

Recursion

Luca Abeni

`luca.abeni@santannapisa.it`

Execution as Evaluation

- Functional program: composition of pure functions
 - Recursion is used instead of iteration
- “Executed” by evaluating the expressions obtained from the functions
- Usual example: factorial!

```
unsigned int fact(unsigned int n)  
{  
    return n == 0 ? 1 : n * fact(n - 1);  
}
```

- Note the “arithmetic if” ($p ? a : b$)
- $\text{fact}(4) = ?$

Example of Evaluation

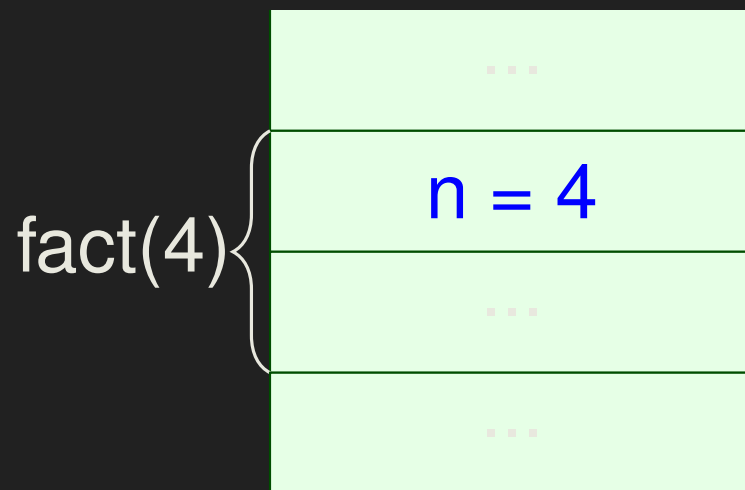
`fact (4)` = ... “`n == 0 ? 1: n * fact(n - 1)`”,
replacing “`n`” with “`4`”

- `(n == 0 ? 1: n * fact(n - 1))(4)`
- So, 2 different replacements: replace “`fact`” with its definition, and then replace “`n`” with “`4`”

```
fact (4) = (4 == 0) ? 1 : 4 * fact (3) =  
4 * fact (3) =  
4 * ((3 == 0) ? 1 : 3 * fact (2)) =  
4 * 3 * fact (2) =  
4 * 3 * ((2 == 0) ? 1 : 2 * fact (1)) =  
4 * 3 * 2 * fact (1) =  
4 * 3 * 2 * ((1 == 0) ? 1 : 1 * fact (0)) =  
4 * 3 * 2 * 1 * 1 = 24
```

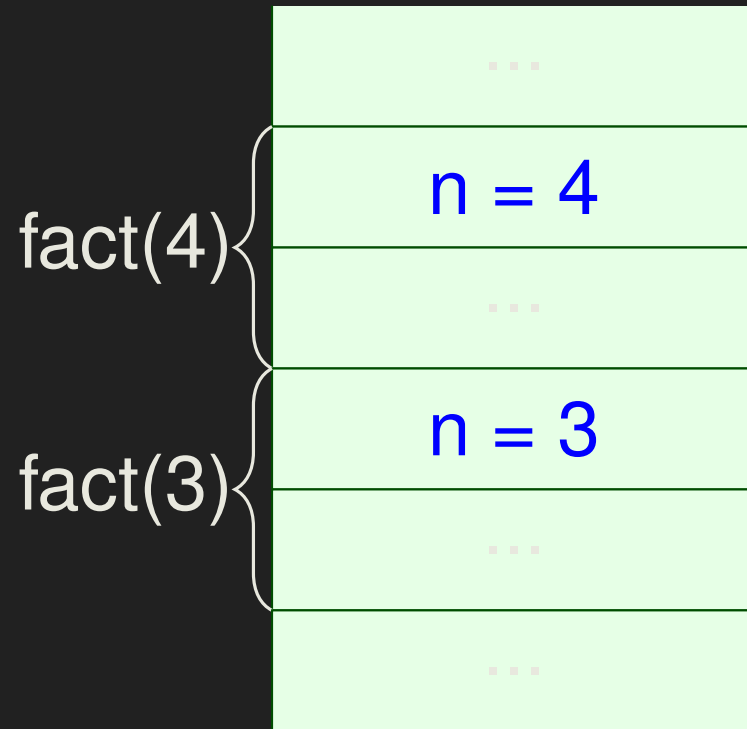
What About the Stack?

- Function invocation \rightarrow activation record (stack frame) allocated on the stack...
- With recursion, this can be interesting!
- `fact(4)`: new stack frame containing:
 - The formal parameter $n = 4$
 - Link to previous stack frames
 - Some space for the return value



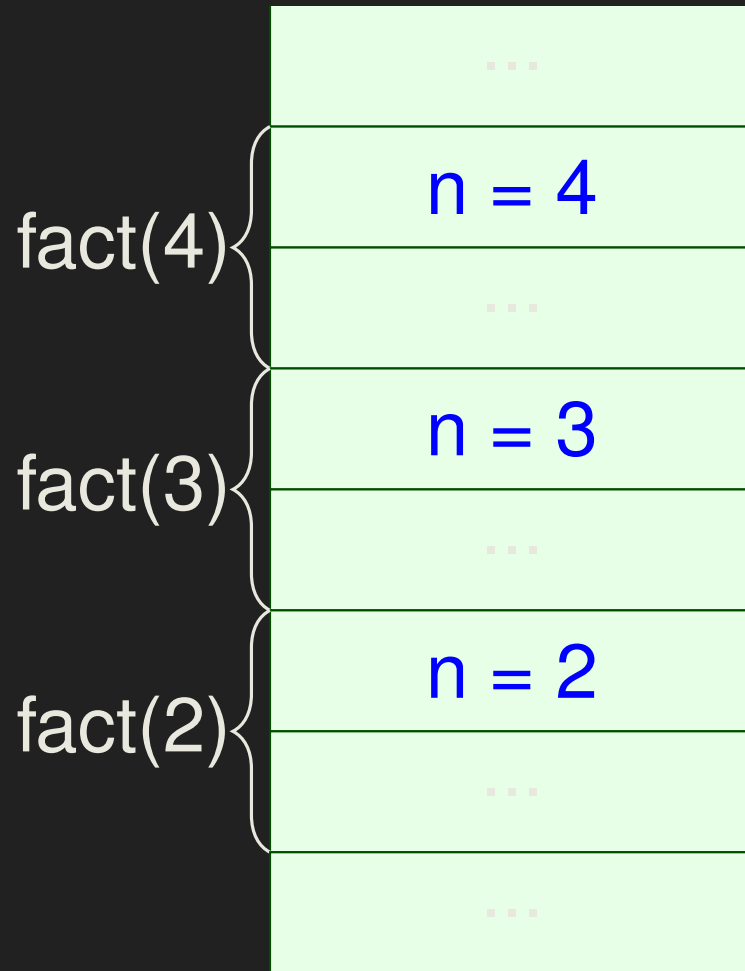
Stack Frame - 2

- $4 * \text{fact}(3)$



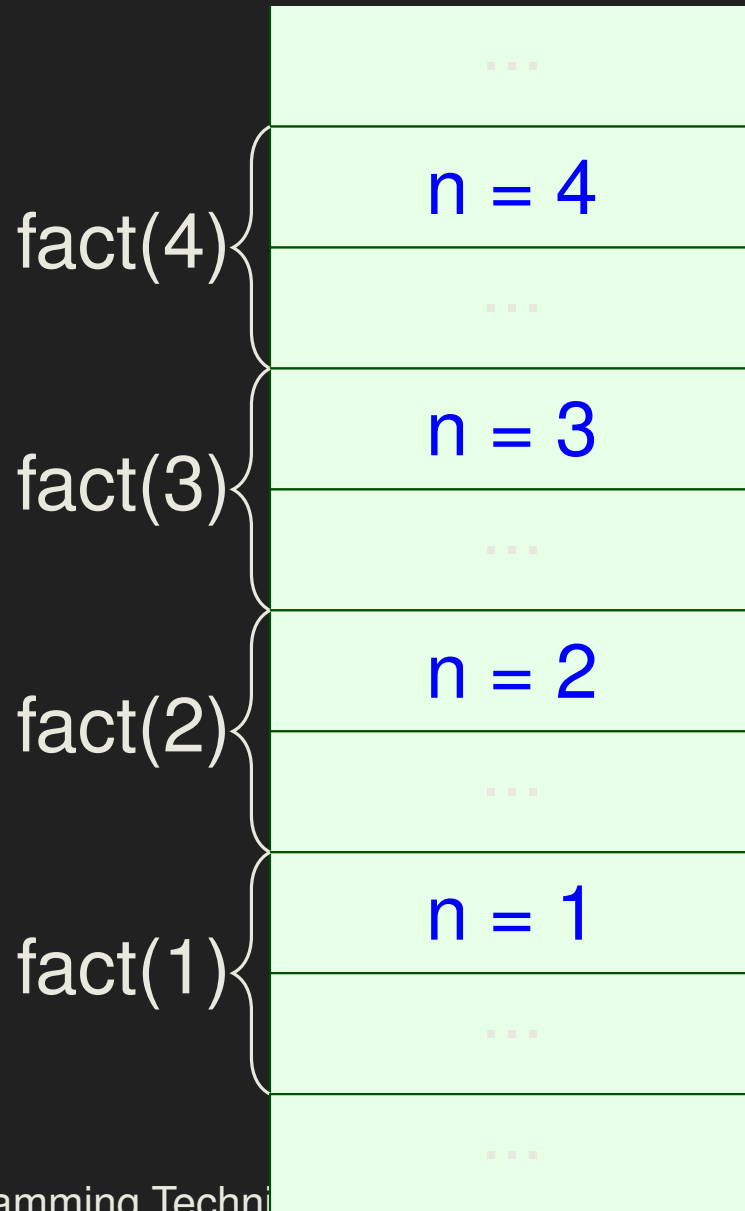
Stack Frame - 3

- `4 * 3 * fact(2)`



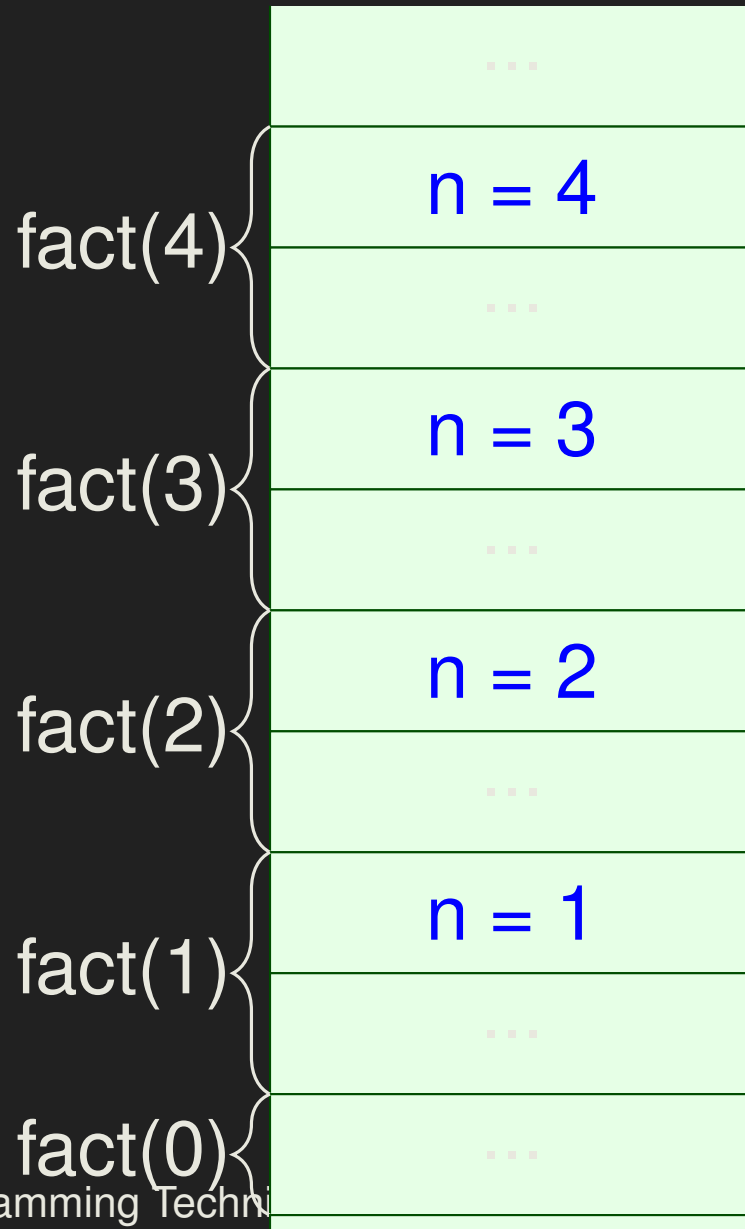
Stack Frame - 4

- 4 * 3 * 2 * fact(1)



Stack Frame - 5

- 4 * 3 * 2 * 1 * fact(0)



Summing Up...

- When `fact(0)` is evaluated, the previous stack frames contain the numbers to be multiplied...
- These stack frames are removed one after the other when the `fact()` instances return, and the multiplications are performed
- When `fact(n - 1)` returns, `fact(n)` still needs to perform a multiplication by `n`
 - It cannot immediately return!
- The stack frames are hence needed until the corresponding `fact()` instance returns, and **they cannot be removed from the stack before that**
 - Recursion \Rightarrow high stack usage!
 - Possible **stack overflow**

Recursion and Stack Usage

- Is stack usage the price to be paid for using recursion?
- Let's consider this factorial implementation:

```
unsigned int fact1(unsigned int n,  
                  unsigned int res)  
{  
    return n == 0 ? res : fact1(n - 1, n * res);  
}  
unsigned int fact(unsigned int n)  
{  
    return fact1(n, 1);  
}
```

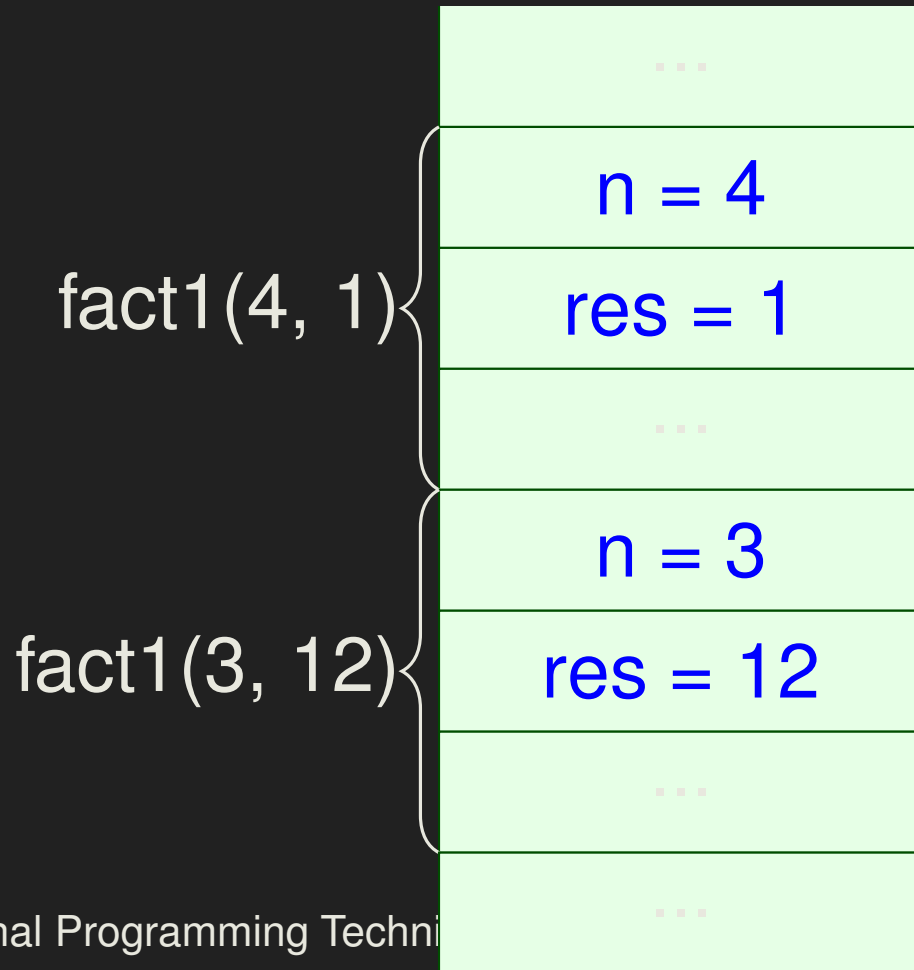
- What's the second formal parameter???

Evaluation

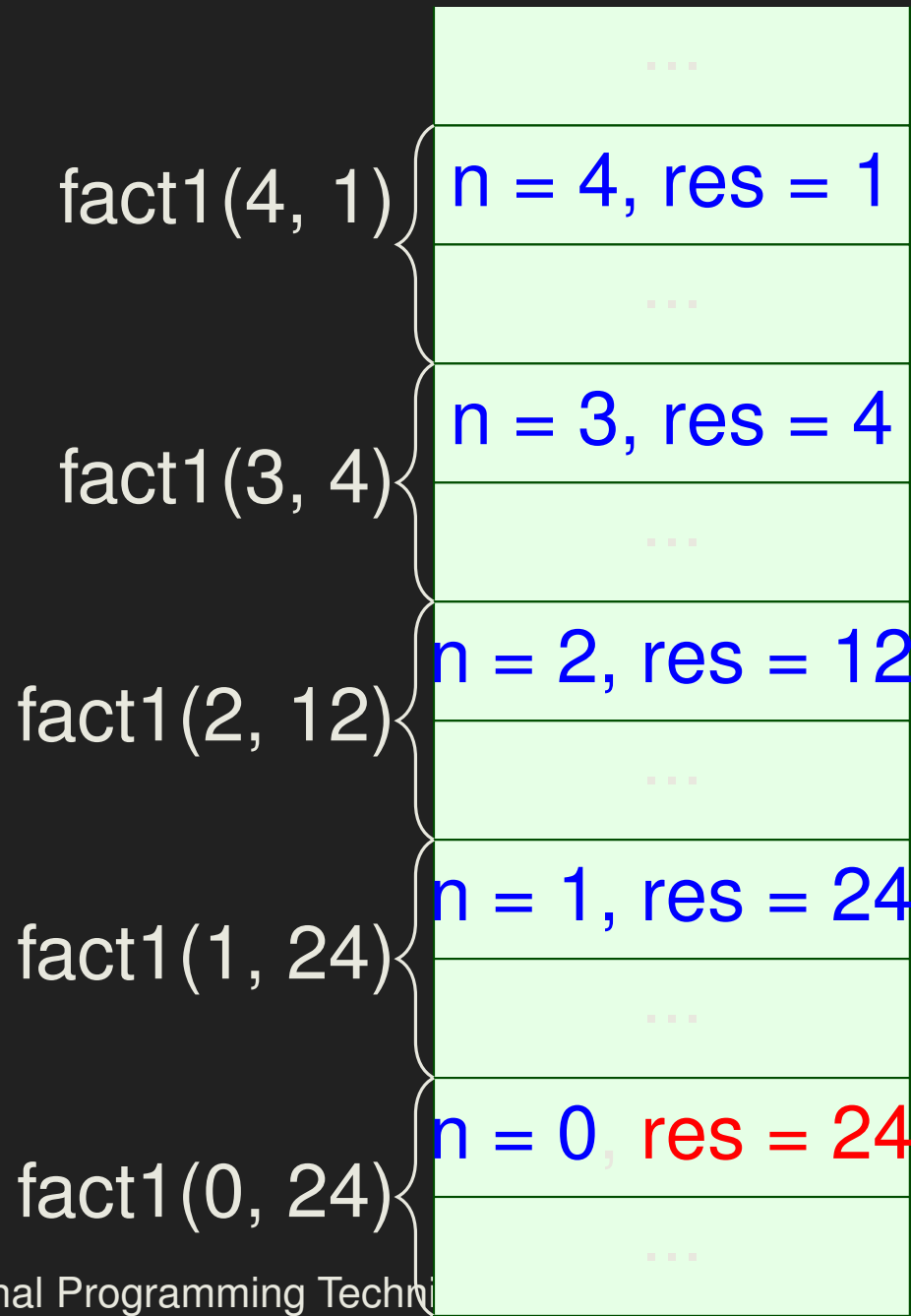
```
fact (4) =  
fact1 (4, 1) =  
(4 == 0) ? 1 : fact1 (3, 4 * 1) =  
fact1 (3, 4) =  
(3 == 0) ? 4 : fact1 (2, 3 * 4) =  
fact1 (2, 12) =  
(2 == 0) ? 12 : fact1 (1, 2 * 12) =  
fact1 (1, 24) = fact1 (0, 1 * 24) = 24
```

Stack Frames, Again

- No operations to be performed when `fact1(n-1, ...)` returns...
- The stack frame of `fact(n-1, ...)` already contains the data to return!



Stack Frames - 2



So...

- When `fact1(0, ...)` is evaluated, data from previous stack frames is not reused...
- Stack frames are removed when the `fact1()` instances return, without having to execute additional operations
- When `fact1(n - 1, ...)` returns, `fact1(n, ...)` returns its value directly
 - `fact1(n - 1, ...)` can immediately return to the `fact1(n, ...)` caller!
- Hence, stack frames can be removed from the stack when recursion is invoked (*before* the function returns)
 - Recursion \Rightarrow no additional stack usage
 - **No stack overflow!**