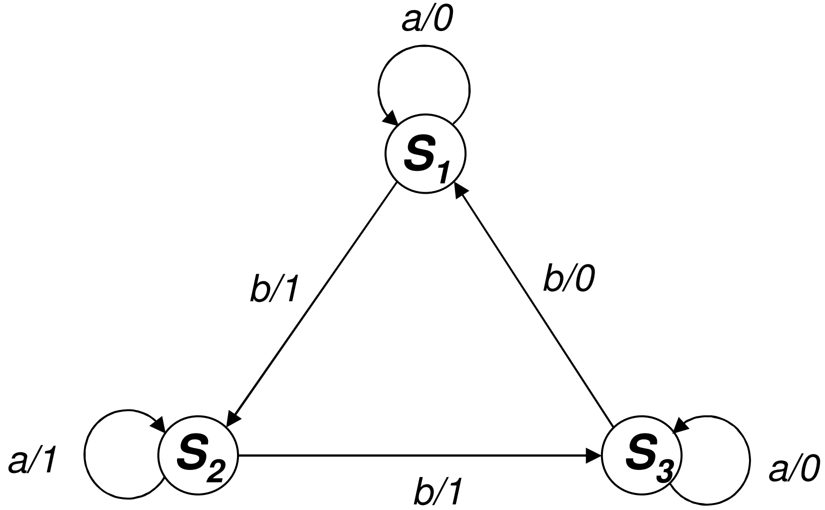
2. Suppose the tree is built up to level k: to build level k+1

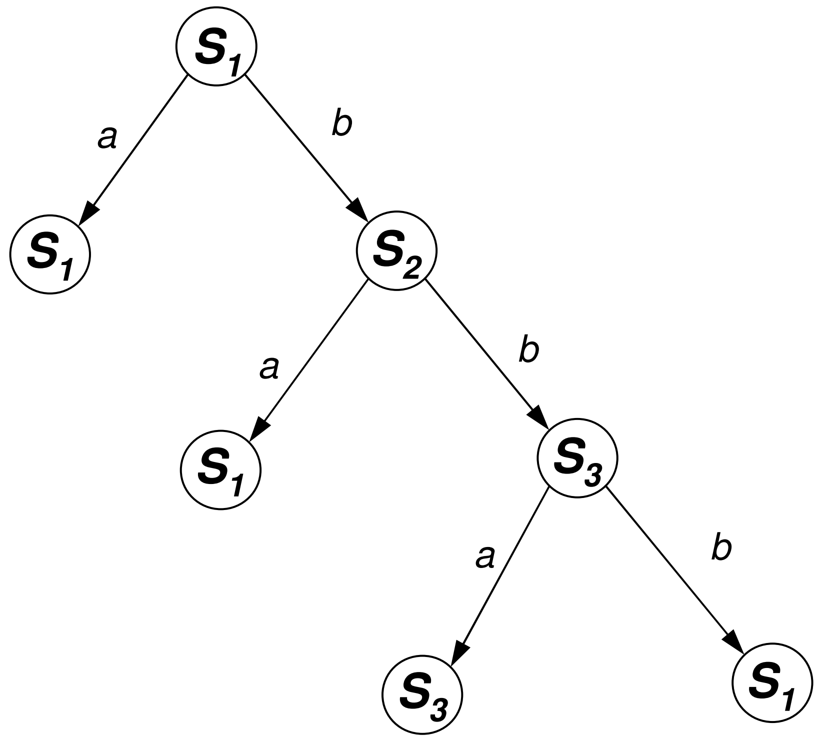
   1. For all nodes t at level k

   2. If the node t is equal to another node in T at level j with $j \leq k$, then t is a leaf of T

   3. Otherwise, let s_i be the label of t. For every input x, if M_s goes from s_i to s_j, attach a branch to t with label x and a successor node s_j

Example of transition cover set





Characterizing set

- The **characterizing set** of M_s is a set **W** of input sequences such that, for every pair $(s_i,s_j) \in S$ there exists a sequence $x \in W$ such that $\lambda(s_i, x) \neq \lambda(s_j, x)$
  - W is also called separating set
  - $x \in W$ are called separating sequences
- The choice of W is not unique, the fewer are the elements in W, the longer are the sequences.

Building a W set

1. **Partition the states S into blocks B_i, i=1..r**
2. **$W \leftarrow \{\}$, r=1, B_1=S**
3. **Repeat until every B_i is a singleton (and r=n)**
   1. **Take two states $s,t \in B_i$ and build their separating sequence x (algorithm shown in previous slides)**
   2. **Add x to W**
   3. **Partition The states s_{ik} in every B_j into smaller blocks based on their outputs $\lambda(s_{ik},x)$**

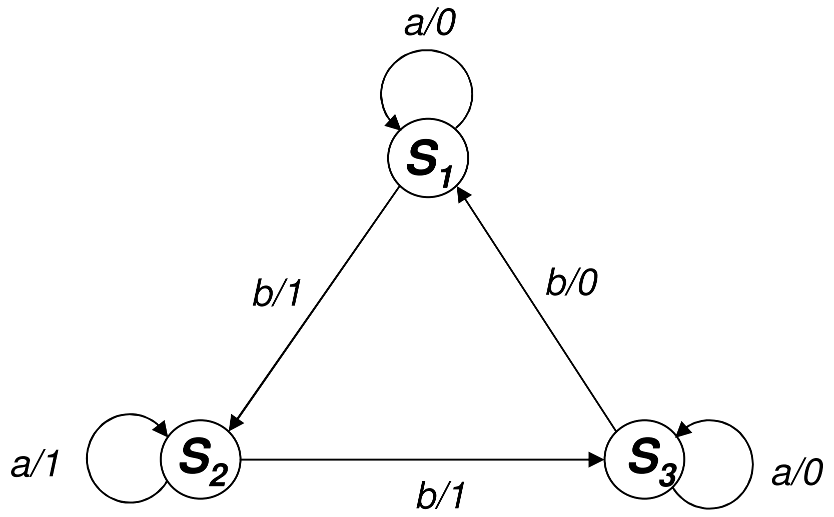
Note: there are no more than n-1 partitions, and no more than n-1 sequences in W

Using P sets and W sets (the W method)

- The method consists in using the set W in place of the status message

- Use the set of P sequences to test all transitions.
- At the end of each sequence x_P, apply all the sequences of W (x_W).
- Apply a reset after each pair x_{Pi}, x_{Wj}
- The total number of sequences is given by the cardinality of PxW

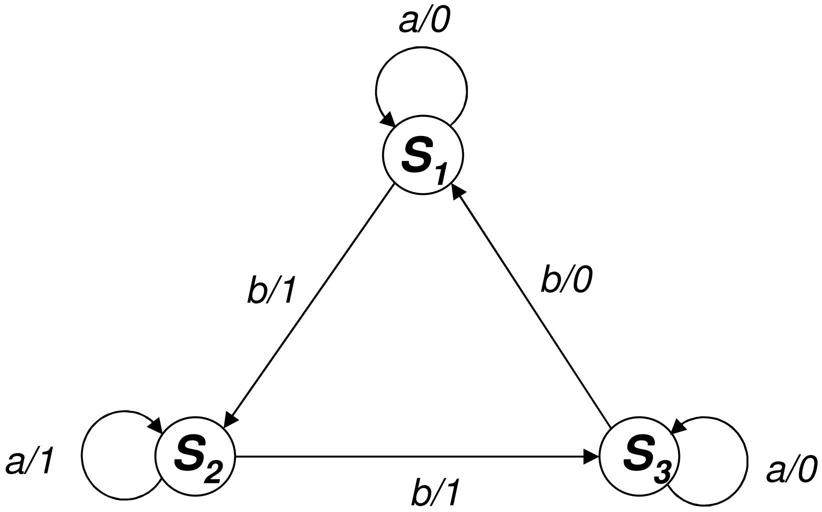
Example

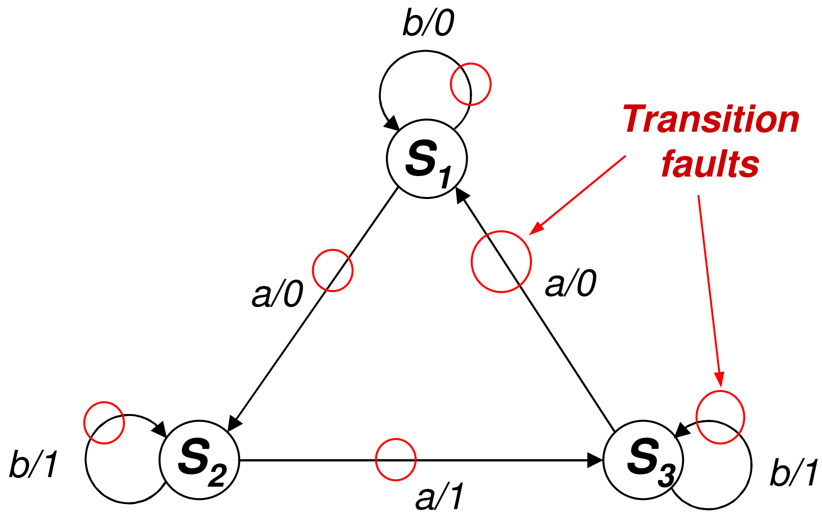


- The set W is simply {a,b} (a distinguishes s_2 from both s_1 and s_3, b distinguishes s_1 from s_3)
- The set P is {ε, a, b, ba, bb, bba, bbb}

P	ε		*a*		*b*		*ba*		*bb*		*bba*		*bbb*	
r.P.W	ra	rb	raa	rab	rba	rbb	rbaa	rbab	rbba	rbbb	rbbaa	rbbab	rbbba	rbbbb
trans test	$s_1 \to (\varepsilon)$		$s_1 \to (a/0)\ s_1$		$s_1 \to (b/1)\ s_2$		$s_2 \to (a/1)\ s_2$		$s_2 \to (b/1)\ s_3$		$s_3 \to (a/0)\ s_3$		$s_3 \to (b/0)\ s_1$	
output	0	1	00	11	11	11	111	111	110	110	1100	1100	1100	1101

Example





P	ε		*a*		*b*		*ba*		*bb*		*bba*		*bbb*	
r.P.W	ra	rb	raa	rab	rba	rbb	rbaa	rbab	rbba	rbbb	rbbaa	rbbab	rbbba	rbbbb
trans test	$s_1 \rightarrow (\varepsilon)$		$s_1 \rightarrow (a/0)\ s_1$		$s_1 \rightarrow (b/1)\ s_2$		$s_2 \rightarrow (a/1)\ s_2$		$s_2 \rightarrow (b/1)\ s_3$		$s_3 \rightarrow (a/0)\ s_3$		$s_3 \rightarrow (b/0)\ s_1$	
output	0	1	00	01	11	11	111	111	110	110	1100	1100	1100	1101

P	ε		*a*		*b*		*ba*		*bb*		*bba*		*bbb*	
r.P.W	ra	rb	raa	rab	rba	rbb	rbaa	rbab	rbba	rbbb	rbbaa	rbbab	rbbba	rbbbb
trans test	$s_1 \rightarrow (\varepsilon)$		$s_1 \rightarrow (a/0)\ s_1$		$s_1 \rightarrow (b/1)\ s_2$		$s_2 \rightarrow (a/1)\ s_2$		$s_2 \rightarrow (b/1)\ s_3$		$s_3 \rightarrow (a/0)\ s_3$		$s_3 \rightarrow (b/0)\ s_1$	
output	0	0	01	01	00	00	001	001	000	000	0001	0001	0000	0000

The Wp method

- The partial W or Wp method has the advantage of reducing the length of the test suite wrt the W method.
- The conformance test is split in two phases
  - During the first phase we test that every state that exists in M_S also exists in M_I
  - During the second phase we check that all transitions (not already checked during the first phase) are correctly implemented

The Wp method

- Phase 1: test that every state that exists in M_S also exists in M_I
- The Wp method uses a State cover set (as opposed to a transition cover set) or Q set
- The state cover set is a set Q of input sequences such that for each s in S there exists an input sequence x in Q that takes the machine to s, that is $\sigma(s_1, x) = s$
- A Q set can be easily built by performing a breadth-first visit of the transition graph of MS

The Wp method

- Phase 2: check that all transitions (not already checked during the first phase) are correctly implemented
- For the second phase, the Wp method uses an identification set W_i specific of each state si instead of a generic characterizing set W for all states ($W_i \subset W$).
- An identification set of state s_i is a set W_i of input sequences such that, for each state $s_j \in S$, there exists an input sequence $x \in W_i$ such that $\lambda(s_i,x) \neq \lambda(s_j,x)$ and no subset of W_i has this property.
  - $\cup W_i = W$

The Wp method

- Phase 1:The input sequences for phase 1 consist in the concatenation of every $q \in Q$ with every $w \in W$ after a reset
  - Every state is checked with a W set
- If the input sequences do not uncover any fault during this phase, we can conclude that every state in M_S has a similar state in the implementation (produces the same output for all the sequences in W)
- This is not sufficient to prove that it is equivalent
  - (we need to check all transitions in the next stage)

The Wp method

- Phase 2: To test all transitions, Wp uses the identification sets.
- For every transition from s_j to s_i on input a, we apply a sequence x (after *reset*) that takes the machine to s_j along transitions already verified in phase 1.
- Then we apply the input a that takes the machine to s_i we verify the correctness of the output, and we apply one identification sequence of W_i
- We repeat the previous for all the sequences in W_i and, if these tests do not uncover faults, by applying them to every transition, we can verify that M_I conforms to the specification.

Bibliography

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